Einstein–Podolsky–Rosen entanglement in time-dependent broadband pumping frequency non-degenerate optical parametric amplifier*

Zhao Chao-Ying(赵超樱)[a)b)†] and Tan Wei-Han(谭维翰)[c)]

[a)] *The College of Science, Hangzhou Dianzi University, Hangzhou* 310018, *China*
[b)] *State Key Laboratory of Quantum Optics and Quantum Optics Devices, Institute of Opto-Electronics, Shanxi University, Taiyuan* 030006, *China*
[c)] *Department of Physics, Shanghai University, Shanghai* 200444, *China*

(Received 7 April 2010; revised manuscript received 14 June 2010)

This paper investigates quantum fluctuations characteristic of time-dependent broadband pumping frequency non-degenerate optical parametric amplifier for below and above threshold regions. It finds that a high squeezing and entanglement can be achieved.

Keywords: time dependent broadband pumping, entanglement, frequency non-degenerate optical parametric amplifier

PACS: 03.65.Ud, 42.50.Lc, 42.65.Yj **DOI:** 10.1088/1674-1056/20/1/010305

1. Introduction

Quantum information experiments usually adopt external coherent field driving mode-locked single-model laser or ultra short pulse laser to output the ideal narrow-band monochromatic light and produce large squeezed quadrature component.[1−3] The entanglement states of light field stem from their squeezing. Up to now, the interest in continuous variable entanglement has been aroused by successful experiments on multi-channel quantum cryptographic communication,[4] quantum holographic teleportation,[5] quantum dense coding.[6,7] The more important thing is that a high squeezing cannot be expected to last for a long time with constant pump.[8−15] Our works aim at improving the degree of squeezing and entanglement, we have investigated the nonlinear effects making influences on quantum fluctuations characteristic of frequency degenerate optical parametric amplifier (FDOPA), including time dependent, phase mismatched, loss, pump depletion, nonlinear driven.[16−19] We also find that single frequency periodic pumping can achieve a high entanglement for frequency non-degenerate optical parametric amplifier (FNOPA).[20] Considering the actual pump light has certain width, in this paper, we investigate quantum fluctuations characteristic of time dependent multi-frequency broadband pumping FNOPA, in which broadband pumping is not generated by single frequency modulation, but generated by optical source itself. We find that a higher squeezing and entanglement than periodic pumping can be achieved.

2. Hamiltonian and equations of motion

The total Hamiltonian for FDOPA is given by[20]

$$H = H_0 + V + W,$$

where $H_0 = \hbar\omega_0 a_0^\dagger a_0 + \hbar\omega_1 a_1^\dagger a_1 + \hbar\omega_2 a_2^\dagger a_2 + \mathrm{i}\hbar(E\,\mathrm{e}^{-\mathrm{i}\omega_L t}a_0^\dagger - E^*\,\mathrm{e}^{\mathrm{i}\omega_L t}a_0) + \sum_{j,i=1,2}\hbar\omega_j B_{ji}^\dagger B_{ji}$,

$$V = \hbar\sum k_j(B_{j1}a_1^\dagger + B_{j2}a_2^\dagger) + \hbar\sum k_j^*(B_{j1}^\dagger a_1 + B_{j2}^\dagger a_2),$$

$$W = \frac{\mathrm{i}\hbar}{2}(2\varepsilon a_1^\dagger a_2^\dagger - 2\varepsilon^* a_1 a_2), \tag{1}$$

where $a_0^\dagger(a_0)$, $a_1^\dagger(a_1)$, $a_2^\dagger(a_2)$ denote the creation and annihilation operators at frequencies ω_0, ω_1, ω_2 referring to the pump, signal and idler field, respectively. We assume $\hbar\omega_1 = \hbar\omega_2 = \hbar\omega_0/2$, H_0 describes the Hamiltonian of pump, signal, idler light and heat bath,

*Project supported by the State Key Laboratory of Quantum Optics and Quantum Optics Devices, Shanxi University, China (Grant No. 200904).
†Corresponding author. E-mail: zchy49@sohu.com
© 2011 Chinese Physical Society and IOP Publishing Ltd

E is the amplitude of coherent driving field with frequency $\omega_{\rm L} \approx \omega_0$, V indicates the interaction Hamiltonian of signal, idler field and heat bath, W represents the interaction Hamiltonian between pump field (included in gain factor $\varepsilon = \chi\alpha_0$) and signal, idler field.

Setting $b_1 = (a_1 + a_2)/\sqrt{2}$, and $b_2 = (a_1 - a_2)/\sqrt{2}$, we can write the time dependent Fokker–Planck equation for FDOPA as[20]

$$\begin{aligned}\frac{\partial p_1}{\partial t} &= \left[k\left(\frac{\partial}{\partial b_1}b_1 + \frac{\partial}{\partial b_1^*}b_1^*\right) - \left(\varepsilon b_1^*\frac{\partial}{\partial b_1} + \varepsilon^* b_1\frac{\partial}{\partial b_1^*}\right)\right.\\ &\quad \left. + \frac{1}{2}\left(\varepsilon\frac{\partial^2}{\partial b_1^2} + \varepsilon^*\frac{\partial^2}{\partial b_1^{*2}}\right)\right]p_1,\\ \frac{\partial p_2}{\partial t} &= \left[k\left(\frac{\partial}{\partial b_2}b_2 + \frac{\partial}{\partial b_2^*}b_2^*\right) + \left(\varepsilon b_2^*\frac{\partial}{\partial b_2} + \varepsilon^* b_2\frac{\partial}{\partial b_2^*}\right)\right.\\ &\quad \left. - \frac{1}{2}\left(\varepsilon\frac{\partial^2}{\partial b_2^2} + \varepsilon^*\frac{\partial^2}{\partial b_2^{*2}}\right)\right]p_2. \qquad (2)\end{aligned}$$

Considering the broadband pumping effect, in the positive P representation, substituting the operators b_1 and b_2 by the c numbers α_1 and α_2, we find that the interactions between three cavity modes α_0, α_1, α_2 obey the following equations:[6,20]

$$\begin{aligned}\frac{{\rm d}\alpha_1}{{\rm d}\tau} &= -\alpha_1 + \frac{\chi}{\gamma_1}\alpha_0\alpha_2^*,\\ \frac{{\rm d}\alpha_2}{{\rm d}\tau} &= -\alpha_2 - \frac{\chi}{\gamma_1}\alpha_0\alpha_1^*,\\ \frac{{\rm d}\alpha_0}{{\rm d}\tau} &= -\gamma_{\rm r}\alpha_0 + \frac{\bar{\varepsilon}'}{\gamma_1} - \frac{\chi}{2\gamma_1}(\alpha_1^2 - \alpha_2^2), \qquad (3)\end{aligned}$$

where τ is the time, χ is the coupling constant, γ_0 and γ_1 are the damping rates, $\gamma_{\rm r} = \gamma_0/\gamma_1$ is the decay ratio, $\bar{\varepsilon}'/\gamma_1 = \bar{\varepsilon}/\gamma_1 \times F$, $\bar{\varepsilon}/\gamma_1 = \mu\sqrt{\gamma_{\rm r}/2\eta}$, μ is pump factor, $\eta = \chi^2/2\gamma_1\gamma_0$,[19,20] $F = \{\cos[\omega_0\tau]\exp[-(\Delta\omega(f-3)/2)^2]\}^2$ is Gaussian pump factor, ω_0 is the central frequency for a broadband light beam, $\Delta\omega$ is the spectrum width, $f = Mod[\tau, 6]$ depends on the pulse spacing and pulse number. In fact, the rubidium glass broadband laser source includes many different frequencies, each frequency component can be expressed by $\cos[(\omega_0 + \delta\omega)\tau]\exp[-(\delta\omega/\Delta\omega)^2]{\rm d}\delta\omega/(2\sqrt{\pi}\Delta\omega)$, and attenuation of amplitude is in the form of Gaussian distribution $\exp[-(\delta\omega/\Delta\omega)^2]{\rm d}\delta\omega/(2\sqrt{\pi}\Delta\omega)$, $\delta\omega$ integrates from $-\infty$ to ∞, the spectral distribution of rubidium glass broadband laser source can be approximated to Gaussian distribution, the actual spectral width is 100–200 Å (1 Å = 0.1 nm), wave length is 10000 Å, the influence of broadband is faint. In the following calculation, we increase the spectral width, take $\omega_0 = 0$, $\Delta\omega = 2$.

3. Squeezing characteristic for FDOPA

Assuming α_0 is real and writing the complex parameters α_1, α_2 in the real and imaginary parts $\alpha_1 \to \alpha_1 + {\rm i}\tilde{\alpha}_1$, $\alpha_2 \to \alpha_2 + {\rm i}\tilde{\alpha}_2$, integrating Eq. (3) numerically, we obtain $\tilde{\alpha}_1(\tau)$,[17–20] and can write the variances of the quantum fluctuation of FDOPA as

$$\left\langle(\Delta y_1)^2\right\rangle = \frac{1}{4} + (\tilde{A}_1(\tau) - \tilde{A}_{10}\tilde{\alpha}_1^2(\tau)), \qquad (4)$$

where $\tilde{A}_1(\tau) = \tilde{\alpha}_1^2(\tau)(\tilde{A}_{10} + 1/4) - 1/4 + \tilde{\alpha}_1^2(\tau)k/2\int_{\tau-\tau_0}^{\tau}\tilde{\alpha}_1^{-2}(t){\rm d}t$, $\tilde{A}_{10} = \mu/4(1+\mu)$. The squeezing calculation is shown in Fig. 1. Here $\eta = 1/1000$, $\gamma_{\rm r} = 10$, $\tau_0 = 2$, $\mu = 2$.

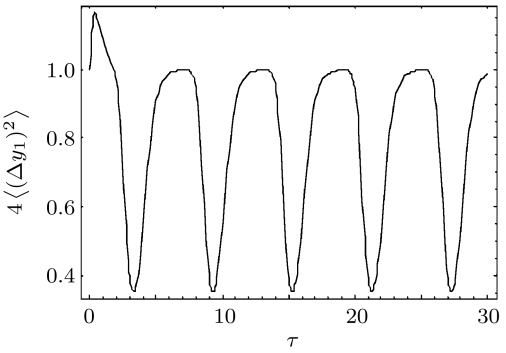


Fig. 1. Squeezing $4\left\langle(\Delta y_1)^2\right\rangle$ versus τ.

Comparing it with periodic pumping,[20] we find that the broadband pumping results in platform broadening, namely, squeezing can last for a longer time. The maximum squeezing is close to the linear theory $1/(1+\mu)$.

4. Entanglement characteristic of FNOPA

In this section, we will study frequency non-degenerate case, we have $\omega_1 = \omega_0/2 + \Delta\omega$, $\omega_2 = \omega_0/2 - \Delta\omega$, where $\Delta\omega$ is frequency detuning. The variance $\bar{V}_{1A}$ for the FNOPA is[20–22]

$$\bar{V}_{1A} = \frac{V_1}{1 - \sin^2[2\varphi]\dfrac{\left\langle\Delta X_1\Delta Y_2\right\rangle^2}{\left\langle(\Delta X_1)^2\right\rangle^2}}, \qquad (5)$$

where $\varphi = \Delta\omega t$ denotes phase mismatched, $\left\langle(\Delta X_1)^2\right\rangle$, $\left\langle(\Delta Y_2)^2\right\rangle$, $\left\langle\Delta X_1\Delta Y_2\right\rangle^2$ are the quantum fluctuations for the FDOPA, and we have $\left\langle(\Delta X_1)^2\right\rangle = \left\langle(\Delta Y_2)^2\right\rangle$, the variance for the FDOPA is $V_1 = \left\langle(\Delta X_1)^2\right\rangle -$

$\langle \Delta X_1 \Delta Y_2 \rangle^2 / \langle (\Delta Y_2)^2 \rangle$,[20] where we can see that the variance $\bar{V}_{1A}$ is always greater than the variance V_1.

In the following, we investigate phase mismatched effect making an influence on entanglement: (a) $\mu = 0.5$ for below threshold; (b) $\mu = 1$ for at threshold; (c) $\mu = 2$ for above threshold; (d) $\mu = 4$ for far above threshold.

Figure 2 depicts the minimum variances $\bar{V}_{1A}$ as a function of time τ without phase mismatched effect, with the increase of pump μ, the minimum variances $\bar{V}_{1A}$ decrease from 0.958, 0.87, 0.66 to 0.40, the peak values increase from 1.02 to 1.60, the general feature of curves is unchanged.

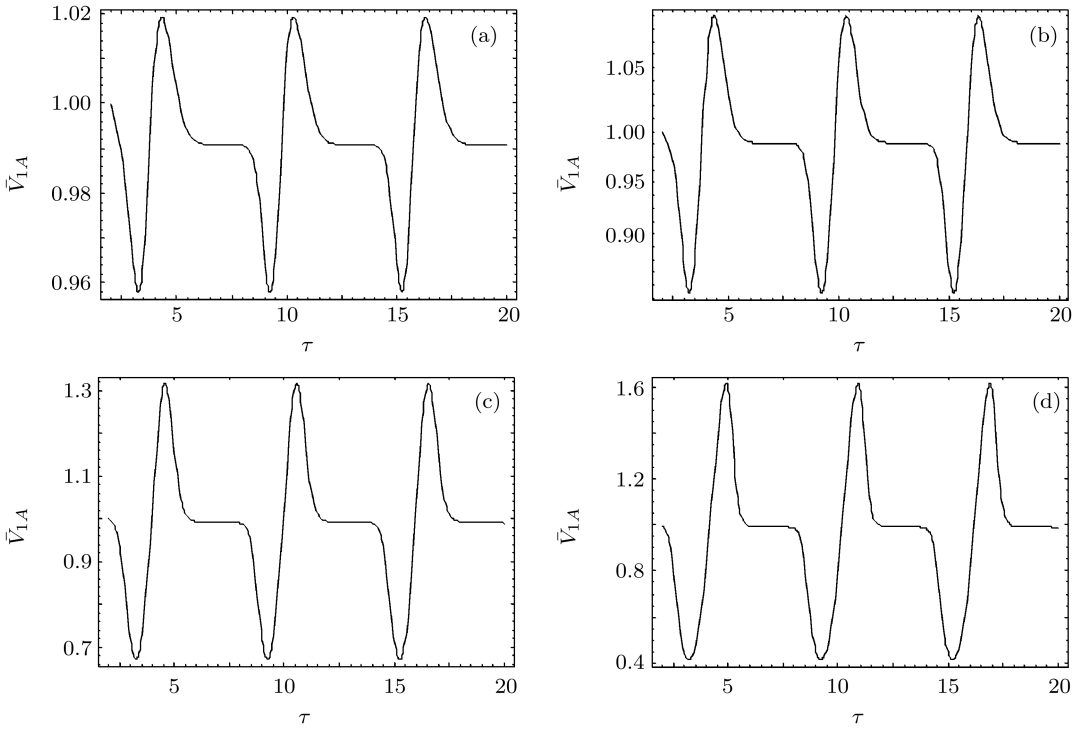
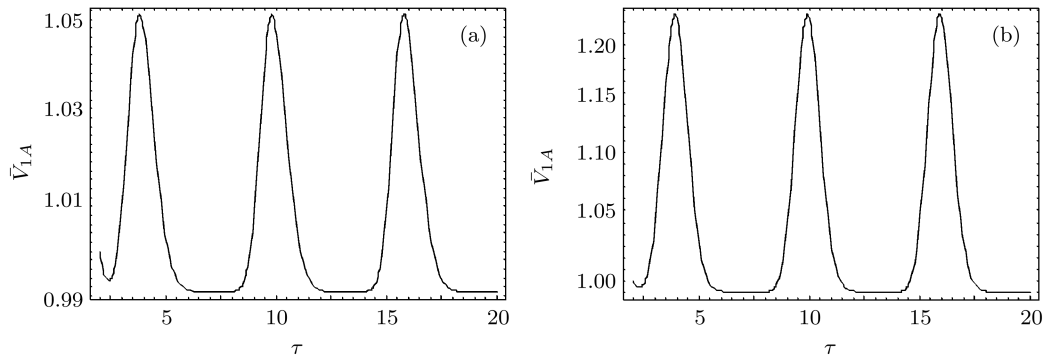


Fig. 2. Minimum variances $\bar{V}_{1A}$ versus time τ with $\varphi = 0$: (a) $\mu = 0.5$; (b) $\mu = 1$; (c) $\mu = 2$; (d) $\mu = 4$.

In the presence of phase mismatched effect, comparing Fig. 3 with Fig. 2, with the increase of pump μ, we find that the minimum values decrease from 0.99, 0.995, 0.998 to 0.80, the peak values increase from 1.064 to 3.120, entanglement disappears. The shapes of curves change a lot. Comparing Figs. 2 and 3 with periodic pumping,[20] we find that the broadband pumping results in platform broadening and pulse numbers reduce per unit time, namely, entanglement can last for a longer time when small φ is adopted.



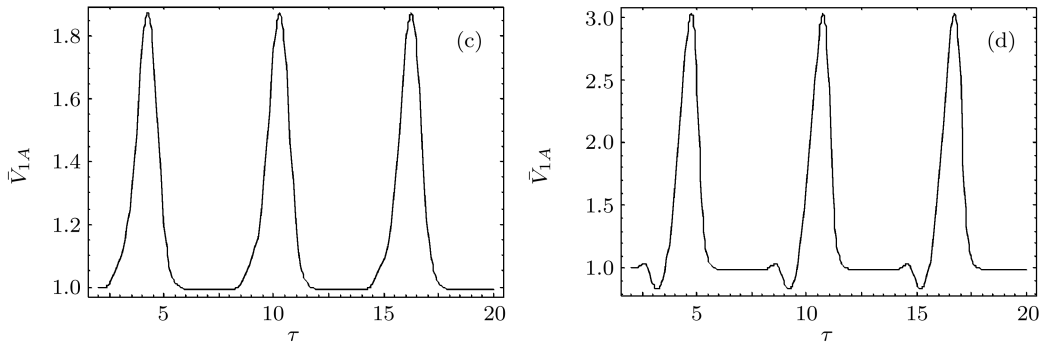


Fig. 3. Minimum variance $\bar{V}_{1A}$ versus time τ with $\varphi = 0.4$: (a) $\mu = 0.5$; (b) $\mu = 1$; (c) $\mu = 2$; (d) $\mu = 4$.

5. Conclusions

We investigate the quantum fluctuation characteristic of time-dependent Gaussian pumping FNOPA for below and above the threshold regions. By analytical and numerical calculations, we find that both the squeezing and entanglement depend on spectral width $\Delta\omega$, pump μ, phase-mismatched factor φ and loss $\gamma_{\rm r}$. The maximum squeezing $4\left\langle(\Delta y_1)^2\right\rangle$ approaches the result $1/(1+\mu)$ from the linear theory, due to accumulation effect of the quantum fluctuations, the envelope of $\bar{V}_{1A}$ curve decreases with the increase of pump μ, the minimum variance attains to 0.40. The broadband pumping can achieve a high squeezing and entanglement.

References

[1] Stappaerts E A, Komine H and Long Jr W H 1980 *Opt. Lett.* **5** 4

[2] Hétet G, Glöckl O, Pilypas K A, Harb C C, Buchler B C, Bachor H A and Lam P K 2007 *J. Phys.* B: *At. Mol. Opt. Phys.* **40** 221

[3] Golubeva T, Ivanov D and Golubev Yu 2008 *Phys. Rev.* A **77** 052316

[4] Braunstein S L and Loock P V 2005 *Rev. Mod. Phys.* **77** 513

[5] Furusawa A, Sørensen J L, Braunstein S L, Fuchs C A, Kimble H J and Polzik E S 1998 *Science* **282** 706

[6] Li X Y, Pan Q, Jing J T, Zhang J, Xie C D and Peng K C 2002 *Phys. Rev. Lett.* **88** 047904

[7] Zhang J, Xie C D and Peng K C 2003 *Europhys. Lett.* **61** 579

[8] Hirano T, Kotani K, Ishibashi T, Okude S and Kuwamoto T 2005 *Opt. Lett.* **30** 1722

[9] Takeno Y, Yukawa M, Yonezawa H and Furusawa A 2007 *Opt. Express* **15** 4321

[10] Vahlbruch H, Mehmet M, Chelkowski S, Hage B, Franzen A, Lastzka N, Goβler S, Danzmann K and Schnabel R 2008 *Phys. Rev. Lett.* **100** 033602

[11] Kim C and Kumar P 1994 *Phys. Rev. Lett.* **73** 1605

[12] Werner M J, Raymer M G, Beck M and Drummond P D 1995 *Phys. Rev.* A **52** 4202

[13] Yu C X, Haus H A and Ippen E P 2001 *Opt. Lett.* **26** 669

[14] Wenger J, Tualle-Brouri R and Grangier P 2004 *Opt. Lett.* **29** 1267

[15] Eto Y, Tajima T, Zhang Y and Hirano T 2007 *Opt. Lett.* **32** 1698

[16] Zhao C Y and Tan W H 2006 *J. Mod. Opt.* **53** 1965

[17] Zhao C Y and Tan W H 2006 *J.Opt. Soc. Am.* B **23** 2174

[18] Zhao C Y and Tan W H 2007 *J. Mod. Opt.* **54** 97

[19] Zhao C Y and Tan W H 2007 *Chin. Phys.* **16** 644

[20] Zhao C Y and Tan W H 2009 *Chin. Phys.* B **18** 4143

[21] Walls D F and Milburn G J 1994 *Quantum Optics* (2nd ed.) (New York: Springer)

[22] Reid M D and Drummond P D 1989 *Phys. Rev.* A **40** 4493