# Time-delay feedback control in a delayed dynamical chaos system and its applications*

Ye Zhi-Yong(叶志勇), Yang Guang(杨 珖)†, and Deng Cun-Bing(邓存兵)

*School of Mathematics and Physics, Chongqing University of Technology, Chongqing* 400054, *China*

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The feedback control of a delayed dynamical system, which also includes various chaotic systems with time delays, is investigated. On the basis of stability analysis of a nonautonomous system with delays, some simple yet less conservative criteria are obtained for feedback control in a delayed dynamical system. Finally, the theoretical result is applied to a typical class of chaotic Lorenz system and Chua circuit with delays. Numerical simulations are also given to verify the theoretical results.

**Keywords:** chaotic system, feedback control, delayed dynamical system, butterfly effect

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## 1. Introduction

Chaos phenomena are widespread in the real world. Apparently, chaos phenomena are irregular, but their regular innate character has been gradually revealed. Because of the complexity of the chaos phenomenon, people always expect to find some way to control the unstable periodic track of the chaos system so that it fits an expected track of stability. By understanding the nature of chaos and the value of its wide-ranging applications in various fields, control of chaos and synchronization research has rapidly developed.[1,2] Recently, control of chaos has been the subject of a wide variety of studies within science and technology communities.[3−5] Feedback control is one of the most interesting forms of chaotic control and a wide variety of approaches have been proposed for it, including: linear and nonlinear feedback controls, time-delay feedback control and many others.[6−8] However, to the best of our knowledge, the aforementioned methods and many other existing control schemes mainly address the control of systems without delays. This feature obviously restricts the complexity of a large class of delayed chaotic systems.

Systems with time delays are fairly ubiquitous in nature. The time delays are usually caused by finite signal transmission speeds and memory effects.[9−11] While many chaotic models are developed in physics, it is believed that those found in economics and biology are formulated in terms of coupled nonlinear oscillators. Time delay plays an important role in controlling such chaotic oscillators. However, literature dealing with the chaotic control of coupled systems with time delays appears to be scarce due to the difficulties associated with the timely and effective control of delayed dynamical systems. Moreover, most of the developed methods are valid only for those chaotic systems with autonomous feedback control. However, in practical situations, the control results of some systems cannot be exactly known and the effect of these uncertainties may actually destroy control results.[12−14] Therefore, it is essential to investigate the control of chaotic systems and in a way that is different from that of previous investigations. In this paper, we shall focus on a controller with delays. To this end, the theoretical results will be applied to a typical class of chaotic Lorenz system and Chua circuit with delays. Numerical simulations are also presented to demonstrate the effectiveness of the approaches.

## 2. Design of controller

In this paper, we consider an n-dimensional delayed dynamical system, which is described by the following differential equations with delays, each of which being a delayed dynamical equation:

$$\dot{x} = \boldsymbol{A}x(t) + f(x(t)) + \boldsymbol{D}x(t-\tau). \qquad (1)$$

Here, $\boldsymbol{x}_i(t) = (x_1(t), x_2(t), \ldots, x_n(t))^{\mathrm{T}} \in \boldsymbol{R}^n$ is the state of the system, $R$ denotes the set of real num-

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†Corresponding author. E-mail: yangguang@cqut.edu.cn

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bers, $\boldsymbol{R}^n = \underbrace{R \times R \times \cdots \times R}_{n}$. For $u \in \boldsymbol{R}^n$, $u^{\mathrm{T}}$ denotes its transpose. The norm of a vector $\boldsymbol{u}$ is defined as $||u|| = (u^{\mathrm{T}}u)^{1/2}$. $\boldsymbol{R}^{n\times n}$ are $n \times n$ real matrices. $f : \boldsymbol{R} \times \boldsymbol{R}^n \longrightarrow \boldsymbol{R}^n$ is a continuously differentiable function with respect to the variable. $\tau$ is the delay and the constant matrix $\boldsymbol{D} = (d_{ij})_{n\times n}$ is the influencing factor of the delayed variable. The initial condition is $x_t(0) = \phi(s)$, where $\tau \in (0, +\infty)$, $s \in [-\tau, 0]$, and $\phi(s) \in C([-\tau, 0], \boldsymbol{R}^n)$. The system with a feedback controller can be constructed as follows:

$$\begin{aligned}\dot{x}(t) = {} & \boldsymbol{A}x(t) + f(x(t)) + \boldsymbol{D}x(t-\tau) \\ & - B(t)x(t-\tau) + u(t), \end{aligned} \tag{2}$$

where $x, u \in \boldsymbol{R}^n$, and $-B(t)x(t-\tau) + u(t)$ is the nonautonomous feedback controller, $B(t) = (b_{ij}(t))_{n\times n}$, $\mid d_{ij} - b_{ij}(t) \mid \leq m_{ij}$, $\boldsymbol{M}$ is a reversible matrix. The feedback controller $-B(t)x(t-\tau) + \boldsymbol{u}(t)$ satisfies $u(t) = Kx(t) + \overline{\theta}\partial V^{\mathrm{T}}/\partial x$.

**Remark 2.1** Many famous chaotic systems can be formulated in terms of system (1), such as a typical class of a chaotic Lorenz system and a Chua circuit with delays.

**Statement 2.1** (Zhou and Chen,[15] Thomas and Morari[16]) Let $\boldsymbol{Q}$ and $\boldsymbol{R}$ be two symmetric matrices, then

$$\begin{pmatrix} \boldsymbol{Q} & S \\ S^{\mathrm{T}} & \boldsymbol{R} \end{pmatrix} > 0$$

is equivalent to $\boldsymbol{R} > 0$, and $\boldsymbol{Q} - S\boldsymbol{R}^{-1}S^{\mathrm{T}} > 0$.

By the statement, we can know, if $1 < \lambda < 2$, $\mid d_{ij} - b_{ij}(t) \mid \leq (m_{ij})_{n\times n}$, and $\boldsymbol{M} = (m_{ij})_{n\times n}, i, j = 1, 2, \ldots, n$, then we will obtain that $-x^{\mathrm{T}}(t)P[\boldsymbol{D} - B(t)]x(t-\tau) - x^{\mathrm{T}}(t-\tau)[\boldsymbol{D} - B(t)]^{\mathrm{T}}Px(t) + x^{\mathrm{T}}(t-\tau)\boldsymbol{Q}x(t-\tau) + \lambda x^{\mathrm{T}}(t)P[\boldsymbol{D} - B(t)]Q^{-1}[\boldsymbol{D} - B(t)]^{\mathrm{T}}Px(t) > 0$.

## 3. Feedback control of delayed dynamical systems

In this section, we focus on the stability analysis of delayed dynamical system (2). It is clear that the stability of a controlled system is related to the inner connecting matrix $\boldsymbol{A}$, the coefficient matrix $\boldsymbol{D}$, the coupling time delay $\tau \in (0, +\infty)$ and the feedback controller $-B(t)x(t-\tau) + u(t)$.

To obtain the feedback control of the delayed dynamical systems, we can define parameters $\theta$ and $\overline{\theta}$ as follows:

Let

$$V(x) = \frac{1}{2}(x^{\mathrm{T}}(t)Px(t)),$$

$$\theta = \sum_{i=1}^{r} \frac{\beta_i^2 ||x(t)||^{2i-2}}{4\gamma_i},$$

$$\overline{\theta}'_t = -\frac{1}{l}\left\| \frac{\partial V^{\mathrm{T}}(x)}{\partial x} \right\| - \sum_{i=1}^{r} \frac{\beta_i^2}{4\gamma_i}(2i-2) \left\| x(t) \right\|^{2i-3} \left\| x \right\|'_t.$$

Consider the following Lyapunov candidate:

$$\begin{aligned} L = {} & \frac{1}{2}(x^{\mathrm{T}}(t)Px(t)) + \frac{1}{2}l(\overline{\theta} + \theta)^2 \\ & + \frac{1}{2}\int_{t-\tau}^{t} x^{\mathrm{T}}(s)\boldsymbol{Q}x(s)\mathrm{d}s. \end{aligned}$$

Taking the differential of $L$ with respect to $t$ and following system (2), we obtain,

$$\left.\frac{\mathrm{d}L(x(t))}{\mathrm{d}t}\right|_{(2)} = x^{\mathrm{T}}(t)P\dot{x}(t) + l(\overline{\theta} + \theta)(\overline{\theta}'_t + \theta'_t) + \frac{1}{2}(x^{\mathrm{T}}(t)\boldsymbol{Q}x(t) - x^{\mathrm{T}}(t-\tau)\boldsymbol{Q}x(t-\tau)).$$

Substituting Eq. (2) and controller $u(t) = \boldsymbol{K}x(t) + \overline{\theta}\partial V^{\mathrm{T}}/\partial x$ into equation $\mathrm{d}L(x(t))/\mathrm{d}t|_{(2)}$ yields

$$\begin{aligned} \frac{\mathrm{d}L(x(t))}{\mathrm{d}t}|_{(2)} = {} & \frac{1}{2}x^{\mathrm{T}}(t)P(2\boldsymbol{A} + 2\boldsymbol{K} + P^{-1}\boldsymbol{Q})x(t) + l(\overline{\theta} + \theta)(\overline{\theta}'_t + \theta'_t) + \frac{1}{2}(2x^{\mathrm{T}}(t)P(\boldsymbol{D} - B(t)) \\ & \times x(t-\tau) - x^{\mathrm{T}}(t-\tau)\boldsymbol{Q}x(t-\tau)) + x^{\mathrm{T}}(t)Pf(x(t)) + x^{\mathrm{T}}(t)P\overline{\theta}\frac{\partial V^{\mathrm{T}}(x)}{\partial x}. \end{aligned}$$

Then, substituting $||f(x)|| < \sum_{i=1}^{r} \beta_i ||x||^i$ into the above equation, and using $V(x) = (x^{\mathrm{T}}(t)Px(t))/2$, we obtain,

$$\begin{aligned} \frac{\mathrm{d}L(x(t))}{\mathrm{d}t}|_{(2)} < {} & \frac{1}{2}x^{\mathrm{T}}(t)\boldsymbol{P}(2\boldsymbol{A} + 2\boldsymbol{K} + \boldsymbol{P}^{-1}\boldsymbol{Q})x(t) + l(\overline{\theta} + \theta)(\overline{\theta}'_t + \theta'_t) + \overline{\theta}\left\| \frac{\partial V^{\mathrm{T}}(x)}{\partial x} \right\|^2 \\ & + \left\| \frac{\partial V^{\mathrm{T}}(x)}{\partial x} \right\| \sum_{i=1}^{r} \beta_i ||x(t)||^i + \frac{1}{2}\lambda\, x^{\mathrm{T}}(t)\boldsymbol{P}(\boldsymbol{D} - B(t))\boldsymbol{Q}^{-1}(\boldsymbol{D} - B(t))^{\mathrm{T}}\boldsymbol{P}x(t). \end{aligned}$$

Taking the symmetric part of $\boldsymbol{A}$ and using $\boldsymbol{A}^{\mathrm{s}} = (\boldsymbol{A} + \boldsymbol{A}^{\mathrm{T}})/2$: symmetric part of $\boldsymbol{A}$, we have

$$\left.\frac{\mathrm{d}L(x(t))}{\mathrm{d}t}\right|_{(2)} \leq \frac{1}{2}x^{\mathrm{T}}(t)\boldsymbol{P}(\boldsymbol{A}+\boldsymbol{K}+\lambda\boldsymbol{M}\boldsymbol{Q}^{-1}\boldsymbol{M}^{\mathrm{T}}\boldsymbol{P})x(t) + \frac{1}{2}x^{\mathrm{T}}(t)\boldsymbol{P}(\boldsymbol{A}^{s}+\boldsymbol{K} + \boldsymbol{P}^{-1}\boldsymbol{Q})x(t) + \sum_{i=1}^{r}\gamma_i||x(t)||^2.$$

Then, taking the minimum eigenvalue of the $\boldsymbol{P}$ and the maximum eigenvalue of the $\boldsymbol{E}$, $\boldsymbol{A}^s + \boldsymbol{K} + \boldsymbol{P}^{-1}\boldsymbol{Q} = E < 0$, where $\boldsymbol{A}, \boldsymbol{K}, \boldsymbol{Q}, \boldsymbol{E}, \boldsymbol{P}$ are $n \times n$ matrices, with $\boldsymbol{K} < 0$ and being a constant matrix or a variable matrix, $\boldsymbol{P}$ and $\boldsymbol{Q}$ being positive definite diagonal matrices.

$$\frac{\mathrm{d}L(x(t))}{\mathrm{d}t}|_{(2)} \leq \frac{1}{2}x^{\mathrm{T}}(t)\boldsymbol{P}(\boldsymbol{A}+\boldsymbol{K}+\lambda\boldsymbol{M}\boldsymbol{Q}^{-1}\boldsymbol{M}^{\mathrm{T}}\boldsymbol{P})x(t) + \sum_{i=1}^{r}\gamma_i||x(t)||^2 + \frac{1}{2}\lambda_{\min}(\boldsymbol{P})\lambda_{max}(E)||x(t)||^2,$$

where we have $\dot{L} < 0$, which means that if $1 < \lambda < 2, P[D - B(t)]Q^{-1}[D - B(t)]^{\mathrm{T}}P \leq PMQ^{-1}M^{\mathrm{T}}P, PA^s = (PA)^s$, $||f(x)|| < \sum_{i=1}^{r}\beta_i||x||^i$, $\lambda_{\min}(\boldsymbol{P})\lambda_{\max}(E)/2 + \sum_{i=1}^{r}\gamma_i < 0$ and $\boldsymbol{A} + \boldsymbol{K} + \lambda\boldsymbol{M}\boldsymbol{Q}^{-1}\boldsymbol{M}^{\mathrm{T}}\boldsymbol{P} < 0$, then we obtain the time-delay control method of the system (1).

**Remark 3.1** If $\boldsymbol{D}$ is a reversible matrix and $\boldsymbol{K}$ is an arbitrary matrix, then the controller can be replaced by $-B(t)x(t) + u(t)$.

# 4. Application examples and simulation results

As an application of the above theoretical criteria, the feedback control of a chaotic system with time delay is discussed in this section. Numerical examples are given to verify and also visualize the theoretical results.[17,18]

**Example 4.1** As is well known, the Lorenz equation has become a classic and popular model for the analysis of nonlinear phenomena. It has also been intensively investigated in the last two decades due to its intrinsic and complex nonlinear dynamical behaviours such as its fractal basin boundary between competing attractors, hopping cross-well chaotic states and so on.[19,20] We now consider a Lorenz system with delays as follows:

$$\dot{x_1} = -a(x_1 - x_2) - e_1x_1(t-\tau), \quad (3)$$

$$\dot{x_2} = (b - x_3)x_1 - x_2 - e_2x_2(t-\tau), \quad (4)$$

$$\dot{x_3} = x_1x_2 - cx_3 - e_3x_3(t-\tau), \quad (5)$$

where parameter $e_i$ is the coefficient of the variable with delays. Let $\boldsymbol{D} = \mathrm{diag}(e_1, e_2, e_3)$. In Fig. 1, we obtain that the solution trajectory of system (3)–(5) is chaotic attractor by numerical simulation. So the Lorenz system with delays is also a chaotic system. The Lorenz system with delays can be formulated in terms of system (1), so we can use the above method to control it.

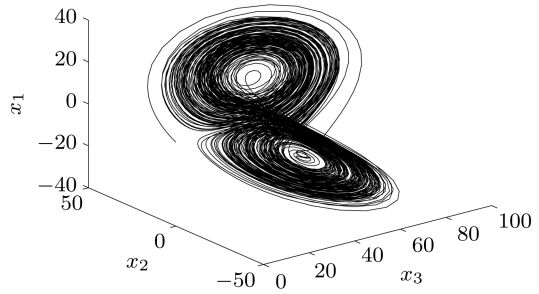



**Fig. 1.** Butterfly-effect chaotic attractor (3)–(5) with the parameters $a = 16$, $b = 45.92$, $c = 4$ and $\tau = 1$.

**Remark 4.1** In Fig. 1, the system exhibits chaotic behaviour at parameters $a = 16$, $b = 45.92$, $c = 4$, $e_1 = 1.2$, $e_2 = 0.75$ and $e_3 = 1$ when the starting value is $[0.01, 0.03, 0.002]$. $x_1, x_2, x_3$ constructs a famous butterfly-effect chaotic strange attractor, see Fig. 1.

Due to the condition of the above method, $\boldsymbol{A} + \boldsymbol{K} + \lambda\boldsymbol{M}\boldsymbol{Q}^{-1}\boldsymbol{M}^{\mathrm{T}}\boldsymbol{P} < 0, \boldsymbol{K} < 0$, $\lambda_{\min}(P)\lambda_{\max}(E)/2 + \sum_{i=1}^{r}\gamma_i < 0$, and $\mid d_{ij} - b_{ij}(t) \mid \leq m_{ij}$, are all satisfied, where $\boldsymbol{A}$ is an arbitrary constant matrix, $||f(x)|| \leq \sum_{i=1}^{r}\beta_i||x(t)||^i (r = 2)$, and let $\boldsymbol{P}$ and $\boldsymbol{Q}$ be diagonal matrixes in which all the elements of the principal diagonal are 1. If $\boldsymbol{K}$ is small enough, the above conditions can be satisfied. $\mid e_i - b_i(t) \mid \leq m_i$, $B(t)$ is chosen as $\mathrm{diag}(b_1(t), b_2(t), b_3(t))$, $\boldsymbol{M}$ is chosen as $diag(m_1, m_2, m_3)$, and $m_i$ is a constant.By choosing $B(t)$, we can control $\boldsymbol{M}$ to make $m_i$ be between 0 and 1.5. In the above system, we can add the feedback controller as follows:

$$B(t)x(t-\tau)$$

$$= \text{diag}(0.0002, 0.0001, 0, 00015)\sin(t)x(t-\tau), \quad (6)$$

$$u(t)$$

$$= -\text{diag}(0.02, 0.02, 0.02)\exp(t)x(t). \quad (7)$$

It is shown in Eqs. (6) and (7) that $u(t)$ and $B(t)$ are the coefficients of the controller, $u(t)$ is the controller without time delay and $B(t)$ is the coefficient of the controller with delay.

It can be observed from Fig. 2 that by adding the controller $-B(t)x(t-\tau)+u(t)$ the controlled system with delays achieves stability. It is shown that the feedback control of the delayed dynamical system is very effective.

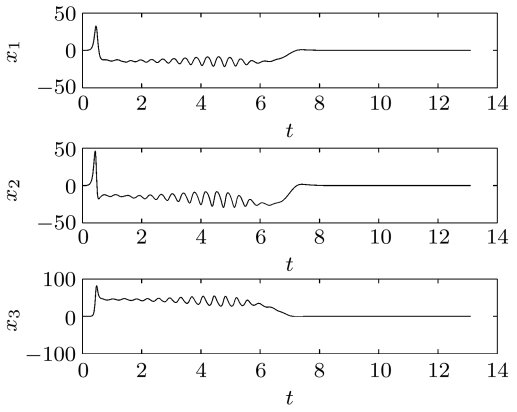



**Fig. 2.** Time response of the state variables in the delayed dynamical system in time interval [0,14], $\tau = 1$.

**Example 4.2** It is well known that Chua's circuit is an extremely simple system, but one that exhibits complicated dynamics of bifurcation and chaos, and is therefore a famous chaotic system. When we add time delays into Chua's circuit, it becomes a time-delay system as given by

$$\dot{x_1} = p_1(x_2 - x_1 - f(x_1)) - d_1x_1(t-\tau), \quad (8)$$

$$\dot{x_2} = x_1 - x_2 + x_3 - d_2x_2(t-\tau), \quad (9)$$

$$\dot{x_3} = -p_2x_2 - p_3x_3 - d_3x_3(t-\tau). \quad (10)$$

Here, the system exhibits chaotic behaviour at parameters $p_1 = 10, p_2 = 15, p_3 = 0.0385$, and $D = \text{diag}(d_1, d_2, d_3)$. The contact function is denoted as $f(x)$, $a$ and $b$ are parameters of function $f(x)$, where $a = -1.27$, $b = -0.68$ and

$$f(x) = bx + \frac{1}{2}(a+b)(\mid x+1 \mid - \mid x-1 \mid). \quad (11)$$

We can see that the time-delayed system is also a chaotic system by obtaining the solution trajectory of systems (8)–(10) through numerical simulation. The solution trajectory is shown in Fig. 3.

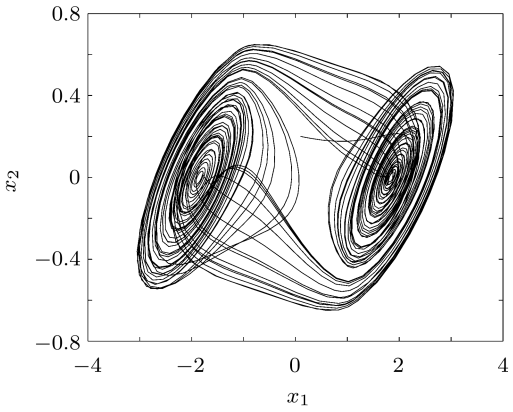



**Fig. 3.** Double-scroll chaotic attractor of Chua's circuit (8)–(10), $\tau = 1$.

Using the above method, it is easy to show that Chua's circuit with delays satisfies $\boldsymbol{A} + \boldsymbol{K} + \lambda \boldsymbol{M}\boldsymbol{Q}^{-1}\boldsymbol{M}^{\mathrm{T}}\boldsymbol{P} < 0$, $\mid d_{ij} - b_{ij}(t) \mid \leq m_{ij}$, $\lambda_{\min}(P)\lambda_{\max}(E)/2 + \sum_{i=1}^{r}\gamma_i < 0$ and $\boldsymbol{K} < 0$. Here, $\boldsymbol{A}$ is an arbitrary constant matrix, $||f(x)|| \leq \sum_{i=1}^{r}\beta_i||x(t)||^i (r=1)$, and let $\boldsymbol{P}$ and $\boldsymbol{Q}$ be diagonal matrixes in which all the elements of the principal diagonal are 1. If $K$ is small enough, the above conditions can be satisfied. $\mid d_i - b_i(t) \mid \leq m_i$, $B(t)$ is chosen as $\text{diag}(b_1(t), b_2(t), b_3(t))$, $M$ is chosen as $\text{diag}(m_1, m_2, m_3)$, $m_i$ is a constant, and $0 < m_i < 1.8$. By choosing $B(t)$, we can control $M$. We can easily construct the feedback controller as follows:

$$u(t)$$

$$= -\text{diag}(0.02, 0.02, 0.02)\exp(t)x(t), \quad (12)$$

$$B(t)x(t-\tau)$$

$$= \text{diag}(0.0002, 0.0001, 0, 00015)\sin(t)x(t-\tau). \quad (13)$$

Taking the initial conditions of the delayed system and the unknown parameters of the controlled system as follows: $\phi(s) = [0.1, 0.13, 0.04]^{\mathrm{T}}$, $s \in [-\tau, 0]$; $d_1 = 1.5, d_2 = 1$ and $d_3 = 0.6$, the numerical simulations show that feedback control is successfully achieved.

The simulation results are displayed in Fig. 4, where we can see that the system quickly becomes stable. The above numerical simulations were done using the Delay Differential Equations (DDEs) Solver in Matlab. The conclusion is obviously consistent with the theoretical results of this paper. Therefore, the approaches developed here further extend the ideas and techniques presented so far in the literature.

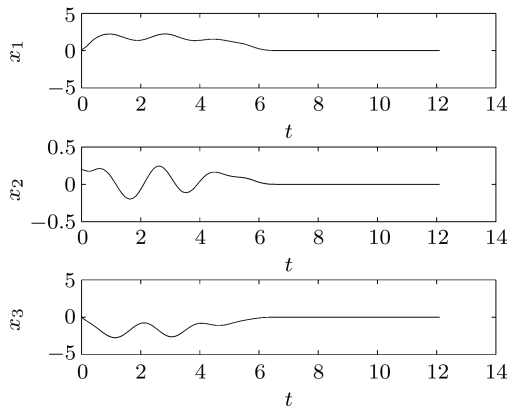



**Fig. 4.** Time response of the state variables in the delayed dynamical system in time interval [0,14] and at $\tau = 1$.

# 5. Conclusions

In this paper, we have further investigated the nonautonomous control dynamics of a general model of complex delayed dynamical systems. Some simple, yet less conservative, criteria for delay-independent feedback control of the systems have been derived analytically. The obtained results have been applied to some typical chaotic systems with delays. Both theoretical and numerical analyses indicate that under some conditions, if the appropriate matrices are selected, the nonlinear chaotic systems can be stable under feedback control. To this end, the theoretical results can be applied to a typical class of chaotic delayed system. Numerical experiment shows the effectiveness of the proposed methods. A possible application of the proposed methods is in secure energy transportation. It is believed that these techniques are essential to chaos economic systems in practical design and engineering applications.

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