Large-scale cluster state generation with nuclear spins in diamonds*

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The cluster state is an indispensable resource for one-way quantum computing (1WQC). We propose a practical scheme for constructing cluster states among nuclear spins in nitrogen-vacancy defect centres (NV centres) in different diamonds. The entanglement of nuclear spins within an NV centre is made by hyperfine coupling via electron spin, and the entanglement between remote NV centres is accomplished using the parity projection of emitted photons. We discus the possibility to build large-scale nuclear-spin cluster states with diamonds.

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1. Introduction

A frequently mentioned measurement-based quantum computing (QC) is the one-way QC (1WQC),[1] which works by consuming the cluster state[2−4] generated in advance. Since the 1WQC was proposed, there have been many studies for efficiently producing cluster states on the large scale.[5−8] The two-dimensional (2D) cluster state was also considered as a basic building block to accomplish genuine QC tasks.[9]

The nitrogen-vacancy defect centres (NV centres) in diamonds are promising to achieve universal QC at room temperature.[10] As a result, QC studies with NV centres have attracted considerable attention, although they are still pretty challenging experimentally.[11] Several favourable properties of NV centres have been exploited, such as the availability of single electron-spin rotation and detection,[12] characteristics of coherence and entanglement of electron and nuclear spins via hyperfine interaction.[13,14] Besides, by means of the electron spin, the nuclear spins within an NV centre could be manipulated efficiently.[15−17] Using this property, we may have a fast and high-fidelity readout from the electron spin.[18] On the other hand, the simple and fully controllable characteristics of the NV centres also provide us with an excellent test bed for investigating spin–bath interaction.[19]

For the case of more spins involved, there have been some proposals (e.g., Ref. [5] and the references therein) to set up entanglement between remote NV centres through using photonic emission and parity projection. Moreover, NV centres in different diamonds could also be entangled by coupling to a single or an array of cantilevers.[20,21] Furthermore, embedding NV centres into separate cavities, we could achieve entanglement with higher success probability between remote NV centres.[22,23]

In this work we combine the ideas of entanglement generation using hyperfine interaction and using photonic parity projection. For convenience of our description, we introduce the terminologies 'broker qubit and client qubit' from Ref. [24]: the broker qubit denoting the electron spin and the client qubit corresponding to the nuclear spin. Specifically, our client qubits include two ^{13}C nuclear spins and one ^{15}N nuclear spin in the first coordination shell of the vacancy,

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and the broker qubit is the electron spin localized at the defect site. This design is based on the fact from a recent experimental report,[14] and is for constructing purely nuclear-spin graph states. We show that four-qubit (i.e., the broker qubit plus the three client qubits) cluster state within a single NV centre could be generated deterministically at room temperature. For scalability, the broker qubits in different NV centres negotiate for building a larger-sized cluster-state via a risky optical protocol, which generates multi-qubits cluster state of nuclear spins in a probabilistic fashion. To reduce the overhead in our scheme, we generate chain-shaped cluster state in each NV centre, instead of the star-shaped with the electron-spin in the centre. This could prevent the entanglement of the nuclear spins from being spoiled in the failed case of the two-NV entanglement.

The rest of the present paper is organized as follows. The next section is for the entanglement of spins within a single NV centre, and then we extend our discussion to more NV centres in Section 3. The operations in these two sections above can be accomplished at room temperature. Due to the broad phonon sideband relevant to photonic emission, however, we should seek for other ways in Section 4 to improve the zero-phonon-line (ZPL) photonic emission, which guarantees entanglement generation between separate NV centres by parity projection of photons and needs cryogenic condition. We discuss the feasibility with current techniques to achieve large-scale nuclear spin cluster states with diamonds.

2. A single NV centre

2.1. System and Hamiltonian

The NV centres we are studying are negatively charged with two unpaired electrons located at the vacancy, and usually treated as electron spin-1. So the ground state is a spin triplet state labeled as ^3A (see Fig. 1), with the levels $m_{\rm S} = \pm 1$ degenerated and a zero-field splitting ($D = 2.87$ GHz) between $m_{\rm S} = 0$ and $m_{\rm S} = \pm 1$.[25] The excited state ^3E is also

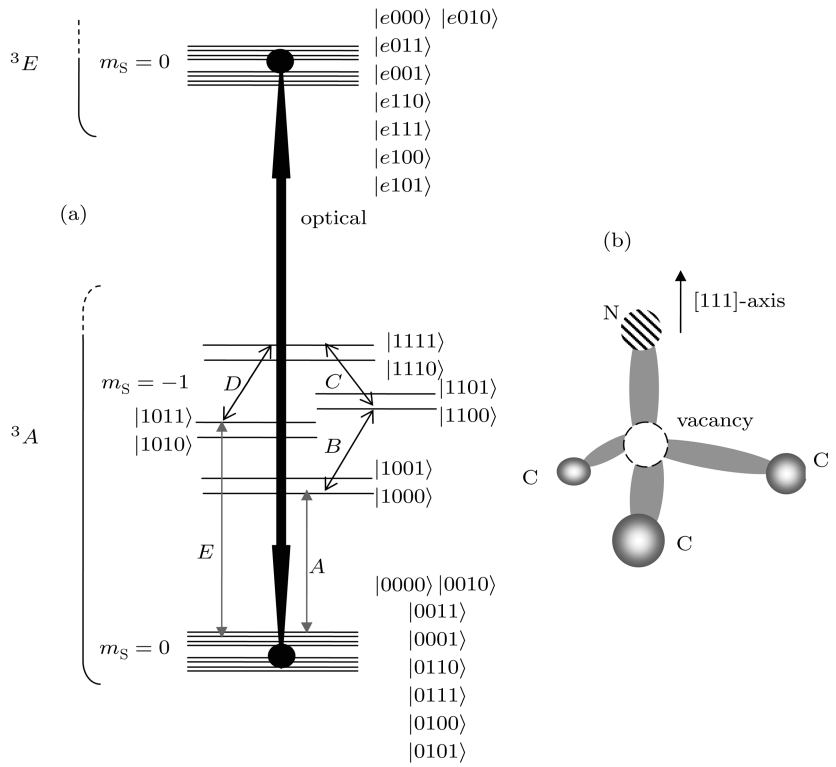


Fig. 1. (a) Energy-level scheme and the relevant transitions in an NV centre, where only two of the hyperfine splittings (i.e., $m_{\rm S} = 0, -1$) of the electron-spin ground state are depicted, with the denotation $|m_{\rm S}, I_{\rm C}, m_{I_{\rm C}}, m_{I_{\rm N}}\rangle$. For ^3A part, the levels with their respective energies are calculated by the second-order perturbation theory. Quantum transitions in Eq. (4) are labeled by solid arrows (the grey for electron spins and the thin arrows for nuclear spins) with A, B, ..., E corresponding to the steps from the first to the last, respectively. The electron-spin state can be mapped to a photon via an optically-allowed transition in black thick arrows. (b) Atomic structure of the NV centre with a broker qubit (i.e., the electron spin) and three client qubits (i.e., the nuclear spins of ^{13}C and ^{15}N).

a spin triplet, associated with a broadband photoluminescence emission with ZPL of 1.945 eV, which allows optical detection of individual NV defects using confocal microscopy. Therefore, high-fidelity polarization and detection of individual electron spins are feasible in this system. For single nuclear spin localized close to the defect, hyperfine couplings make it possible to manipulate and detect the nuclear spins via the electron spin.[13,15] In this section, we try to set up entanglements using hyperfine couplings between the electron spin and the nuclear spins of ^{13}C and ^{15}N in the first shell of a single NV centre, based on the Hamiltonian under the magnetic field B_z,

$$H_{\rm exact} = g_{\rm e}\beta_{\rm e}\hat{S}_z B_z + \hat{S}_z D\hat{S}_z + \sum_i (\hat{\underline{S}}\cdot\underline{\underline{A}}_i\cdot\hat{\underline{I}}_i - g_{\rm n}\beta_{\rm n}I_{zi}B_z), \quad (1)$$

where the electron spin is $\underline{S}=1$, the nuclear spins of ^{13}C and ^{15}N are both $\underline{I}=1/2$; $\beta_{\rm e}$ is the Bohr magneton; $\beta_{\rm n}$ the nuclear magneton; the constants are $D=2.87$ GHz, $g_{\rm e}=2.0028$ and $g_{\rm n}=1.40483$.[14,26,27] $\underline{\underline{A}}$ is the hyperfine interaction tensor related to the coupling between the electronic and the nuclear spins. We have omitted the interaction between the nuclear spins because they are negligible compared with other terms.

Equation (1) can be rewritten by neglecting the small splitting terms of the nuclear spins,

$$H = g_{\rm e}\beta_{\rm e}\hat{S}_{\rm z}B_{\rm z} + D\hat{S}_{\rm z}{}^2 + A_{\rm C}S_{\rm z}I_{\rm z}^{\rm C} + +A_{\rm N}S_{\rm z}I_{\rm z}^{\rm N}, \quad (2)$$

$$H' = (1/2)[A_{\rm C}(S_+I_-^{\rm C} + S_-I_+^{\rm C}) + A_{\rm N}(S_+I_-^{\rm N} + S_-I_+^{\rm N})], \quad (3)$$

where the hyperfine parameters $A_{\rm C}$ and $A_{\rm N}$ are assumed to be isotropic for simplicity, with $A_{\rm C}=150$ MHz[17] and $A_{\rm N}=3.05$ MHz.[14] As in Ref. [14], we only consider the electronic states $m_{\rm S}=0$ and -1, and employ the total spin quantum numbers $I_{\rm C}$ and $m_{I_{\rm C}}$ of the two ^{13}C nuclear spins as well as $m_{I_{\rm N}}$ of the ^{15}N nuclear spin as good quantum numbers. By the denotation $|m_{\rm S}, I_{\rm C}, m_{I_{\rm C}}, m_{I_{\rm N}}\rangle$, we may label $|-1,1,-1,-(1/2)\rangle = |1111\rangle$, $|-1,1,0,-(1/2)\rangle = |1101\rangle$, $|-1,0,0,-(1/2)\rangle = |1011\rangle$, $|-1,1,1,-(1/2)\rangle = |1001\rangle$ and so on, as show in Fig. 1.

Since H' is much smaller than H, we may solve the model using second-order perturbation as listed in Table 1,

Table 1. Solutions of Eqs. (2) and (3) by second-order perturbation theory, where the parameters are defined in the text.

levels	energy
$\|1111\rangle$	$D - g_{\rm e}\beta_{\rm e}B + A_{\rm C} + \frac{A_{\rm N}}{2}$
$\|1110\rangle$	$D - g_{\rm e}\beta_{\rm e}B + A_{\rm C} - \frac{A_{\rm N}}{2} + A_3^{\rm N}$
$\|1101\rangle$	$D - g_{\rm e}\beta_{\rm e}B + \frac{A_{\rm N}}{2} + A_2^{\rm C}$
$\|1100\rangle$	$D - g_{\rm e}\beta_{\rm e}B - \frac{A_{\rm N}}{2} + A_1^{\rm C} + A_1^{\rm N}$
$\|1011\rangle$	$D - g_{\rm e}\beta_{\rm e}B + \frac{A_{\rm N}}{2}$
$\|1010\rangle$	$D - g_{\rm e}\beta_{\rm e}B - \frac{A_{\rm N}}{2} + A_1^{\rm N}$
$\|1001\rangle$	$D - g_{\rm e}\beta_{\rm e}B - A_{\rm C} + \frac{A_{\rm N}}{2} + A_4^{\rm C}$
$\|1000\rangle$	$D - g_{\rm e}\beta_{\rm e}B - A_{\rm C} - \frac{A_{\rm N}}{2} + A_3^{\rm C} + A_2^{\rm N}$
$\|0000\rangle$ $\|0010\rangle$	0
$\|0011\rangle$	$-A_1^{\rm N}$
$\|0001\rangle$	$-A_2^{\rm N}$
$\|0110\rangle$	$-A_1^{\rm C}$
$\|0111\rangle$	$-A_2^{\rm C} - A_3^{\rm N}$
$\|0100\rangle$	$-A_3^{\rm C}$
$\|0101\rangle$	$-A_4^{\rm C} - A_1^{\rm N}$

where

$$A_1^{\rm C} = A_{\rm C}^2/\left(-g_{\rm e}\beta_{\rm e}B + D - \frac{A_{\rm N}}{2}\right),$$
$$A_2^{\rm C} = A_{\rm C}^2/\left(-g_{\rm e}\beta_{\rm e}B + D + \frac{A_{\rm N}}{2}\right),$$
$$A_3^{\rm C} = A_{\rm C}^2/\left(-g_{\rm e}\beta_{\rm e}B + D - A_{\rm C} - \frac{A_{\rm N}}{2}\right),$$
$$A_4^{\rm C} = A_{\rm C}^2/\left(-g_{\rm e}\beta_{\rm e}B + D - A_{\rm C} + \frac{A_{\rm N}}{2}\right),$$
$$A_1^{\rm N} = A_{\rm N}^2/(-2g_{\rm e}\beta_{\rm e}B + 2D - A_{\rm N}),$$
$$A_2^{\rm N} = A_{\rm N}^2/(-2g_{\rm e}\beta_{\rm e}B + 2D - 2A_{\rm C} - A_{\rm N}),$$
$$A_3^{\rm N} = A_{\rm N}^2/(-2g_{\rm e}\beta_{\rm e}B + 2D + 2A_{\rm C} - A_{\rm N}).$$

2.2. Cluster state preparation

For constructing the desired entanglement, we initialize the electron spin by a laser pulse, setting the system in $m_{\rm S}=0$ state. Then we implement the following steps of operations for the four-qubit cluster state in an NV centre:

(i) Transition from of $|0000\rangle$ into $|1000\rangle$ by a selective $2\pi/3$ microwave (MW) pulse, yielding $(1/2)|0000\rangle + (\sqrt{3}/2)|1000\rangle$. Although the system has been set in the state $m_{\rm S}=0$ during the initialization, the nuclear spin states are undetermined. As a result, if the system has not been in the state $|0000\rangle$, the subsequent MW excitation to $|1000\rangle$ does not work,

yielding no observable signal.[14] In this case, some additional steps are needed for accomplishment of this step;[15]

(ii) Transition to $(1/2)|0000\rangle + (1/2)|1100\rangle - (1/2)|1111\rangle + (1/2)|1011\rangle$ by selective radio frequency (RF) pulses;

(iii) Generation of $(1/2)|0000\rangle + (1/2)|1100\rangle - (1/2)|1111\rangle + (1/2)|0011\rangle$ using a selective MW π pulse on the transition from $|1011\rangle$ to $|0011\rangle$.

The operations above could be summarized as in Table 2.

$$
\begin{aligned}
&|0000\rangle \\
&\xrightarrow{A} \frac{1}{2}\Big|0000\rangle + \frac{\sqrt{3}}{2}\Big|1000\rangle \\
&\xrightarrow{B} \frac{1}{2}\Big|0000\rangle + \frac{\sqrt{3}}{2}\Big|1100\rangle \\
&\xrightarrow{C} \frac{1}{2}\Big|0000\rangle + \frac{\sqrt{3}}{2}\left(\frac{1}{\sqrt{3}}|1100\rangle + \sqrt{\frac{2}{3}}|1111\rangle\right) \\
&\xrightarrow{D} \frac{1}{2}\Big|0000\rangle + \frac{1}{2}\Big|1100\rangle - \frac{1}{2}\Big|1111\rangle + \frac{1}{2}\Big|1011\rangle \\
&\xrightarrow{E} \frac{1}{2}\Big|0000\rangle + \frac{1}{2}\Big|1100\rangle - \frac{1}{2}\Big|1111\rangle + \frac{1}{2}\Big|0011\rangle.
\end{aligned}
\tag{4}
$$

Assisted by an Hadamard (H) transformation $H = 1/\sqrt{2}(\sigma_x + \sigma_z)$ on the first (i.e., the electronic) and last (i.e., the ^{15}N nuclear) spins, the entangled state in the last step of Eq. (4) is actually equivalent to the four-qubit cluster state $|\phi_4\rangle = (1/4)\otimes_{a=1}^{4}(|0\rangle + |1\rangle\sigma_z^{(a+1)})$, with $\sigma_z^5 = 1$.

Table 2. Pulse sequences for generating a four-qubit cluster state with a single electron spin and three nuclear spins.

cluster state	frequency / transition	pulse length
$\|\phi_4\rangle$	laser (initialization into $m_S = 0$)	$3\mu s$
	MW/$\|0000\rangle \leftrightarrow \|1000\rangle$	$2\pi/3$
	RF/$\|1000\rangle \leftrightarrow \|1100\rangle$	π
	RF/$\|1100\rangle \leftrightarrow \|1111\rangle$	$2\arccos(\sqrt{1/3})$
	RF/$\|1111\rangle \leftrightarrow \|1011\rangle$	$3\pi/2$
	MW/$\|1011\rangle \leftrightarrow \|0011\rangle$	π

As the nuclear spins couple to the electron spin but without coupling with each other within our model, the straightforward way to an entanglement generation is to produce a star-shape entanglement with the electron spin in the centre. But as shown in the next section, such a chain-shaped cluster state in Eq. (4) is essential to the generation of a larger-sized nuclear-spin cluster state with minimum overhead, although some operations in Eq. (4) are with double-quantum process, implying very weak transition probabilities.

Another point is that the H gate is the unconditional gate with needing multiple pulses. This can be easily operated on electron spins. To accomplish the H gate efficiently on the nuclear spins, however, we should first swap the states between the electron spin and the ^{15}N nuclear spin, and then swap them again after the H gate has been performed on the electron spin. This corresponds to an H gate on the ^{15}N nuclear spin.

3. More NV centres entangled by brokerage

To be scalable, the four-qubit cluster states in different NV centres should be linked with each other to form a larger-sized cluster state. To this end, we implement entanglement operation (EO)[5] using the electron-spin qubits in separated NV centres acting as brokers (see Fig. 2). By emitting photons conditional on their respective electronic states, we may obtain entangled NV centres after detecting the emitted photon. This is because the 'which-path' information is erased with the photon passing through a beam splitter. So our detection of the photons maps the entanglement from the photonic states to the electron states of the NV centres.

The quantum information stored in the client qubits is fully insulated from the EO failure during our state fusion, because the chain-shape cluster state makes it possible that the EO failure event only disen-

tangles the electron spin from the original four-qubit entangled state. To continue the EO for the two separate NV centres, we only need to have the electron spins be entangled with the nuclear spins again, and then repeat the parity projection of the photons. Specifically, assume that we have already prepared four-qubit cluster states in several NV centres, respectively. The following strategy is for fusing two of these four-qubit cluster states:

1) The EO of two remote electron-spin qubits, respectively, in two NV centres by projecting the photonic entanglement;

2) In the successful case of the EO, implement X measurement on two electron-spin qubits in the basis states $|\pm\rangle \equiv |0\rangle \pm |1\rangle$. Then a six-nuclear-spin cluster state can be obtained simply by applying an H gate to one of our nuclear-spin qubits;

3) If the EO is unsuccessful, the edge of each original cluster state will be reduced with the electron spin being disentangled. The entanglement of the electron spin with the nuclear spins again can be carried out by preparing the electron-spin qubit in the state $|+\rangle \equiv |0\rangle + |1\rangle$, executing a controlled-phase flip (CPF) gate targeting on the electron spin conditional on the nearby ^{13}C nuclear spin state.

4) Perform step 1) again.

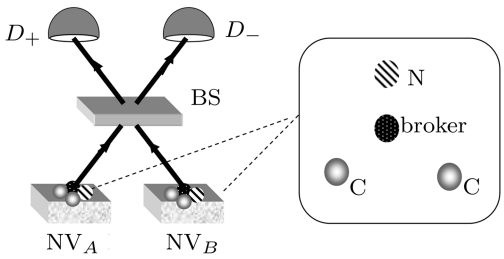


Fig. 2. Schematic of the apparatus to create entanglement between distant NVs using a beam splitter for erasing which-path information. The inset shows the qubits (depicted by the balls: electron spin (black with dots) and the ^{13}C (grey), ^{15}N (black with oblique lines) nuclear spins) in a single NV centre. The broker qubit (electron spin) helps to entangle nuclear spins within a single NV centre by hyperfine interaction, and in different NV centres by parity projection of photons.

To clarify further, we take an example in Fig. 3 for joining two four-qubit cluster states in separate NV centres into a six-qubit cluster state. Before

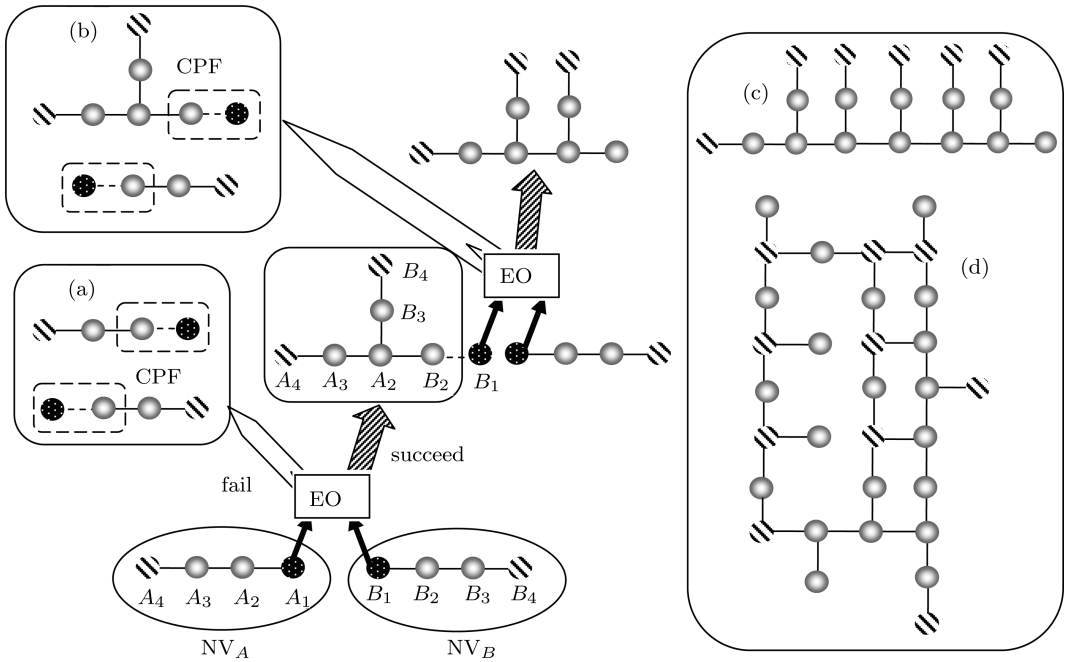


Fig. 3. Schematic for steps of entanglement generation (EO). The left-hand part demonstrates simple strategy for growing a multi-qubit cluster state, where the entanglement between qubits (depicted by balls) is denoted by the solid line between the balls. As described in the text, the EOs are made on the electron spins. If our strategy succeeds, a six-qubit (with nuclear spins only) cluster state is formed by two NV centres. Otherwise, the electron spins are removed from the original cluster states. The dash boxes mean the operations to take the electron spin back to the cluster state after the failure event. The right-hand box shows the possible cluster state configurations, where panel (c) shows a typical linear growth of the cluster state by the steps in the left-hand part; panel (d) is for a higher-dimensional network formed by additional EOs between ^{15}N nuclear spin qubit and electron spin qubit, with respect to panel (c).

implementing any fusion operation, we can write the states as

$$(|0\rangle_{A_1}+|1\rangle_{A_1}Z_{A_2})(|0\rangle_{B_1}+|1\rangle_{B_1}Z_{B_2}) \times (|0\rangle_{A_2}+|1\rangle_{A_2}Z_{A_3})(|0\rangle_{B_2}+|1\rangle_{B_2}Z_{B_3})|X\rangle, \quad (5)$$

where A_1 (B_1) means the electron spin in the NV centre A (B); A_k (B_k) with $k=2,3,4$ denote the nuclear spins in different NV centres; $|X\rangle$ represents the remaining qubits beyond the spins 1, 2 and 3; Z_{A_i} (Z_{B_i}) is the usual Pauli z operator acting on the qubit A_i (B_i). Following the steps in Ref. [5], prior to measurement, we make a σ_x operation on the qubit A_1 and wait for a photo detection event in either D_+ or D_-. This operation is probabilistic because only two projection operators $|10\rangle_{A_1B_1A_1B_1}\langle 10|\pm|01\rangle_{A_1B_1A_1B_1}\langle 01|$ are associated with our desired measurement result. If the EO succeeds, we have

$$(|1\rangle_{A_1}|0\rangle_{B_1}\pm|0\rangle_{A_1}|1\rangle_{B_1}Z_{A_2}Z_{B_2}) \times (|0\rangle_{A_2}+|1\rangle_{A_2}Z_{A_3}) \times (|0\rangle_{B_2}+|1\rangle_{B_2}Z_{B_3})|X\rangle, \quad (6)$$

in which the case with a minus sign needs an additional Z operation on one of the electron-spin qubits. Followed by X measurements on two electron-spin qubits in the basis $|\pm\rangle\equiv|0\rangle\pm|1\rangle$, the whole state turns to

$$(|0\rangle_{A_2}|0\rangle_{B_2}+|1\rangle_{A_2}|1\rangle_{B_2}Z_{A_3}Z_{B_3})|X\rangle, \quad (7)$$

which is actually the final 2D six-qubit cluster state in Fig. 3 after an H gate has been made on the qubit B_2.

So far, we have generated the cluster state without involving any brokers. It only consists of nuclear spins in entanglement. Taking the brokers back would be considered when we hope to further expand our cluster state. As shown in the left-hand part of Fig. 3, we take the electron spins back to the entanglement, and then execute the EO repeatedly, thereby yielding a larger-sized cluster state.

4. Feasibility

4.1. Estimate of experimental conditions

The first part of our scheme is devoted to the deterministic entanglement of the qubits in each NV centres. By comparison with a recent experiment[14] with entangling the electron spin with two ^{13}C nuclear spins, we have involved an additional qubit, i.e., the ^{15}N nuclear spin in the entanglement. As the coupling between the nuclear spins is negligible and omitted under our model, after we remove the electron spin from the four-qubit chain-shaped cluster state, the remaining state is really entangled by quantum correlation,[28] instead of by coupling.

From the achieved experimental results,[14] we may estimate for our case that the radio frequency (RF) driven one-quanta transition takes 1 μs and the two-quanta transition driven by the RF pulse takes about 10 μs. Since the operations by microwave (MW) pulses are usually faster than those by RF pulses, accomplishment of our cluster state in Eq. (4) should microseconds take tens of microseconds. In this sense, we may involve more nuclear spins in the entanglement if more ^{13}C nuclear spins can be found and manipulated near the vacancy, because the operational time for entanglement is much shorter than the decoherene time.[14]

The second part of our scheme is intrinsically probabilistic. Although similar to the atomic cases, our case here is quite different due to some peculiar characteristics of the NV centres. First, only linearly polarized photons can be produced in the laser excitation. Without the circularly polarized photons, we cannot generate entangled NV centres by coincident detection of photons, but only by a click on one of the detectors.[5,11] This would probably lead to some errors in the entanglement generation because of indistinguishability of the photon loss due to the detector inefficiency in the two-photon emission from the real case of the single-photon detection. A simple estimate of the problem could be as below. Assume the success rate of detecting a single photon to be p. We have success rate p^2 for detecting two photons simultaneously. With currently available single-photon detector, the efficiency of the detection in our case can be reduced to 10^{-6} due to photon loss and dark count. As a result, considering that the spontaneous emission time is 10 ns,[29] we have the time for entangling two NV centres by parity projection of the emitted photons to be

$$10\ \text{ns}/[(1-10^{-6})\times(10^{-3}-10^{-6})]\approx 10\ \mu\text{s},$$

in which we have supposed $p=10^{-3}$ by means of the confocal microscopes for collecting the emitted photons.[29] This entangling time is much shorter than the decoherence time of the electron spin in the NV centre, e.g., $T_1=380$ s,[30] and $T_2=350$ μs.[31] For an array of NV centres labeled as 1, 2, 3, ..., n, we may first entangle in parallel pairs 1 and 2, 3 and 4,

$\ldots, n-1$ and n, and then pairs 2 and 3, 4 and 5, $\ldots$, $n-2$ and $n-1$, simultaneously. So two steps could entangle all the NV centres, which take the time

$$10 \text{ ns}/[(1-10^{-6}) \times (10^{-3}-10^{-6})]^2 \approx 10 \text{ ms}.$$

Since we need to entangle the electron spins with the nuclear spins again after each EO, the total implementation time 10 ms would not affect the generated entanglement.

4.2. Experimental challenges

To effectively produce entanglement of NV centres by parity projection of photons, we require the 'which-path' information to be removed after the photons have passed through the beam splitter due to interference. However, experimental reports so far have shown that 96% of the emitted photons reside in broad photon sidebands to the resonant ZPL even in a cryogenic situation.[11] This implies that most emitted photons from the NV centres cannot effectively interfere in the beam splitter because 'nature' could distinguish their paths from the associated phonon sidebands. As a result, no entanglement can be achieved even if we receive a click at one of the detectors. A possible solution is to locate the NV centres in different optical cavities, e.g., in photonic crystal cavities, respectively, because the Purcell effect could enhance approximately 95% photonic emission into the ZPL.[22] Since the Purcell enhancement is proportional to Q/V with Q and V the Q factor and the size of the cavity, respectively, smaller cavity with higher Q factor is expected.

We notice that the diamond photonic chip[23] has been explored with coupling to planar photonic crystal waveguides. This would be a promising way to the achievement of the second part of our scheme. Other relevant cavity techniques include the thin and single-crystal diamond membranes,[32] whispering-gallery mode resonators[33] and photonic band-gap microcavities.[34] To have the remote NV centres coupled to those cavity (or resonator) modes, however, a number of technical difficulties have to be overcome.

4.3. Comparison with the previous idea

A good candidate system for producing cluster states should be the qubits that are well controlled and suffer negligible decoherence. This is because that the one-way QC strongly depends on the quality of the generated cluster state and it takes time to carry out the single-qubit operations in series for implementing the necessary operations. Nuclear spins meet the requirement very well because they are more robust to decoherence with respect to other qubit candidates such as electron spins or charge spins. In our proposal, since it is still pretty hard with current techniques to obtain large-scale entanglement, we consider here as many as possible nuclear spins to be entangled in a single NV centre. Compared with Ref. [24] with an electron spin and an nuclear spin involved in every NV, the present paper deals with three nuclear spins plus an electron spin in cluster state in each NV centre, which could carry out few-qubit QC at room temperature, for example, to accomplish a four-qubit refined Deutsch–Jozsa algorithm[35] and Grover search.[36] These are non-trivial quantum tasks. Moreover, the four-qubit (or three-qubit) cluster state in an NV centre considered in our scheme can allow the correction of errors induced by decoherence if the electron spin or one of the nuclear spins has been affected.[37] This is evidently helpful for implementing 1WQC in high fidelity by a number of NV centres in cluster states.

Another difference between our scheme and that in Ref. [24] is that the electron spins have been in entanglement with the nuclear spins before we produce entanglement of two NV centres. In contrast, the entanglement between two spatially separate NV centres in Ref. [24] were assumed to serve as the pre-preparation step, whose intrinsic probability of success was not counted into the procedure for entangling the nuclear spins by local hyperfine interaction with the electron spins. In fact, if we disentangle the electron spins from the nuclear spins after the local nuclear spins have been in cluster states, we may in principle follow the steps in Ref. [24], but generate a larger-scale cluster state than that in Ref. [24].

5. Conclusion

We have specifically designed the operations to generate large-scale cluster states with nuclear spins in different NV centres. The operations in a single NV centre are deterministic and almost in the reach by currently available techniques. This makes it possible with a single NV centre to carry out few-qubit quantum information processing at room temperature. In

contrast, the fusion of different NV centres is intrinsically probabilistic and only works well at low temperature. With reference to the probabilistic operations, the advantage of our scheme is the protection of the entanglement among the nuclear spins from the failure events in fusing two NV centres. As the electron spins act as the brokers in our scheme, using hyperfine interaction repeatedly, we may produce graph states in different sizes and with different configurations.

We have noticed some other new ideas for overcoming the difficulties in achieving entanglement of many NV centres. For example, using cantilevers,[20,21] an array of NV centres could be entangled by coupling to the same vibrational mode. Recent experiment has also demonstrated the sharp ZPL relevant to photonic emission from some nanodiamonds.[38] We would like to mention that no matter which method is employed to entangle separate NV centres, electron spins should take the role for readout or for remote-qubit entanglement and the nuclear spins are excellent qubits for storing quantum information. Therefore our scheme would be helpful for experimentally generating different graph states using NV centres.

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