The spin evolution of spin-3 ^{52}Cr Bose–Einstein condensate*

Situ Shu-Ping(司徒树平) and He Yan-Zhang(贺彦章)†

State Key Laboratory of Optoelectronic Materials and Technologies, and School of Physics and Engineering, Sun Yat-Sen University, Guangzhou 510275, *China*

(Received 16 May 2010; revised manuscript received 27 June 2010)

This paper studies theoretically the spin evolution of a Bose–Einstein condensate starting from a mixture of two or three groups of ^{52}Cr (spin-3) atoms in an optical trap. The initial state is so chosen that the condensate has total magnetization zero so that the system does not distinguish up and down. It is assumed that the system is very dilute (particle number is very small), the temperature is very low, and the frequency of the harmonic trap is large enough. In these situations, the deviation caused by the neglect of the dipole–dipole interaction and by using the single-mode approximation is reduced. A theoretical calculation beyond the mean field theory is performed and the numerical results are helpful for the evaluation of the unknown strength g_0.

Keywords: Bose–Einstein condensate, spin evolution, short-ranged interaction strength
PACS: 03.75.Mn, 03.75.Kk, 03.75.Nt, 05.30.Jp **DOI:** 10.1088/1674-1056/20/1/010310

1. Introduction

Bose–Einstein condensate (BEC) of atoms with nonzero spin has greatly attracted the interest of both experimentalists and theorists[1−8] in recent years. Five years ago, the Bose gas of ^{52}Cr atoms with electronic spin $J = 3$ and nuclear spin $I = 0$ was condensed successfully.[9] Experimentally, the atomic spins of ^{52}Cr were frozen in magnetic trap, but freed in optical trap.[10] For these spin-3 bosons, the short-ranged interaction between two atoms is specified by the strengths g_S, where $S = 0$, 2, 4 and 6 are the total spins of the pair. All g_S except the one for $S = 0$ have been determined experimentally.[11] To fully understand the character of ^{52}Cr, it is important to measure g_0 with the help of theory. Among all the rich physics of BEC, one attractive phenomenon is the spin evolution.[12−16] It was found that, for ^{87}Rb and ^{23}Na, the evolution of the average populations of spin components depends on the strengths of interaction sensitively.[17,18] Therefore, the interaction can be determined (or confirmed) by observing the evolution.

In this paper, we consider the spin evolution of the ^{52}Cr atoms. It is designed that these atoms are first divided into three groups, the Z-components of spin, μ, of the atoms in each group are the same. However, those of different groups are different. Then, these atoms are put together. Simultaneously, an optical trap is created and the evolution begins. What we are interesting is the effect of the unknown strength g_0 on the evolution. In the theoretical aspect, the single-mode approximation (SMA, namely, the spatial wave functions of all the atoms are approximately considered as the same), has been extensively used by a number of authors to simplify the calculation. However, for the ^{52}Cr atoms, the SMA is generally not a very good approximation because they have a larger spin and therefore a stronger dipole–dipole interaction $V_{\rm dd}$ occurs. This interaction would cause a stronger coupling of spatial and spin degrees of freedom, and therefore the SMA is spoiled.[19] It is reminded that, when the spins of a pair of spin-3 atoms are coupled to a two-body spin-state with spin S and component M denoted as $\chi_{\rm SM}$, we have the matrix element:[20,21]

$$\begin{aligned}\langle\chi_{S'M'}|V_{\rm dd}|\chi_{SM}\rangle = & -252\frac{T_{\rm d}}{r^3}\sqrt{4\pi(2S'+1)}C^{2,0}_{1,0,1,0}\\ & \times\begin{Bmatrix}1&1&2\\3&3&S\\3&3&S'\end{Bmatrix}\\ & \times C^{S,M}_{2,M-M',S',M'}Y_{2,M-M'}(\hat{r}),\end{aligned}\quad(1)$$

where $T_{\rm d}$ is the strength, and the Clebsch–Gordan and 9-j coefficients together with the spherical harmonics are included. From this formula, it is clear that $V_{\rm dd}$ will inevitably cause a spatial excitation of d-wave.

*Project supported by the National Natural Science Foundation of China (Grant Nos. 10874249 and 11075223).
†Corresponding author. E-mail: heyanzh@mail.sysu.edu.cn
© 2011 Chinese Physical Society and IOP Publishing Ltd

However, it is recalled that the energy of the d-wave excitation of a particle in harmonic oscillator ground state is $2\hbar\omega$, where ω is the frequency of the harmonic trap. For the case that the temperature is very low (so that nearly all the particles are in the ground state), the condensate is very dilute (so that the average separation of a pair of particles is not small), and ω is large enough, the d-wave excitation will be suppressed by the gap $2\hbar\omega$, and therefore the effect of $V_{\rm dd}$ can be suppressed. In this special case, the use of the SMA is reasonable. In this paper, we consider a very dilute condensate with the particles number being very small ($N = 20$), the temperature is assumed to be very low, and ω is sufficiently large. Accordingly, SMA is used in the following.

In general, spin-evolutions depend on the total magnetization of the system. From the previous experience, we know that the evolution of a polarised system is pale (because a spin-flip can not be caused by the contact interaction in a collision with two spin-parallel particles), but is swifter if the total magnetization is zero. In the following discussion, the special initial condition with total magnetization zero is chosen. Accordingly, the system does not distinguish up and down.

Since the particle number N under consideration is so small, the mean field theory might not work very well. Thus a method beyond the mean field theory is used in the following.

2. Hamiltonian and the eigenstates

For a ^{52}Cr condensate with N atoms, when the dipole force is neglected, the interaction between a pair of spin-3 bosons i and j is denoted as $U_{ij} = O_{ij}\delta(\boldsymbol{r}_i - \boldsymbol{r}_j)$, where $O_{ij} = g_0P_0 + g_2P_2 + g_4P_4 + g_6P_6$ and P_0 to P_6 are the projection operators of the S-channels. Based on the SMA, each boson has the same spatial state $\phi(\boldsymbol{r}_k)$. When integration over the spatial degrees of freedom has been performed, the Hamiltonian reads

$$H = \sum_{i<j} U_{ij} = f\sum_{i<j} O_{ij}, \quad f = \int |\phi(\boldsymbol{r})|^4 {\rm d}\boldsymbol{r}. \quad (2)$$

After the integration, only the spin degrees of freedom are left in the Hamiltonian. To diagonalize the Hamiltonian, we use the following Fock-states as the basis functions:

$$|\alpha\rangle = |N_{\bar{3}}N_{\bar{2}}N_{\bar{1}}N_0N_1N_2N_3\rangle, \quad (3)$$

where α indicates the set $\{N_{\bar{3}}, \ldots, N_3\}$, N_μ is the number of bosons with spin component μ, and $\bar{\mu}$ means $-\mu$. There exist two restrictions on the N_μ as follows.

$$\begin{cases} N = N_{\bar{3}} + N_{\bar{2}} + N_{\bar{1}} + N_0 + N_1 + N_2 + N_3, \\ M = -3N_{\bar{3}} - 2N_{\bar{2}} - N_{\bar{1}} + N_1 + 2N_2 + 3N_3, \end{cases} \quad (4)$$

where M is the total magnetization (a constant). In the Fock space, by using the factional parentage coefficient,[22] the matrix elements of Hamiltonian read

$$\begin{aligned}
\langle\beta|H|\alpha\rangle &= \langle N'_{\bar{3}}N'_{\bar{2}}N'_{\bar{1}}N'_0N'_1N'_2N'_3|f\sum_{i<j}O_{ij}|N_{\bar{3}}N_{\bar{2}}N_{\bar{1}}N_0N_1N_2N_3\rangle \\
&= 2f\sum_{\mu\le\nu;\mu'\le\nu'}\delta_{\mu'+\nu',\mu+\nu}\sum_{S=0,2,4,6} g_S C^{S,(\mu'+\nu')}_{3,\mu',3,\nu'} C^{S,(\mu+\nu)}_{3,\mu,3,\nu}\Bigg[\delta_{\mu'\nu'}\delta_{\mu\nu}\frac{N_\mu(N_\mu-1)}{4}\delta^{\ldots,N'_\mu-2,\ldots}_{\ldots,N_\mu-2,\ldots} \\
&\quad + (1-\delta_{\mu'\nu'})\delta_{\mu\nu}\frac{\sqrt{N'_{\mu'}N'_{\nu'}N_\mu(N_\mu-1)}}{2}\delta^{\ldots,N'_{\mu'}-1,\ldots,N'_{\nu'}-1,\ldots}_{\ldots,N_\mu-2,\ldots} \\
&\quad + (1-\delta_{\mu\nu})\delta_{\mu'\nu'}\frac{\sqrt{N_\mu N_\nu N'_{\mu'}(N'_{\mu'}-1)}}{2}\delta^{\ldots,N'_{\mu'}-2,\ldots}_{\ldots,N_\mu-1,\ldots,N_\nu-1,\ldots} \\
&\quad + (1-\delta_{\mu\nu})(1-\delta_{\mu'\nu'})\sqrt{N_\mu N_\nu N'_{\mu'}N'_{\nu'}}\delta^{\ldots,N'_{\mu'}-1,\ldots,N'_{\nu'}-1,\ldots}_{\ldots,N_\mu-1,\ldots,N_\nu-1,\ldots}\Bigg],
\end{aligned} \quad (5)$$

where $C^{S,(\mu+\nu)}_{3,\mu,3,\nu}$ is the Clebsch–Gordan coefficient. In the label $\delta^{\ldots,N'_{\mu'}-1,\ldots,N'_{\nu'}-1,\ldots}_{\ldots,N_\mu-2,\ldots}$, the superscript denotes a revised set of $\{N'_{\bar{3}}, \ldots, N'_3\}$ by reducing both $N'_{\mu'}$ and $N'_{\nu'}$ by 1, and the subscript denotes a revised set of $\{N_{\bar{3}}, \ldots, N_3\}$ by reducing N_μ by 2. When the two revised sets are one-to-one identical, the label is 1, otherwise

it is zero.

When both N and M are given, the dimension of the matrix is finite. After the Hamiltonian is diagonalized, the j-th eigenenergy E_j and the corresponding eigenstate ψ_j are obtained. The ψ_j can be expanded by the basis functions (or vice versa) as follows.

$$\psi_j = \sum_\alpha c_\alpha^j |\alpha\rangle \quad \text{or} \quad |\alpha\rangle = \sum_j c_\alpha^j \psi_j, \tag{6}$$

where the coefficient $c_\alpha^j = \langle\alpha|\psi_j\rangle$ is real. These eigenstates are used in the following description of evolution.

3. Evolution of population of spin components and spin-correlation

The spin evolution begins when the three groups of atoms are mixed together. In this paper, all the atoms of the first group have $\mu = 3$, those of the second one have $\mu = -3$ and those of the third one have $\mu = 0$. Obviously, the initial state is just a Fock-state $|I\rangle = |N_{\bar{3}}, 0, 0, N_0, 0, 0, N_3\rangle$ where N_3, $N_{\bar{3}}$, and N_0 are the number of atoms in the first, second and third groups, respectively. The $|I\rangle$ can be expanded by the series of ψ_j with the coefficients c_I^j. Thus the time-dependent solution $\Psi(t)$ of the Schrödinger equation describing the evolution reads

$$\begin{aligned}\Psi(t) &= \mathrm{e}^{-\mathrm{i}Ht/\hbar}|I\rangle = \sum_j c_I^j \mathrm{e}^{-\mathrm{i}E_j t/\hbar}\psi_j \\ &= \sum_\alpha C_\alpha(t)|\alpha\rangle. \end{aligned}\tag{7}$$

where $C_\alpha(t) = \sum_j c_I^j c_\alpha^j \mathrm{e}^{-\mathrm{i}E_j t/\hbar}$ is time-dependent. Since c_I^j are known constants determined by $|I\rangle$, E_j and ψ_j are also known. So the evolution can be fully understood.

From $\Psi(t)$, we define the time-dependent population $P_\mu^I(t)$ which is the probability of an atom in spin component μ at t. It reads

$$P_\mu^I(t) = \frac{1}{N}\langle\Psi(t)|a_\mu^+ a_\mu|\Psi(t)\rangle = B_\mu^I + O_\mu^I(t), \tag{8}$$

where a_μ^+ and a_μ are the creation operator and annihilation operator of an atom in μ, respectively. The B_μ^I and $O_\mu^I(t)$ read

$$B_\mu^I = \sum_j (c_I^j)^2 \sum_\alpha (c_\alpha^j)^2 \frac{N_\mu^\alpha}{N}, \tag{9}$$

$$\begin{aligned}O_\mu^I(t) = 2\sum_{j<j'} \cos\left[\frac{(E_{j'}-E_j)t}{\hbar}\right] c_I^j c_I^{j'} \\ \times \sum_\alpha c_\alpha^j c_\alpha^{j'} \frac{N_\mu^\alpha}{N},\end{aligned}\tag{10}$$

where N_μ^α is the number of atoms in μ within $|\alpha\rangle$. Equation (8) contains two terms. The first one B_μ^I is only determined by $|I\rangle$ and is time-independent and therefore, appears as a background of oscillation. The second one $O_\mu^I(t)$ contains the time-dependent factor $\cos[(E_{j'} - E_j)t/\hbar]$ which implies an oscillation upon the above background.

In the correlation point of view, P_μ^I corresponds to the one-body density of coordinate space. This means that it is not sufficient for understanding how the particles are coupled so that additional quantity is needed to investigate the complicated spin-structure. Extracting from Eq. (7), we further define $Q_m^I(t)$ which is the probability of the pair of particles in m component,[23]

$$\begin{aligned}Q_m^I(t) = \sum_\alpha |C_\alpha(t)|^2 \sum_\mu \delta_{2\mu,m} \frac{N_\mu^\alpha(N_\mu^\alpha - 1)}{N(N-1)} \\ + \sum_\alpha |C_\alpha(t)|^2 \sum_{\mu\neq\nu} \delta_{\mu+\nu,m} \frac{N_\mu^\alpha N_\nu^\alpha}{N(N-1)},\end{aligned}\tag{11}$$

where $|C_\alpha(t)|^2$ is the weight of $|\alpha\rangle$ in $\Psi(t)$. The $Q_m^I(t)$ is contributed by two items. The first one comes from the Fock-state containing a pair of particles with same spin component. The second one comes from the Fock-state containing a pair of particles with different spin component. Obviously, the quantity $Q_m^I(t)$ is a measure of spin-correlation which corresponds to the two-body density of coordinate space. As well as the latter, $Q_m^I(t)$ can greatly help to understand the structure of the pair of particles. Since $Q_m^I(t)$ contains the time-dependent factor $C_\alpha(t)$, it is also an oscillation term.

At the beginning of evolution (*i.e.*, $t = 0$), $\Psi(0) = |I\rangle$. This directly leads to $P_\mu^I(0) = N_\mu^I/N$. And from Eq. (7), we have $C_\alpha(0) = \delta_{\alpha I}$, so that Eq. (11) can be simplified as

$$Q_m^I(0) = \frac{1}{N(N-1)} \sum_\mu (N_\mu^I N_{m-\mu}^I - \delta_{2\mu,m} N_\mu). \tag{12}$$

Above relations indicate that no matter the strengths are, evolutions of both P_μ^I and Q_m^I begin at the constants which depend only on initial state. However, when time goes on (*i.e.*, $t > 0$), the different cases

of g_0 can generally impact the coupling among particles so that they choose the most favoured spin states from the aspect of lower energy. Therefore, the chosen states form the different evolution patterns of $P^I_\mu(t)$ and $Q^I_m(t)$. This is the reason why these two quantities can help to measure g_0 in experimentation. Moreover, if the initial state has the symmetry $N^I_{\bar{\mu}} = N^I_\mu$, the evolutions are also symmetric where $P^I_{\bar{\mu}}(t) = P^I_\mu(t)$ and $Q^I_{\bar{m}}(t) = Q^I_m(t)$. This is because in such cases, the Z-axis can be reversed.

In the following calculation, $N = 20$ is used. The initial states are given as $N^I_{\pm 3} = (N - N^I_0)/2$ and $N^I_{\pm 2} = N^I_{\pm 1} = 0$, where N^I_0 is even and is ranged from 0 to N. In this choice, $|I\rangle$ is uniquely determined by N^I_0. Obviously, the system has the up-down symmetry and the total magnetization M is zero, those conditions are in favour of the SMA. In what follows 10^{-8} meV, Å (1 Å=0.1 nm) and sec are used as units for energy, length and time, respectively. The strengths g_2, g_4 and g_6 are taken from Ref. [24], namely, -3.88, 30.80 and 59.64, respectively, whereas g_0 will be given at a number of testing values. The average density is given as $f = 2 \times 10^{-11}$. This value is simply evaluated under a model that the density is uniform inside a sphere. A slight deviation of f does not affect the following qualitative results.

3.1. The background B^I_μ

Li *et al.* proved that the background of oscillation in spin-evolutions of spin-1 condensates does not depend on the interaction.[25] However, their argument is based on the uniqueness of the Fock-state with a given N, M, and N_0. Obviously, the uniqueness holds no more for spin-3 systems. Therefore, the knowledge of interaction might generally be obtained by observing B^I_μ. This is shown in Fig. 1 where the dependency on the initial state and on g_0 is revealed. It is also shown that B^I_μ would depend on g_0 rather weakly if g_0 is positive. In this case, the structures of low-lying eigenstates would be dominated by $S = 2$ pairs (because only in this kind of pairs, the two atoms are mutually attracted). However, B^I_μ would depend on g_0 rather strongly if g_0 is negative and close to g_2. In this case, the structures of the eigenstates would vary sensitively with g_0 due to the competition of the $S = 2$ and $S = 0$ pairs.[23] As a result, there is a domain of sensitivity. If the realistic g_0 turns out to fall in this domain, it could be determined by observing B^I_μ.

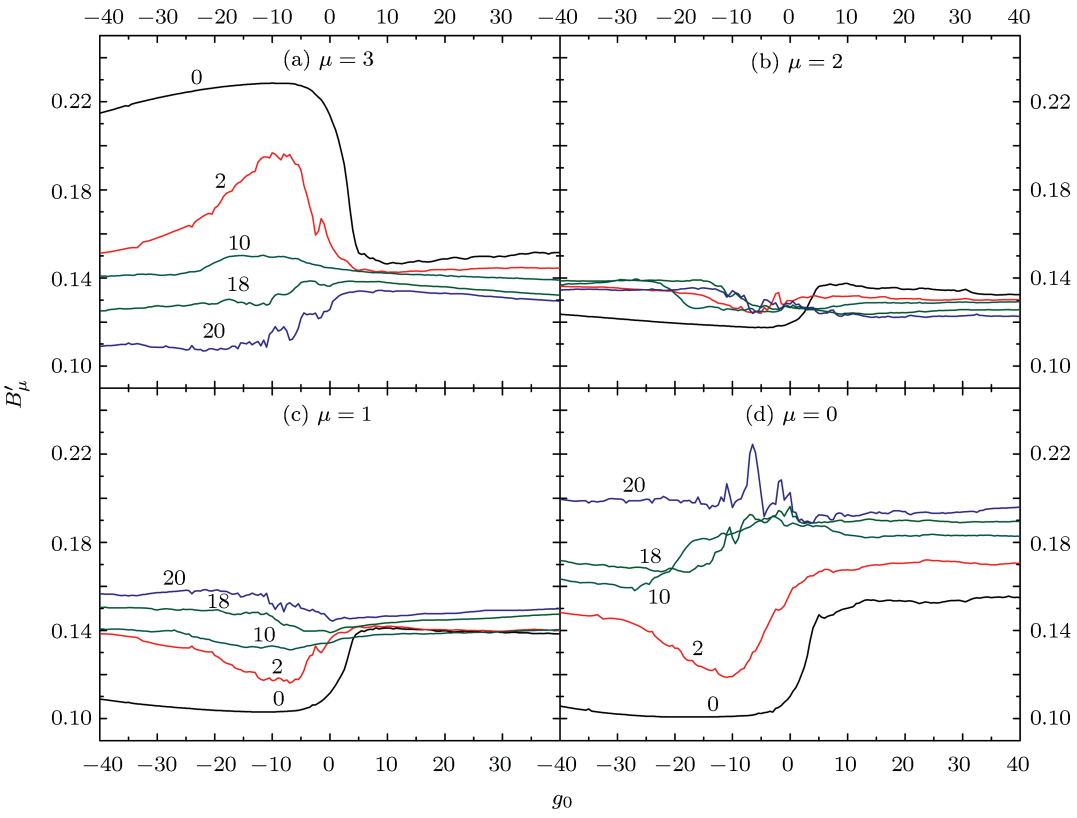


Fig. 1. (Colour online) The background of spin evolution B^I_μ, against g_0 where (a) $\mu = 3$, (b) $\mu = 2$, (c) $\mu = 1$ and (d) $\mu = 0$, respectively. Five cases of initial states where $N^I_0 = 0$, 2, 10, 18 and 20, respectively, are given and marked by the curves.

The background can be rewritten as $B^I_\mu = \sum_j W^I_j F^j_\mu$, where $W^I_j = (c^j_I)^2$ is the weight of ψ_j in $|I\rangle$, $F^j_\mu = \sum_\alpha (c^j_\alpha)^2 N^\alpha_\mu / N \equiv \langle \psi_j | a^+_\mu a_\mu | \psi_j \rangle / N$ is the probability of an atom in μ within ψ_j. Note that the curve with $N^I_0 = 0$ is much higher in Fig. 1(a), but much lower in Fig. 1(d). In this state, all the spins are either up ($\mu = 3$) or down ($\mu = -3$) initially. Therefore, those ψ_j with a larger $F^j_{\pm 3}$ (i.e., having averagely more atoms for $\mu = \pm 3$) would have a larger weight W^I_j. This fact explains why the curve with $N^I_0 = 0$ is the highest in Fig. 1(a), where the $\mu = 3$ atoms are observed. Meanwhile, those ψ_j with a larger F^j_0 would have a smaller weight W^I_j, which explains why the curve with $N^I_0 = 0$ is the lowest in Fig. 1(d).

3.2. The oscillation $O^I_\mu(t)$

For a ^{52}Cr condenstate, the spin component of a pair of two particles can be from -6 to 6. In Eq. (10), the time factor $(E_{j'} - E_j)/\hbar$ is in general not a multiple of integer among all pairs of j and j'. Therefore $P^I_\mu(t)$ is non-periodic which is illustrated in Fig. 2.

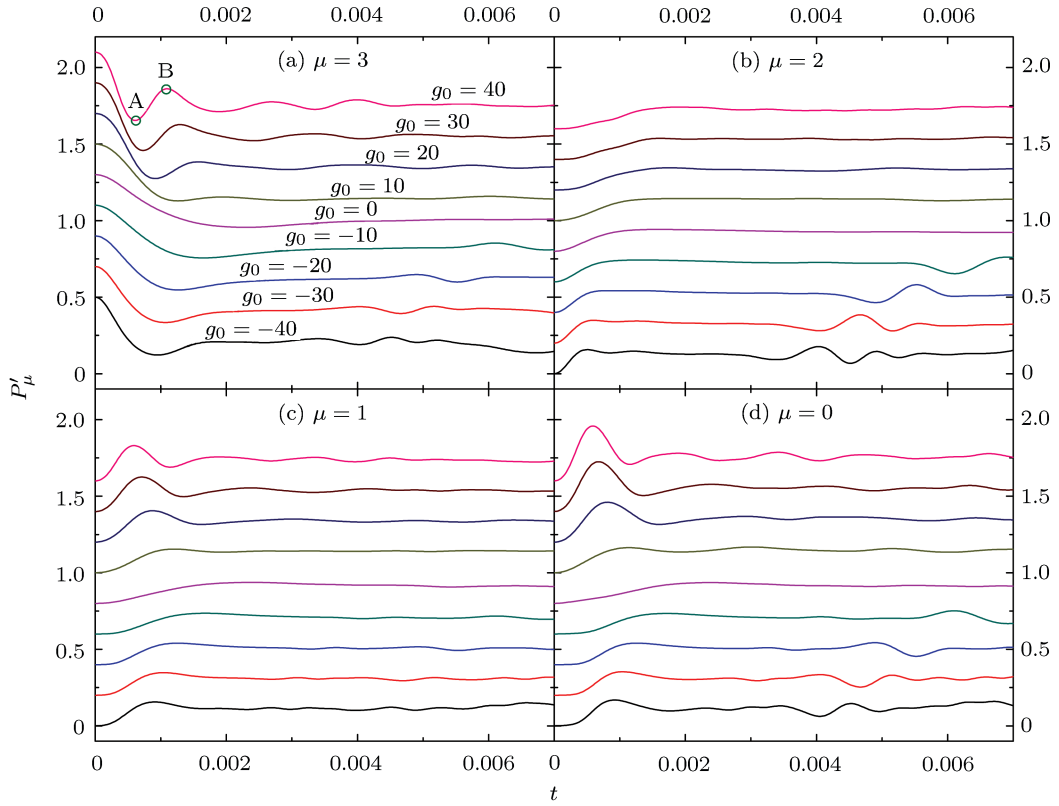


Fig. 2. (Colour online) The populations of spin evolution, $P^I_\mu(t)$, against t (in second) where (a) $\mu = 3$, (b) $\mu = 2$, (c) $\mu = 1$ and (d) $\mu = 0$, respectively. The initial state has $N^I_0 = 0$. In every subfigure, the curves from the lowest to the highest have g_0 from -40 to 40 with a step 10 as marked in (a). Each curve has been shifted upward by 0.2 more than its lower adjacent to guide the eyes.

First, we focus on the case where $N^I_0 = 0$. This case also implies that $P^I_{\pm 3}(0) = 0.5$ at the beginning. Afterward, due to the appearance of other components, $P^I_{\pm 3}(t)$ go down from the maximum 0.5 as shown in Fig. 2(a), while others $P^I_\mu(t)$ go up from 0. And they fluctuate around the backgrounds. Let the first minimum of $P^I_\mu(t)$ in Fig. 2(a) be denoted as A located at $t = T^I_{\mu,A}$, and the second maximum in the figure be denoted as B located at $t = T^I_{\mu,B}$. Related data are given in Table 1. It is clearly shown that from the locations of the maximum and minimum obtained via the theoretical calculation, the strength g_0 can be determined once the realistic locations are experimentally measured. Additional information can also be extracted from Figs. 2(b) and 2(d). For example, the first peak of $P^I_\mu(t)$ with $\mu = 1$ or 0 appearing in the earliest stage of evolution can help to discriminate g_0.

Table 1. The first minimum and the second maximum of $P_3^I(t)$ denoted as $P_{3,A}^I$ and $P_{3,B}^I$, respectively, for $N_0^I = 0$. The $T_{3,A}^I$ and $T_{3,B}^I$ are their locations (in ms).

g_0	-40	-30	-20	-10	0	10	20	30	40
$P_{3,A}^I$	0.12	0.13	0.15	0.16	0.16	0.13	0.07	0.06	0.05
$T_{3,A}^I$	0.94	1.08	1.26	1.66	2.30	1.26	0.92	0.74	0.60
$P_{3,B}^I$	0.21	0.21	0.25	0.25	0.21	0.15	0.18	0.23	0.26
$T_{3,B}^I$	1.84	2.62	4.88	6.10	5.60	1.94	1.58	1.30	1.10

Secondly, for comparison, the evolutions for both $N_0^I = 10$ and 20 are illustrated in Fig. 3. It is shown that the evolutions are no more sensitive to g_0. Thus we conclude that $N_0^I = 0$ as shown in Fig. 2 is a much better choice.

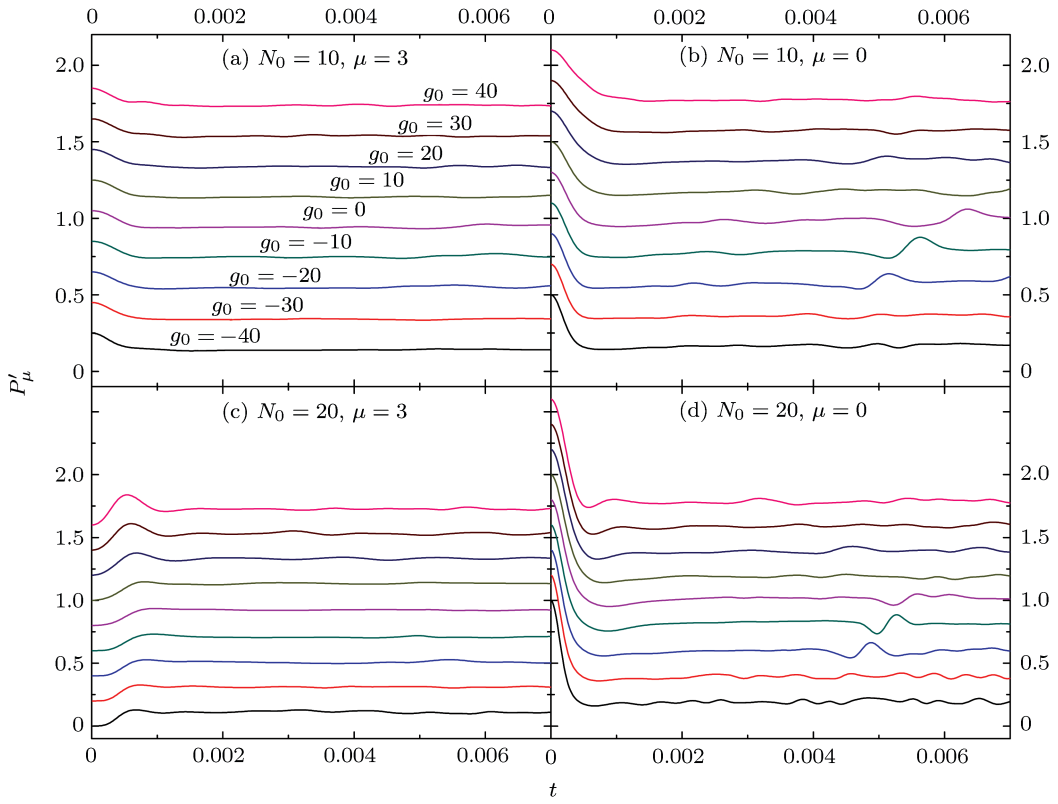


Fig. 3. (Colour online) The same as Fig. 2 except that $N_0^I = 10$ for (a) $\mu = 3$ and (b) $\mu = 0$, and $N_0^I = 20$ for (c) $\mu = 3$ and (d) $\mu = 0$, respectively.

3.3. The spin-correlation $Q_m^I(t)$

Since the time factor $E_j/\hbar$ in $C_\alpha(t)$ from Eq. (11) is not a multiple of integer among all eigenstates, $Q_m^I(t)$ is non-periodic. This is illustrated in Figs. 4 and 5.

In the case of $N_0^I = 0$ and from Eq. (12), $Q_0^I(0) = 10/19 \approx 0.526$, $Q_{\pm6}^I(0) = 9/38 \approx 0.237$ while $Q_i^I(0) = 0$ ($i = \pm1, \ldots, \pm5$). Since g_6 ($>> g_2$) is too positive to keep pair in lower energy, pairs are not likely to stay in channel with $S = 6$. Accordingly, $m = \pm6$ are the unfavourable spin state. Therefore, pairs prepared in $m = \pm6$ would transform to other states when time goes on. This indicates that $Q_{\pm6}^I$ decreases with time as illustrated in Fig. 4(a) no matter what g_0 will be. As well as g_6, g_4 is also much larger than g_2. Thus the $S = 4$ is neither a favourable spin state. Additionally, $Q_{\pm4}^I = 0$. So the probability of pair being in $m = \pm4$ always keeps in lower level. This is demonstrated in Fig. 4(b) where $Q_{\pm4}^I < 0.07$. Compared with g_6 and g_4, we find that the negative g_2 describes the attraction and can help to decrease the energy. Thus $S = 2$ is a more favourable spin state. Correspondingly, $Q_{\pm2}^I$ (note that it contains the $(S = 2, m = 2)$, $(S = 4, m = 2)$ and $(S = 6, m = 2)$ states but the last two states have low probabilities) increases to a high value (about 0.11 as shown in Fig. 4(c)) with time.

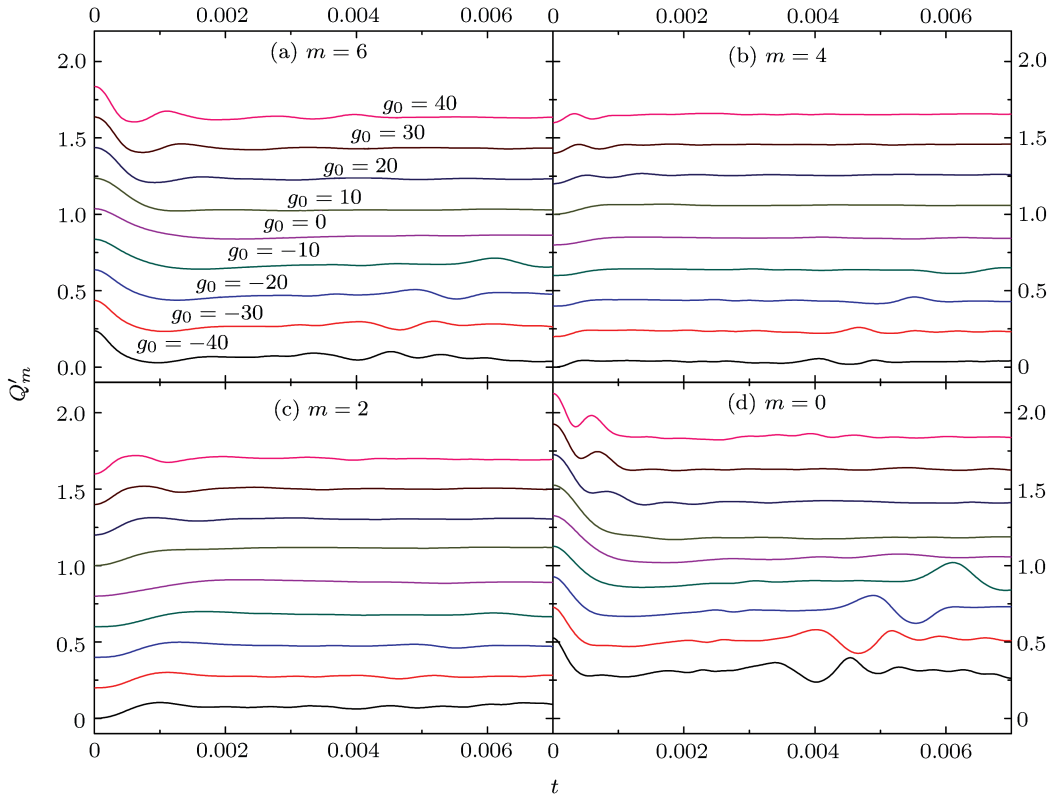


Fig. 4. (Colour online) The evolution of spin-correlation, $Q^I_m(t)$, versus t (in second) where (a) $m = 6$, (b) $m = 4$, (c) $m = 2$ and (d) $m = 0$, respectively. The initial state has $N^I_0 = 0$. In every subfigure, the curves from the lowest to the highest have g_0 from -40 to 40 with a step 10 as marked in (a). Each curve has been shifted upward by 0.2 more than its lower adjacent to guide the eyes.

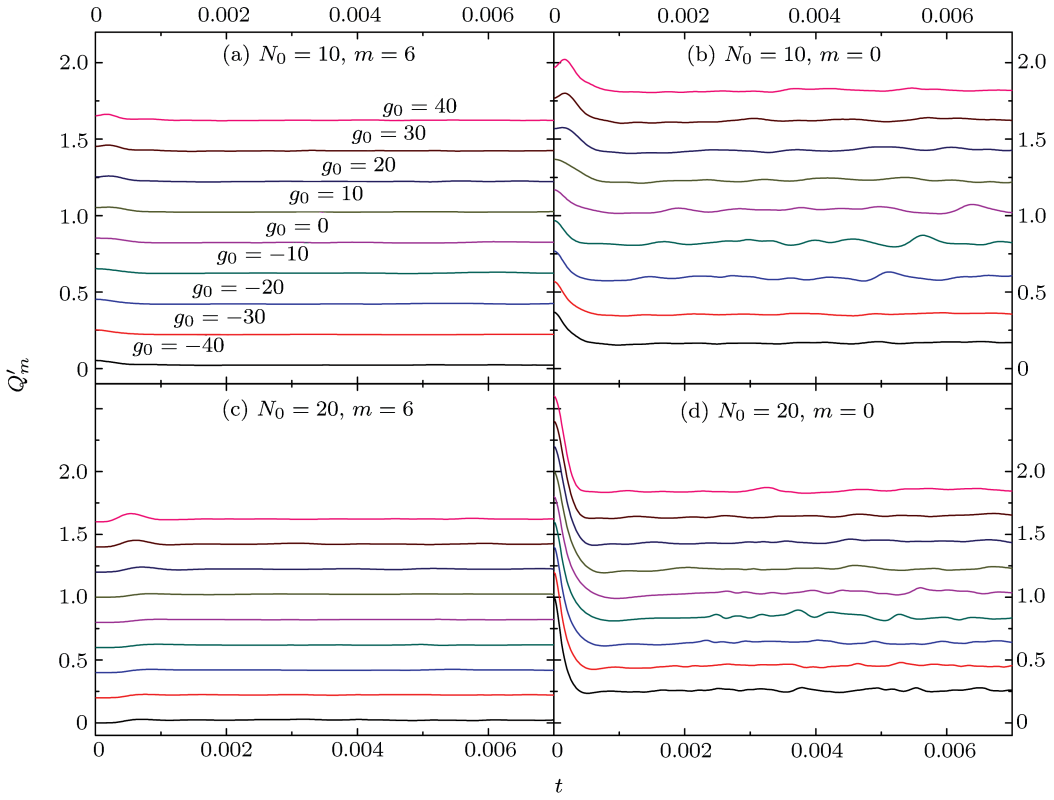


Fig. 5. (Colour online) The same as Fig. 4 except that $N^I_0 = 10$ for (a) $m = 6$ and (b) $m = 0$, and $N^I_0 = 20$ for (c) $m = 6$ and (d) $m = 0$, respectively.

Moreover, since the above spin states appear with time, the probability of $S = 0$ pairs state (same for $m = 0$ state) definitely decreases which is illustrated in Fig. 4(d). However, in this subfigure, there exists some extra oscillation upon the evolution. They appear at the early stage of evolution when $g_0 > 0$ (the upper curves) while at the later stage when $g_0 < 0$ (the lower curves). Implicitly, this favours the determination of the character (repulsive or attractive) for g_0 in experiment.

For comparison, the evolutions for both $N_0^I = 10$ and 20 are illustrated in Fig. 5. As well as P_μ^I, Q_m^I is no more sensitive to g_0.

4. Conclusion

The aim of the present paper is to evaluate the strength g_0 via the spin evolution. We do find some cases where P_μ^I depends on g_0 sensitively. It is recalled that, when a particle with spin-component μ collides with another particle with ν, g_0 would play a role only if $\nu = -\mu$. This is because the $S = 0$ channel would emerge only in this case. As an example, in Fig. 2(a), we have $N/2$ up-particles ($\mu = 3$) and $N/2$ down-particles ($\mu = -3$). Thus, among the $N(N-1)/2$ possible collisions, $N^2/4$ of them depend on g_0, while $N(N-2)/4$ do not depend on g_0. The ratio is $N/(N-2)$, which is larger than 1. On the other hand, in Fig. 3(a), we have $N/4$ up-particles, $N/4$ down-particles, and $N/2$ $\mu = 0$ particles. Thus the ratio is $(3N-4)/(5N-4)$, which is considerably smaller than 1. From the ratios we suggest that the curves in Fig. 2(a) should be more sensitive to g_0 than those in Fig. 3(a). This suggestion is confirmed by the numerical data (say, the curve with $g_0 = 40$ is very different from that with $g_0 = 0$ in Fig. 2(a), but they are quite similar in Fig. 3(a)). On the other hand, the different resumed value of g_0 impacts the competition of S pairs and the competition leads to the different character of evolution for spin population and spin-correlation.

We study the spin evolution starting from a mixture of three groups of ^{52}Cr atoms with a total magnetization of zero. The total number of particles is 20. A method beyond the mean-field theory and under the SMA is used. The numerical results show that the knowledge on g_0 could be extracted if the initial state has been appropriately chosen. Nonetheless, the above theoretical calculation can only provide a rough evaluation of g_0. For an accurate determination, more precise theory beyond the SMA and with the dipole force taking into account is necessary. This will lead to a great complexity, and hopefully can be realized in the near future.

References

[1] Ho T L 1998 *Phys. Rev. Lett.* **81** 742
[2] Ohmi T and Machida K 1998 *J. Phys. Soc. Jpn.* **67** 1822
[3] Stamper-Kurn D M, Andrews M R, Chikkatur A P, Inouye S, Miesner H J, Stenger J and Ketterle W 1998 *Phys. Rev. Lett.* **80** 2027
[4] Stenger J, Inouye S, Stamper-Kurn D M, Miesner H J, Chikkatur A P and Ketterle W 1998 *Nature* (London) **396** 345
[5] Law C K, Pu H and Bigelow N P 1998 *Phys. Rev. Lett.* **81** 5257
[6] Görlitz A, Gustavson T L, Leanhardt A E, Löw R, Chikkatur A P, Gupta S, Inouye S, Pritchard D E and Ketterle W 2003 *Phys. Rev. Lett.* **90** 090401
[7] Lewenstein M, Sanpera A, Ahufinger V, Damski B, Sen De A and Sen U 2007 *Adv. Phys.* **56** 243
[8] Han J R, Liu X F and Wang Y J 2009 *Chin. Phys.* B **18** 5301
[9] Griesmaier A, Werner J, Hensler S, Stuhler J and Pfau T 2005 *Phys. Rev. Lett.* **94** 160401
[10] Diener R B and Ho T L 2006 *Phys. Rev. Lett.* **96** 190405
[11] Stuhler J, Griesmaier A, Koch T, Fattori M, Pfau T, Giovanazzi S, Pedri P and Santos L, 2005 *Phys. Rev. Lett.* **95** 150406
[12] Schmaljohann H, Erhard M, Kronjäger J, Sengstock K and Bongs K 2004 *Appl. Phys.* B: *Lasers Opt.* **79** 1001
[13] Chang M S, Hamley C D, Barrett M D, Sauer J A, Fortier K M, Zhang W, You L and Chapman M S 2004 *Phys. Rev. Lett.* **92** 140403
[14] Chang M S, Qin Q, Zhang W, You L and Chapman M S 2005 *Nature Physics* (London) **1** 111
[15] Pu H, Law C K, Raghavan S, Eberly J H and Bigelow N P 1999 *Rhys. Rev.* A **60** 1463
[16] Diener R B and Ho T L 2006 arXiv:cond-mat/0608732v1 [cond-mat.other]
[17] Luo M, Bao C G and Li Z B 2008 *Phys. Rev.* A **77** 043625
[18] Chen Z F, Bao C G and Li Z B 2008 arXiv:0802.0822v1 [cond-mat.other]
[19] Yi S, Müstecaplioğlu Ö E, Sun C P and You L 2002 *Phys. Rev.* A **66** 011601(R)
[20] Li Z B, Chen Z F, He Y Z and Bao C G 2010 *Phys. Rev.* A **82** 032708
[21] O'Dell D H J, Giovanazzi S and Eberlein C 2004 *Phys. Rev. Lett.* **92** 250401
[22] Bao C G 2004 *Acta Sci. Nat. Univ. Sunyatseni* **46** 70
[23] Bao C G 2009 *Few-Body Syst.* **46** 87
[24] Werner J, Griesmaier A, Hensler S, Stuhler J, Pfau T, Simoni A and Tiesinga E 2005 *Phys. Rev. Lett.* **94** 183201
[25] Li Z B, Bao C G and Katriel J 2008 *Phys. Rev.* A **77** 023614