Dark soliton in one-dimensional Bose–Einstein condensate under a periodic perturbation of trap*

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The perturbation of a confining trap leads to the collective oscillation of a Bose–Einstein condensate, thereby the propagation of a dark soliton in the condensate is affected. In this study, periodic perturbation is employed to match the soliton oscillation. We find that the soliton dynamics depends sensitively on the coupling between the moving direction of the trap and that of the soliton. The soliton energy/depth evolves periodically, and a relevant shift in the soliton trajectory occurs as compared with the unperturbed case. Overall, the soliton oscillation frequency changes little even if the perturbation amplitude and frequency vary.

Keywords: Bose–Einstein condensate, dark soliton

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1. Introduction

The presence of repulsive atomic interactions in Bose–Einstein condensates (BECs) leads to rich and intriguing nonlinear phenomena. A key example is the formation of dark soliton.[1−9] Dark solitons are localised one-dimensional (1D) defects characterised by a notch in the ambient condensate density. Many investigations show that dark soliton, accompanied by a phase jump, results from a balance between the defocusing dispersion and the focusing repulsive nonlinear interaction. Thus, one of the main properties of dark soliton is preserving its localised shape during propagation.[10−13] In fact, dark soliton in 1D system can display particle-like characters. For example, a dark soliton under harmonic confinement is expected to oscillate back and forth in the trap.[10,11] If a stationary dark soliton is initially created off-centre in the trap, it will experience a force causing it to accelerate towards the trap centre. Recently, Parker *et al.*[14,15] and Proukakis *et al.*[16] have considered some modifications about the harmonic trap in order to demonstrate the possible soliton behaviours. They[14] embedded a tight harmonic trap in a weaker harmonic trap, by which sound waves could be controlled to escape. And they[15] added an optical lattice to the harmonic trap to perturb the dark soliton. Furthermore, they[16] have considered the parametric driving as well as the damping mechanism. The influence of the finite temperature effect on dark soliton has also been revealed.[17] Similarly, Bilas and Pavloff have investigated the propagation of a dark soliton in a quasi-1D BEC with a random potential[18] and the propagation of a dark soliton encountering an obstacle on its way.[19] Certainly, some other factors should be taken into consideration in the real environment.

Motivated by the above studies, we consider a periodic perturbation on the whole confining trap. We assume that the whole trap is not fixed but oscillating. In fact, we cannot assure that there is not any vibration in real environment. For instance, the whole experimental setup vibrates and the confining trap follows. The mechanical vibration will cause small displacements of the BEC, nudging it away from the trap centre, thereby giving it a velocity and making it prone to the collective oscillation. If dark soliton is in this system, the dynamics behaviour would change accordingly.

Here, the vibration of the trap does not create new solitons or cause the BEC to undergo the intense collective oscillation. Thus, the oscillation amplitude of the trap should be very small. Meanwhile, the oscillation frequency should not be equal to the natural frequency of the BEC oscillation, because this frequency would cause the resonance of the BEC. Specially, if the oscillation frequency of the trap is equal to that of the soliton oscillation, the oscillating soliton will be well matched. We can preliminarily estimate two main cases: one is that the trap and the soliton move

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in the same directions, the other is that they move in the opposite directions.

In this study, we perform detailed investigations of the soliton behaviours under various perturbations. We find that the soliton behaviours are sensitively dependent on coupling between the initial moving direction of the soliton itself and that of the trap. On the whole, the soliton energy is smaller than that in the unperturbed case if the soliton moves in the same direction as the trap does. Conversely, the soliton energy is bigger. The perturbation has the soliton trajectory deviating from the unperturbed one. The increase of perturbation amplitude will enlarge the deviation. Furthermore, the experiments which terminate the presented perturbation will be considered.

2. Model and basic equations

At zero temperature, the dynamics of BECs is governed by the Gross–Pitaevskii (GP) equation

$$\mathrm{i}\hbar\frac{\partial}{\partial t}\psi = \left(-\frac{\hbar^2\nabla^2}{2m} + V_{\mathrm{ext}} + g\left|\psi\right|^2\right)\psi, \qquad (1)$$

here, ψ denotes the macroscopic order parameter of the system, V_{ext} the confining potential, m the atomic mass, and $g = 4\pi\hbar^2 a/m$ the scattering amplitude, with a being the s-wave scattering length. In the limit of tight transverse confinement, the BEC dynamics is effectively longitudinal.[20]

Dark soliton solutions are supported by the 1D form of Eq. (1) in the absence of external confinement ($V = 0$).[13,21] For a uniform background density n, a dark soliton with speed v and position $(x - vt)$ has the form,

$$\psi(x,t) = \sqrt{n}\left\{\mathrm{i}\frac{v}{c} + \beta\tanh[\beta\frac{(x-vt)}{\xi}]\right\}\mathrm{e}^{(\mu t/\mathrm{i}\hbar)}, \qquad (2)$$

where $\beta = \sqrt{1 - v^2/c^2}$, $\mu = gn = mc^2$ is the chemical potential, $c = \sqrt{\mu/m}$ is the Bogoliubov speed of sound, and $\xi = \hbar/\sqrt{\mu m}$ is the healing length characterising the size of the soliton.

In numerical experiment, the soliton energy is often calculated by integrating the GP energy function

$$\varepsilon(\psi) = \frac{\hbar^2}{2m}\left|\nabla\psi\right|^2 + V_{\mathrm{ext}}\left|\psi\right|^2 + \frac{1}{2}g\left|\psi\right|^4, \qquad (3)$$

across the soliton region ($X \pm 5\xi$, where X is the instantaneous position of the local density minimum) and subtracting the corresponding contribution of the background fluid.[14,16,22]

Usually, one considers the BEC in a static harmonic potential $V_{\mathrm{har}}(x) = m\omega^2 x^2/2$, where ω is the frequency of the trap. Here, as we consider the perturbation of the whole trap, the trap can be written as

$$V_{\mathrm{ext}}(x,t) = \frac{m\omega^2[x + h\sin(\omega_{\mathrm{d}}t)]^2}{2}, \qquad (4)$$

where h and ω_{d} are the amplitude and the frequency of the perturbation, respectively.

3. Initial conditions

In numerical simulation, one may use the density profile approximated by the Thomas–Fermi (TF) solution as initial condition,

$$n(x,t=0) = \begin{cases} [\mu - V_{\mathrm{ext}}(x,0)]/g & \mu \geq V_{\mathrm{ext}}(x,0) \\ 0 & \text{else} \end{cases}. \qquad (5)$$

For simplicity, we set $\hbar = 1$, the chemical potential $\mu = 1$, the atomic mass $m = 1$, and the scattering amplitude $g = 1$. Thus the spatial extent of the system is characterised by the healing length $\xi = \hbar/\sqrt{m\mu}$, and the time unit is ξ/c. Meanwhile, we have $\omega = (\sqrt{2}/100)(c/\xi)$ for the external potential. By Eq. (5), the BEC has a TF radius of $R_{\mathrm{TF}} = 100\xi$. We assume that the perturbation is very weak. So the amplitude of the trap oscillation should be much less than the TF radius, i.e., $h << R_{\mathrm{TF}}$. The frequency of perturbation approaches the soliton oscillation frequency ω_{sol}, i.e., $\omega_{\mathrm{d}} \sim \omega_{\mathrm{sol}}$, where $\omega_{\mathrm{sol}} = \omega/\sqrt{2}$. Furthermore, to reduce the underlying density fluctuation on the edges, we initially smooth the density profile by propagating the TF wave function in imaginary time.

We now estimate the parameters for a realistic experiment. By integrating out the transverse degrees of freedom, the GP equation reduces to a 1D form with a modified coupling coefficient $g_{1\mathrm{D}} = g/(2\pi l_\perp^2)$, where $l_\perp = \sqrt{\hbar/(m\omega_\perp)}$ is the transverse harmonic oscillator length, with $\omega_\perp$ being the transverse trapping frequency. For a ^{23}Na condensate with $m = 38.18 \times 10^{-27}$ kg and $a = 2.8$ nm,[23] we assume the tight transverse confining frequency $\omega_\perp = 5000 \times 2\pi$ Hz and the 1D peak condensate density $n_{1\mathrm{D}} = 10^8$ m^{-1}. Thus, the longitudinal confining frequency is $39.7 \times 2\pi$ Hz. Our space and time units correspond to 0.4 μm and 5.7×10^{-5} s respectively. The system has the number of atoms, $N_0 \approx 52000$.

4. Results

Our investigation is based on numerical simulations. Figure 1 shows the evolution of soliton energy under various amplitudes of perturbation. Clearly, the moving direction of the soliton and that of the trap determine the evolution of the soliton. In the absence of the perturbation ($h = 0$), our model degenerates to the experimental case where soliton oscillates in a harmonic trap. In this case, the soliton propagates and it becomes asymmetrically deformed and tries to adjust itself to the inhomogeneous background by radiating counterpropagating sound pulses.[14] As is well known, the soliton oscillates in the trap at $\omega_{\rm sol} = \omega/\sqrt{2}$, emitting and reabsorbing sound waves. Overall, the soliton does not decay, but becomes continuously perturbed by fluid motion generated by the soliton itself.

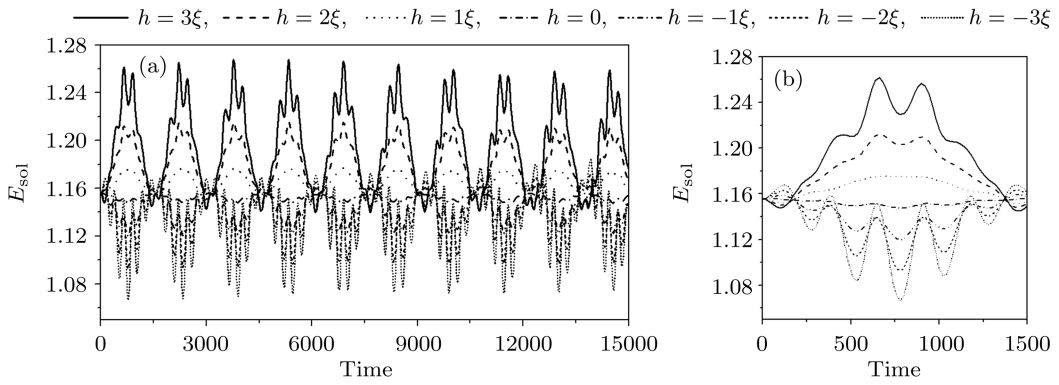


Fig. 1. Soliton energies versus time under various perturbation amplitudes of the trap with initial speed of the solion being $0.3c$ and the initial position being at $x = 0$ (a), and their magnified parts (b). The parameters are $\omega = (\sqrt{2}/100)(c/\xi)$ and $\omega_{\rm d} = \omega/\sqrt{2}$. The energy is in units of μ, and the time is in units of ξ/c. The corresponding units in the following figures are the same as in this picture, unless otherwise stated.

When the trap moves in the reverse direction as the soliton does ($h > 0$), the soliton tends to first gain energy to a peak value. Then, the soliton energy decreases to the original value. Such a gain-loss cycle of the energy may keep on repeating. Here, we provide a possible explanation for this phenomenon. In homogeneous condensate whose density is constant, the slower the dark soliton, the bigger the soliton energy is.[21] When the trap moves in the reverse direction as the soliton does ($h > 0$), the soliton velocity becomes relatively slow. So, the soliton energy tends to increase. However, the soliton energy cannot increase illimitably because the condensate cannot support that kind of soliton. A property of dark soliton is preserving its localised shape during propagation. Thus, the soliton energy returns to the original value in the cycle. The period of the energy evolution approximates to $1516\xi/c$. So in most of the time the soliton energy is bigger than its initial value. Increasing the amplitude of perturbation enhances the abilities of soliton to gain and lose energy in its cycle.

On the contrary, when the trap moves in the same direction as the soliton does ($h < 0$), the soliton tends to lose energy first to a minimum, and then it regains energy to its original value. It forms a lose-gain cycle with a period of $1516\xi/c$. Similarly, increasing the amplitude of the perturbation enables the soliton to lose more energy then its regained energy. Thus in most of the time the soliton energy is smaller than its initial value in this case. In fact, in Fig. 1 compared is the evolution of a dark soliton in the harmonic trap with that in the same trap when it is additionally perturbed by a periodic oscillation.

Figure 2 shows the corresponding evolutions of the soliton position under various amplitude perturbations of the trap. Generally speaking, the BEC will follow the motion of the trap. Because the oscillation amplitude and the oscillation frequency of the trap are very small, the trap oscillation leads to the deviation of the soliton trajectory from that in the unperturbed case ($h = 0$). The oscillation period of the soliton fluctuates under the perturbation. On the whole, the oscillation frequency of the soliton remains around the value of $\omega_{\rm sol} = \omega/\sqrt{2}$. Consequently, this feature ensures the coupling between the moving direction of the trap and that of the soliton.

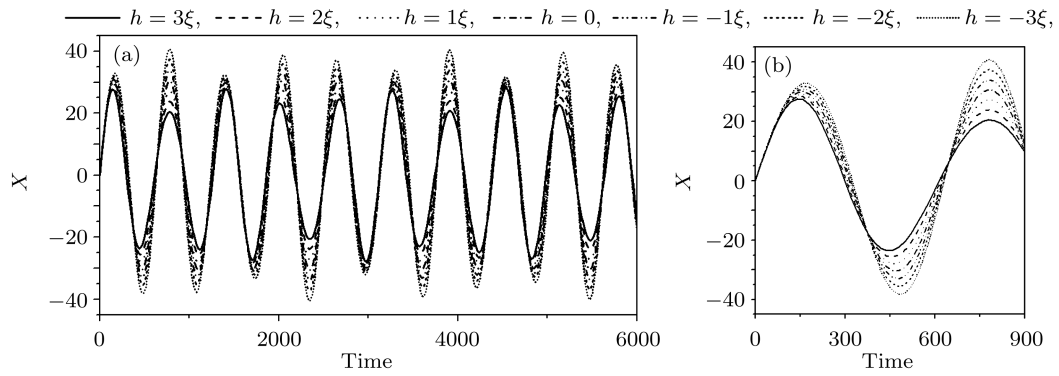


Fig. 2. (a) Soliton trajectories versus time under various perturbation amplitudes of the trap (a), and their magnified parts (b). The length unit is $\xi = \hbar/\sqrt{m\mu}$. The parameters are the same as those in Fig. 1.

When the trap moves in the same direction as the soliton does, the BEC follows the motion. So the soliton is carried and it oscillates off centre farther. The soliton energy tends to be smaller than its original value in Fig. 1. However, when the trap moves in the direction opposite to the direction in which soliton moves, the motion of the BEC relatively reduces the oscillation amplitude of the soliton, and the soliton energy forms the gain-loss cycle. In general, if the perturbation frequency of the trap is equated to the natural frequency of particle oscillation, then the resonance may be induced. Here, although the trap oscillation matches that of the soliton, the soliton does not form the resonance, which the normal particles would display. After all, the soliton is not the normal particle, although lots of investigations have indicated its particle-like characters.

We have demonstrated the evolutions of the soliton energy and the soliton trajectory. Undoubtedly, it is anticipated that the soliton depth will change as the whole BEC moves. To confirm the scenario, in Fig. 3 the corresponding evolutions of the soliton depth plotted. Note that the soliton depth is the difference between the corresponding background BECs and the local minimum density. Obviously, the soliton depth changes as the BEC moves, and the coupling between the moving direction of the trap and that of the soliton plays an important role. Just like the soliton energy in Fig. 1, the soliton depth also tends to first increase and then return to its original when $h > 0$; but when $h < 0$, it tends to first decrease and then return to its original value. Certainly, the gain-loss cycle ($h > 0$) [lose-gain cycle ($h < 0$)] repeats as time goes on. When the trap is fixed ($h = 0$), the fluctuation of the soliton depth derives from the soliton-sound interaction. Furthermore, Fig. 3 indicates that a shallow soliton can be converted into a deeper (shallower) one by controlling the coupling between the moving direction of the soliton and that of the trap.

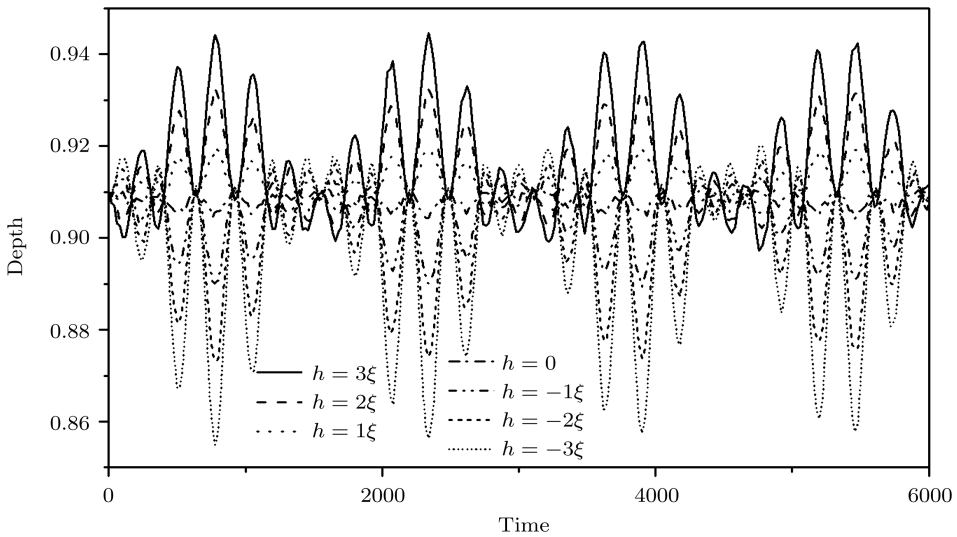


Fig. 3. Soliton depths versus time under various perturbation amplitudes of the trap, with the parameters being the same as those in Fig. 1.

In order to fully explore the soliton behaviours in the above text, we naturally realise the influence of the collective movement of the BEC. Figure 4 shows the evolutions of the whole BEC. In our experiments, we do not want to intensely excite the BEC. In Fig. 4(a), we can see the BEC energy does not increase remarkably. Figure 4(b) shows the corresponding collective oscillation of the BEC. Its oscillation amplitude is much less than the TF radius. Therefore, our model only induces the weak collective oscillation of the BEC. In fact, the perturbation causes the local density of the BEC to increase or decrease weakly. When the soliton tends to move to the density-increasing region, the soliton energy evolves in the gain-loss cycle. Otherwise, the soliton energy evolves in the lose-gain cycle.

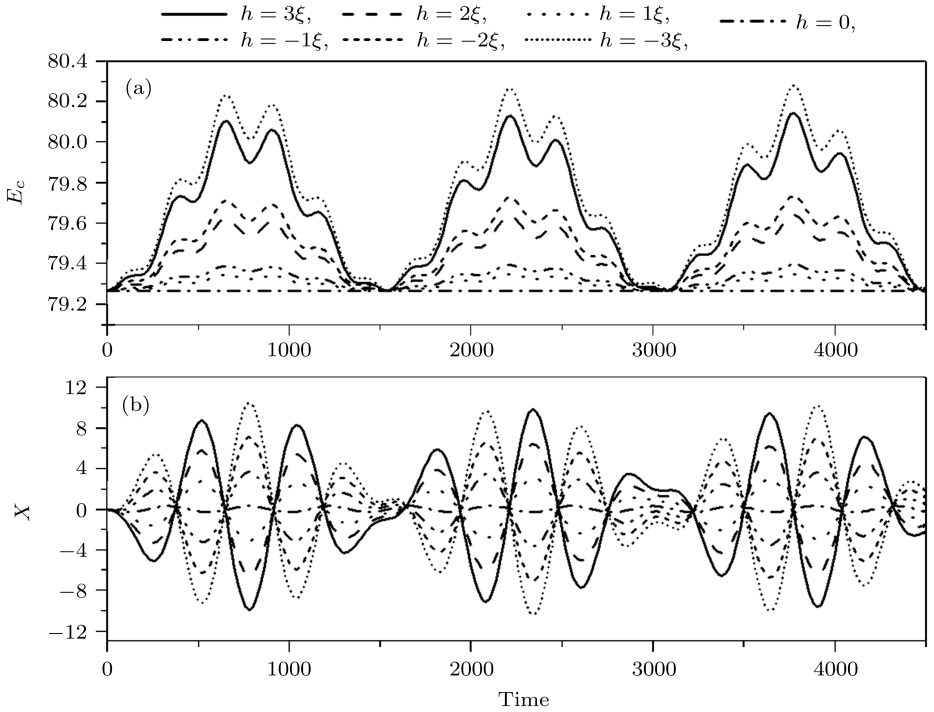


Fig. 4. (a) Evolutions of the BEC energy versus time, and (b) average positions of the BEC versus time, with the parameters being the same as those in Fig. 1.

Because the perturbation leads to the collective oscillation of the whole BEC, the energy of the BEC increases naturally. However, the perturbation does not simply pump energy into the BEC. In the process, the BEC can lose energy to its initial value. Thus an energy cycle with a period of $1516\xi/c$ occurs in Fig. 4(a). According to Fig. 1, the soliton energy tends to lessen as the moving directions of the soliton and the trap are the same. But the perturbation always leads the BEC energy to increase little. For the same amplitude of the perturbation, the BEC even obtains and loses more energy when the soliton moves in the same direction as the trap does. Certainly, if h equals 0, the trap does not oscillate at all and the BEC energy is conserved. And the oscillation of the BEC results only from the periodic motion of the soliton.

In the text above, we have focused on the evolution of soliton with an initial velocity of $0.3c$. Now, we change the initial velocity (see Fig. 5). The coupling between moving directions is obvious. If the soliton initially moves in the direction opposite to the direction in which the trap moves ($v > 0$), it tends to first gain energy (become deeper) and then come back. While if the soliton initially moves in the same direction as the trap does ($v < 0$), it tends to first lose energy (become shallower) and then come back. Those behaviours can repeat as they do in Fig. 1. If the initial velocity of soliton is zero, the soliton trajectory is not fixed due to the motion of the BEC, but its depth changes little. Certainly, the main effect of increasing the soliton speed is the increasing of soliton oscillation amplitude. We have also noted that the soliton trajectories always cross at some special positions (near the origin position) in Fig. 5(a). This indicates that the solitons with various velocities all arrive at these positions, no matter what their initial velocities are. Hence, we can infer that the soliton oscillation frequency remains around an expected value

of $\omega_{\rm sol} = \omega/\sqrt{2}$.

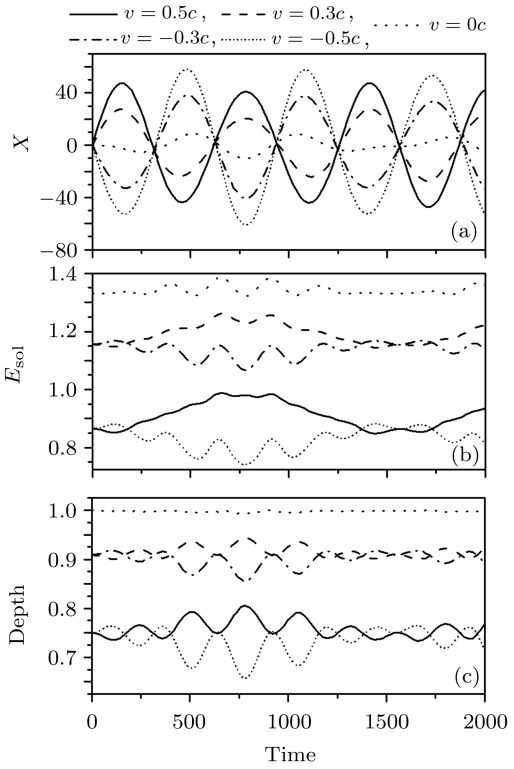


Fig. 5. Soliton trajectories (a), soliton energies (b), and soliton depth (c) versus time for various initial velocities of soliton, with parameters being $h = 3\xi$, $\omega = (\sqrt{2}/100)(c/\xi)$ and $\omega_{\rm d} = \omega/\sqrt{2}$, and the initial position of solitons at $x = 0$.

In real experiment, if the trap is found to be oscillating, the oscillation may be stopped in order to reduce its influence. Here, we propose to terminate the perturbation at a given time. What will happen to the soliton? Figure 6(a) displays the evolutions of the soliton trajectory when the perturbation is turned off at various times. The corresponding soliton energy and soliton depth are shown in Figs. 6(b) and 6(c), respectively. Note that we choose the time when the soliton starts to change its moving direction. At this time, the soliton speed is zero. So, it would be the optimal time for the terminating operation in real experiment. For comparison, the soliton properties under continuous perturbation (t_0 = infinite) are indicated. In Fig. 6(b), the termination of the perturbation at a particular time leads to a breakdown of the energy cycle. Subsequently, the soliton starts a new cycle in the energy. The soliton energy features modulation due to the collective oscillation of the BEC. Therefore, we cannot efficiently fix the soliton energy through terminating operation. In fact, we also test the terminating operation at other times. The amplitudes of these modulations are varying with the point in the cycle at which the perturbation is terminated. In the limit case where the perturbation is terminated at close to the initial soliton energy, these energy modulations are minimal. For intermediate points at the peak soliton energy, the modulations are largest. The above modulation properties also appear in the corresponding soliton depth behaviours (see Fig. 6(c)).

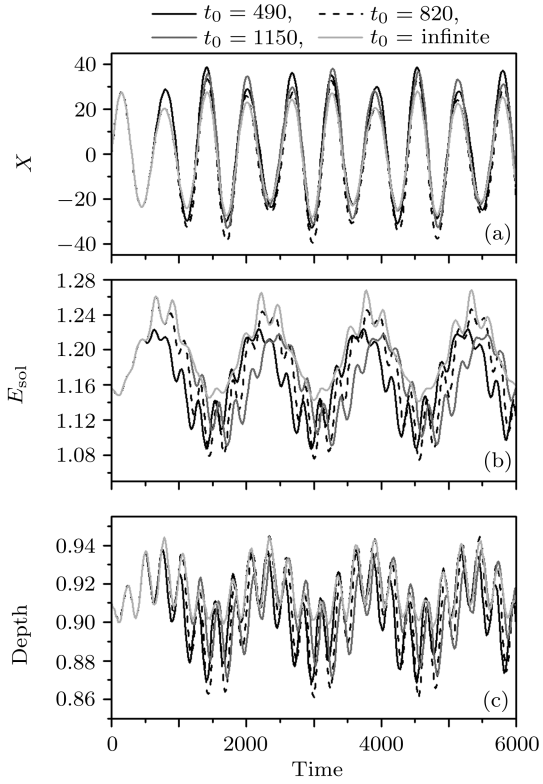


Fig. 6. Effect of terminating the perturbation after various evolution times, showing soliton trajectory (a), soliton energy (b), and soliton depth (c) versus time, with t_0 being the given time when the perturbation is terminated, with parameters being $h = 3\xi$, $\omega = (\sqrt{2}/100)(c/\xi)$ and $\omega_{\rm d} = \omega/\sqrt{2}$, and initial speed of the solion $0.3c$ and initial position at $x = 0$ respectively.

Finally, we mention the perturbation frequencies which mismatch the soliton oscillation frequency (see Fig. 7). For comparison, we also plot the results for the frequency case where $\omega_{\rm d} = \omega_{\rm sol}$. When the perturbation frequency is not equal to the soliton oscillation frequency, the soliton dynamics becomes complicated. In Fig. 7(a), the perturbation changes almost not the soliton oscillation frequency but the oscillation amplitude. Due to the difference in frequency between the soliton oscillation and the perturbation oscillation, the coupling between the moving direction of the trap and that of the soliton changes. Thus, the soliton energy/depth first increases and then decreases down to lower than its initial value in Figs. 7(b) and 7(c). Furthermore, we also find some similar phenomena to

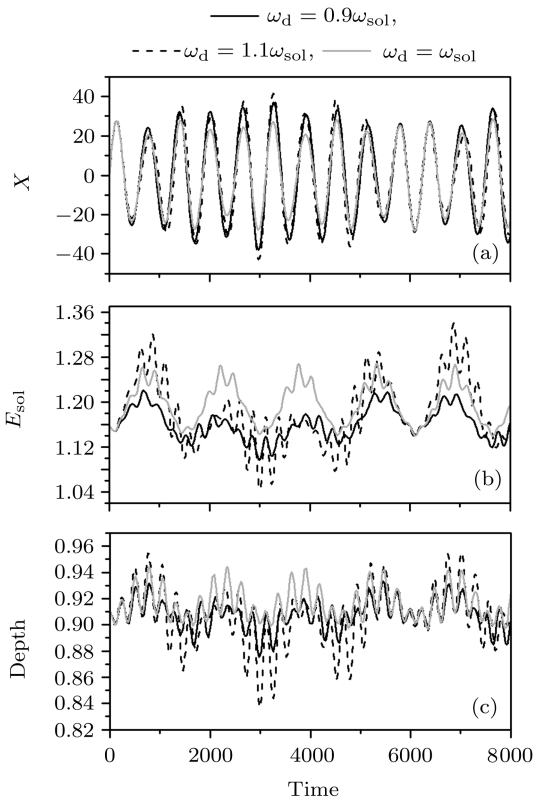


Fig. 7. Evolutions of soliton when the perturbation frequency of the trap is off the soliton oscillation frequency, showing soliton trajectory (a), soliton energy (b), and soliton depth (c) versus time, with parameters being $h = 3\xi$, and $\omega = (\sqrt{2}/100)(c/\xi)$, initial speed of the solion $0.3c$ and the initial position at $x = 0$.

take place when the two frequencies have large mismatch. Note that the initial position of the soliton in our experiments is at $x = 0$. If this position varies, the coupling between the moving direction of the soliton and that of the trap would change accordingly. And the evolutions of the soliton energy and the soliton trajectory will change too.

5. Conclusion

We have studied the perturbation of the trap affecting the evolution of the soliton. The collective oscillation of the whole BEC is enhanced when the perturbation is added. The evolution of the soliton depends sensitively on the coupling between the moving direction of soliton and that of perturbation. Perturbed on condition that the perturbation frequency is equal to soliton oscillation frequency, the soliton is prone to obtain or lose energy first, then returns circularly. Accordingly, the soliton trajectory deflects from the unperturbed one. Increasing the perturbation amplitude only enhances the change in soliton evolution. Note that the soliton oscillation frequency changes little. Furthermore, if the perturbation has existed in the experiment, even terminated, we cannot completely return to the unperturbed case but create a new cycle in the following evolution. The mismatched frequencies lead the soliton evolution to become complicated.

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