Deterministically distinguishing a remote Bell state*

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It has been proven that, with a single copy provided, the four Bell states cannot be distinguished by local operations and classical communications (LOCC). Traditionally, a Bell basis projective measurement is needed to distinguish the four Bell states, which is usually carried out with a local interference between two particles. This paper presents an interesting protocol that allows two remote parties to distinguish four Bell states deterministically. We prove that our protocol of distinguishing remote Bell states is beyond LOCC.

Keywords: Bell basis measurement, LOCC, distinguishability

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1. Introduction

Entanglement plays a unique role for its amazing quantum phenomenon. It also provides an evidence for the nonlocality of the world.[1] The simplest entangled states are the two-qubit maximally entangled states, namely Bell states. The Bell states are not only used to study some fundamental problems of physics, such as Bell inequality,[2,3] but are also applied to quantum information processing (QIP).[4−7] For example, in quantum key distribution (QKD), the two particles in a Bell state may be distributed remotely. So it is interesting whether the two remote parties, called Alice and Bob, can distinguish the four Bell states. Now it is well known that one can easily discriminate the four Bell states by von Neumann measurement which can be performed with a local interference between the two particles. However, when the two particles are faraway separated, the distinguishing task with this way cannot be fulfilled successfully.

Local operation and classical communication (LOCC) is a usual candidate to discriminate the state encoded on distant particles. It has been studied deeply that the discrimination of entangled state with LOCC, no matter whether they are orthogonal or not, sometimes is impossible.[8−13] In the framework of LOCC, both Alice and Bob can operate on their own particles locally and broadcast their results through an authentically classical channel. Mathematically, Alice's and Bob's LOCC can be written in a product form of their local operations. Suppose that Alice's local operator is $U_{\rm A}$ and Bob's local operations can be depicted as $U_{\rm B}$ as an example, the global operator can be written as $U_{\rm A}\otimes U_{\rm B}$. When Alice and Bob share two particles in Bell states, there are quantum correlations between them. Though the whole state of the two particles is a pure state, the state of either particle can be obtained by tracing out the space of the other, which is totally mixed. This local measurement will destroy the superposition, and the quantum correlation between the particles may be broken. Essentially, such local measurements together with classical communication is a positive operator-valued measure (POVM) measurement. It may result in the loss of the nonlocal information about the relative phase between the particles. Therefore, only for a single copy being provided, it is impossible to distinguish the four Bell states with LOCC.

As mentioned above, the essence of the failure in distinguishing the Bell states with LOCC is due to the breakdown of interference between the two particles. Generally speaking, a Bell basis measurement needs an interference between two particles. In principle, the interference appears when an overlap of two wave packets occurs. For example, an optically partial Bell basis measurement needs two photons passing through a beam splitter (BS) at the same time. Naturally, it is interesting to ask whether the remote Alice and Bob, where local interference between their two particles is not available, can find a scheme to distinguish the Bell states. Intuitively, Alice and Bob must do something more than LOCC.

In this paper, we will show how the remote Alice and Bob contrive to complete their distinguishing task in an ingenious way. This paper is arranged as

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follows: First, we prove the faraway separated Alice and Bob can distinguish the four Bell states one from another deterministically. Secondly, we show that our scheme does not contradict the proposition that Alice and Bob cannot distinguish the four Bell states with LOCC. In our scheme, Alice and Bob must operate locally in their own labs. Nevertheless, they can successfully discriminate the four Bell states, which means this scheme beyond the framework of LOCC. Lastly, we give some discussions and conclusions.

2. Bell basis measurement by remote Alice and Bob

Entanglement is an important resource in QIP to realize the function that is impossible for classical information processing. In QIP, when a quantum logical operation is completed, the calculation result should be read out. This readout is a quantum measurement process to obtain the information encoded in a quantum state. If the information is encoded in a distant multi-qubit state, a simple scheme to read the information is that one measures each qubit locally and then obtain the information about the global state. However, if the global state is an entangled state, such as the Bell state of a two-qubit system, this scheme cannot help us obtain all the information about the state in most cases. So, when two remote parties, Alice and Bob, want to read out the information encoded on two particles which are randomly in a Bell state, they should find a scheme beyond LOCC.

Let us suppose that the distant Alice and Bob share two qubits in a Bell state, such as $|\Psi^+\rangle_{\rm AB} = \frac{1}{\sqrt{2}}(|HV\rangle_{\rm AB} + |VH\rangle_{\rm AB})$, where A and B denote the particles held by Alice and Bob, and H and V denote the horizontal and vertical polarisation respectively. Because the four Bell states are all equivalent in the framework of LOCC, our assumption does not loss any generality. Sharing $|\Psi^+\rangle_{\rm AB}$, as we will prove in the following, distant Alice and Bob will have the capacity beyond LOCC to distinguish the Bell states of particles C and D deterministically. Here C and D are randomly prepared on any of the four Bell states

$$
\begin{aligned}
|\Psi^+\rangle_{\rm CD} &= \frac{1}{\sqrt{2}}(|HV\rangle_{\rm CD} + |VH\rangle_{\rm CD}),\\
|\Psi^-\rangle_{\rm CD} &= \frac{1}{\sqrt{2}}(|HV\rangle_{\rm CD} - |VH\rangle_{\rm CD}),\\
|\Phi^+\rangle_{\rm CD} &= \frac{1}{\sqrt{2}}(|HH\rangle_{\rm CD} + |VV\rangle_{\rm CD}),\\
|\Phi^-\rangle_{\rm CD} &= \frac{1}{\sqrt{2}}(|HH\rangle_{\rm CD} - |VV\rangle_{\rm CD}). \qquad (1)
\end{aligned}
$$

Our protocol can be illustrated as follows: (i) Alice and Bob share particles A and B which are in state $|\Psi^+\rangle_{\rm AB}$. (ii) Particles C and D are randomly prepared in one of the four Bell states and distributed to Alice and Bob respectively. (iii) Alice measures particles A and C in the Bell basis. Likewise, Bob also performs a Bell basis measurement on particles B and D. They obtain their measurement results locally. (iv) They communicate their results through an authentically classical channel. (v) Alice and Bob can determine the state of the particles C and D with the results in Eq. (2).

Proof: Alice and Bob first share particles A and B which are in the state $|\Psi^+\rangle_{\rm AB} = \frac{1}{\sqrt{2}}(|HV\rangle_{\rm AB} + |VH\rangle_{\rm AB})$. Suppose that particles C and D are in one of the four Bell states. Then the whole state of particles A, B, C, and D is in one of the following four states: $|\Psi^+\rangle_{\rm AB}\otimes|\Psi^+\rangle_{\rm CD}$, $|\Psi^+\rangle_{\rm AB}\otimes|\Psi^-\rangle_{\rm CD}$, $|\Psi^+\rangle_{\rm AB}\otimes|\Phi^+\rangle_{\rm CD}$ or $|\Psi^+\rangle_{\rm AB}\otimes|\Phi^-\rangle_{\rm CD}$. Since Alice holds particles A and C while Bob holds particles B and D, these four states can be recombined mathematically as follows,

$$
\begin{aligned}
|\Psi^+\rangle_{\rm AB}\otimes|\Psi^+\rangle_{\rm CD} &= \frac{1}{2}[|\Phi^+\rangle|\Phi^+\rangle - |\Phi^-\rangle|\Phi^-\rangle + |\Psi^+\rangle|\Psi^+\rangle - |\Psi^-\rangle|\Psi^-\rangle]_{\rm ACBD},\\
|\Psi^+\rangle_{\rm AB}\otimes|\Psi^-\rangle_{\rm CD} &= \frac{1}{2}[|\Phi^-\rangle|\Phi^+\rangle - |\Phi^+\rangle|\Phi^-\rangle + |\Psi^+\rangle|\Psi^-\rangle - |\Psi^-\rangle|\Psi^+\rangle]_{\rm ACBD},\\
|\Psi^+\rangle_{\rm AB}\otimes|\Phi^+\rangle_{\rm CD} &= \frac{1}{2}[|\Phi^+\rangle|\Psi^+\rangle - |\Phi^-\rangle|\Psi^-\rangle + |\Psi^+\rangle|\Phi^+\rangle - |\Psi^-\rangle|\Phi^-\rangle]_{\rm ACBD},\\
|\Psi^+\rangle_{\rm AB}\otimes|\Phi^-\rangle_{\rm CD} &= \frac{1}{2}[|\Phi^-\rangle|\Psi^+\rangle - |\Phi^+\rangle|\Psi^-\rangle + |\Psi^+\rangle|\Phi^-\rangle - |\Psi^-\rangle|\Phi^+\rangle]_{\rm ACBD}. \qquad (2)
\end{aligned}
$$

Now, Alice and Bob measure their own particles locally in the Bell bases and there are 16 possible measurement outcomes. From Eq. (2), we can see that the Bell state of C and D is uniquely corresponding to the measurement outcomes of Alice's and Bob's Bell bases measurement outcomes. That is, with their local measurement results and classical communications, Alice and Bob can deduce the originally sole state on particles C and D. In this way, we have proven that an unknown Bell state encoded on two remote particles C and D can be discriminated with unity probability.

3. Distinguishing remote Bell states beyond LOCC

It is well known that remote Alice and Bob cannot discriminate the four Bell states by using LOCC when a single copy is provided. In our scheme, however, they can deterministically distinguish the four Bell states with a single copy. So, our scheme is beyond LOCC. One thing should be noticed that, in our scheme, Alice and Bob have shared another maximally entangled state beforehand, which can provide a non-classical correlation between them. Obviously, it is such non-classical correlation which makes our scheme surpass LOCC that is not restricted by LOCC. Thus, our scheme of distinguishing the remote Bell states does not contradict with the framework of LOCC.

In LOCC, only one qubit quantum channel is provided by the maximally entangled EPR pair. And there are two maximally entangled EPR pairs which can provide two qubits quantum channel in our protocol. Thus, a proposition can be concluded and interpreted subsequently that the distant Alice and Bob should share a quantum channel whose quantum capacities at least two bits if they want to distinguish among the four Bell states encoded on separated entangled particles.

It has been proven that the maximally entangled EPR can provide a quantum channel whose quantum information capacity is one bit.[14] And the quantum capacity of a noise quantum channel is numerically equivalent to the maximally coherent information that can be transferred through it, which can be represented as[15]

$$Q(N) = \max_{\rho_{\rm in}} \{H(B) - H(E)\}. \tag{3}$$

Here $\rho_{\rm in}$ is the density matrix of the input quantum state, $H(B)$ is von Neumann entropy for the density matrix of the output state, and $H(E)$ is von Neumann entropy lost in the noise channel. If an ideal quantum channel is used and no eavesdropper appears during the transferring process, $H(E) = 0$. That is, the output state is the same as the input state. Considering that distinguishing some state encoded on separated entanglement EPR pairs between remote partners is the key step in Lo–Chau protocol.[14] And every preparing-and-measuring QKD protocol has its counterpart in entanglement-base QKD protocols. Thus, the ability for the remote Alice and Bob to distinguish among the separated Bell states is determined by the capacity of the quantum channel. In traditional LOCC, a one-qubit quantum channel is provided by the entangled EPR pair, then the maximal quantum information they can exchange at the same time is one bit. It is then an easy task if they are asked to distinguish between any two Bell states which correspond to one bit quantum information. Nevertheless, when the entangled particles are randomly encoded on any of the four Bell states, Alice and Bob require a quantum channel with two-bit capacity to distinguish them. In our protocol, though particles A and B are randomly coded on any Bell state and they do not interact directly with particles C and D, they provide another quantum channel which permits one extra bit quantum information to transfer between Alice and Bob, and makes the four Bell states distinguishable between them. Then our protocol is not within the framework of LOCC.

4. Discussion and conclusion

In this paper, we have presented a protocol for remote Alice and Bob to distinguish the four Bell states deterministically. Traditionally, one needs to perform a Bell basis measurement to distinguish the four Bell states from each other, which requires a local interference between two particles that cannot be utilised by the remote Alice and Bob. In our protocol, remote Alice and Bob have shared a maximally entangled EPR pair A and B before their distinguishing process, which provides an extra quantum channel besides the correlation between particles C and D. Such distinguishing of remote Bell states may be applied in the future QIP,[16–19] such as quantum computation where an entangled state distributed and processed by remote quantum processers needs to be read out remotely.

References

[1] Einstein A, Podolsky B and Rosen N 1935 *Phys. Rev.* **47** 777

[2] Bell J S 1965 *Physics* **1** 195

[3] Clauser J F, Horne M A, Shimony A and Holt R A 1969 *Phys. Rev. Lett.* **23** 880

[4] Ekert A K 1991 *Phys. Rev. Lett.* **67** 661

[5] Bennett C H, Brassard G and Mermin N D 1992 *Phys. Rev. Lett.* **68** 557
[6] Yang W X and Gong Z X 2007 *J. Phys.* B **40** 1245
[7] Wu Q, Fang M F, Cai J W and Hu Y H 2009 *Chin. Phys.* B **18** 5336
[8] Walgate J, Short A J, Hardy L and Vedral V 2000 *Phys. Rev. Lett.* **85** 4972
[9] Walgate J and Hardy L 2002 *Phys. Rev. Lett.* **89** 147901
[10] Ghosh S, Kar G, Roy A, Sen (De) A and Sen U 2001 *Phys. Rev. Lett.* **87** 277902
[11] Horodecki M, Sen(De) A, Sen U and Horodecki K 2003 *Phys. Rev. Lett.* **90** 047902-1
[12] Fan H 2004 *Phys. Rev. Lett.* **92** 177905-1
[13] Hayashi M, Markham D, Murao M, Owari M and Virmani S 2006 *Phys. Rev. Lett.* **96** 040501
[14] Lo H K and Chau H F 1999 *Science* **283** 2050
[15] Schumacher B and Nielsen M A 1996 *Phys. Rev.* A **54** 2629
[16] Deng L, Chen A X, Chen D H and Huang K L 2008 *Chin. Phys.* B **17** 2514
[17] Zhang X Z, Gong W G, Tan Y G, Ren Z Z and Guo X T 2009 *Chin. Phys.* B **18** 2143
[18] Fang M F and Xiao X 2009 *Chin. Phys.* B **18** 4695
[19] Liu J, Wang Q, Kuang L M and Zeng H S 2010 *Chin. Phys.* B **19** 030313