Synchronisation of chaotic systems using a novel sampled-data fuzzy controller*

Feng Yi-Fu(冯毅夫)[a)b)†] and Zhang Qing-Ling(张庆灵)[b)]

[a)] *School of Mathematics, Jilin Normal University, Siping* 136000, *China*
[b)] *Institute of Systems Science, Northeastern University, Shenyang* 110004, *China*

(Received 4 March 2010; revised manuscript received 11 June 2010)

This paper presents the synchronisation of chaotic systems using a sampled-data fuzzy controller and is meaningful for many physical real-life applications. Firstly, a Takagi–Sugeno (T–S) fuzzy model is employed to represent the chaotic systems that contain some nonlinear terms, then a type of fuzzy sampled-data controller is proposed and an error system formed by the response and drive chaotic system. Secondly, relaxed LMI-based synchronisation conditions are derived by using a new parameter-dependent Lyapunov–Krasovskii functional and relaxed stabilisation techniques for the underlying error system. The derived LMI-based conditions are used to aid the design of a sampled-data fuzzy controller to achieve the synchronisation of chaotic systems. Finally, a numerical example is provided to illustrate the effectiveness of the proposed results.

Keywords: chaotic systems, synchronisation, sampled-data systems, parameter-dependent Lyapunov–Krasovskii functional

PACS: 01.20.+x **DOI:** 10.1088/1674-1056/20/1/010101

1. Introduction

Chaotic dynamics can be found in many real-life processes, such as in the turbulence of liquids, human heart beats, weather and brain activity. The synchronisation of chaotic systems has attracted considerable attention due to its great potential application in secure communication, chemical reactions and biological systems.[1] The first method of synchronisation of two chaotic systems with different initial conditions was introduced in Ref. [2]. Many different methods have subsequently been used, both theoretically and experimentally, to synchronise chaotic systems, such as linear and nonlinear feedback control,[3−5] backstepping control,[6−8] variable structure control,[9−12] adaptive control,[13−15] impulsive control,[16−20] active control,[21] etc. In general, synchronisation is achieved by modifying the chaotic behaviour of the response chaotic system using a controller. With the rapid development of digital circuit techniques, powerful microcontroller and digital computers are now available at low cost.[22] A sampled-data fuzzy controller can be implemented using a microcontroller or a digital computer, which reduces implementation cost and time. Therefore, investigating the synchronisation of chaotic systems using a sampled-data fuzzy controller is meaningful in a great number of real-life applications.

Fuzzy-model-based control is a promising approach for dealing with complex nonlinear systems. Recently, a fuzzy-model-based control approach was employed to synchronise chaotic systems,[23−25] which is a useful application in communication to ensure a security. A unified approach to fuzzy modelling and a robust synchronisation of different hyperchaotic systems using a common quadratic Lyapunov function and normal parallel distributed compensation (PDC) control scheme is proposed in Ref. [23]. Based on the Takagi–Sugeno (T–S) fuzzy model and the delayed feedback control scheme, Ref. [24] presents a robust synchronisation strategy for a class of chaotic systems with unknown parameters and disturbances. The synchronisation of chaotic systems using a time-delayed fuzzy-state feedback controller is also investigated in Ref. [25]. It is worth noting that the common quadratic Lyapunov functions have been applied in the above literature[23−25], which is the source of some conservatism. There, there is still much conservatism to be lifted if we change 'something' in either the Lyapunov function, relaxed stabilisation technique or the form of introducing additional variables in the

*Project supported by the National Natural Science Foundation of China (Grant No. 60974004).
†Corresponding author. E-mail: yf19692004@163.com
© 2011 Chinese Physical Society and IOP Publishing Ltd

process of system synthesis.

In this paper, synchronisation of chaotic systems is handled using a the fuzzy-model-based approach. A fuzzy model is first employed to represent the chaotic systems and a sampled-data fuzzy controller is then proposed to synchronise the chaotic systems. Unlike the control signal of the continuous-time fuzzy controller,[23−25] the control signal of the sampled-data fuzzy controller will stay constant during the sampling period.[26] To implement the synchronisation of the chaotic systems, an error system is derived with time-varying input delays and bounded signals. Thus, by using a new parameter-dependent Lyapunov–Krasovskii functional and relaxed stabilisation techniques for the underlying error system, LMI-based synchronisation conditions are derived to aid the design of a sampled-data fuzzy controller to realise less conservative synchronisation. With the use of convex programming techniques, the solution under the LMI-based synchronisation conditions can be acquired efficiently and numerically to realise the design of the sampled-data fuzzy controller.

The findings of this paper can be summarised as follows: (i) a new parameter-dependent Lyapunov–Krasovskii functional is given to further reduce the conservatism that arises from common quadratic functions; and (ii) relaxed stabilisation techniques for the underlying error system (with time-varying input delays) are proposed to provide extra degrees of freedom in the resulting optimisation problems. With the help of the above, relaxed LMI-based synchronisation conditions will be developed for synchronising the response chaotic system and the drive chaotic system.

The rest of the paper is organised as follows. In Section 2, the T–S fuzzy model and the sampled-data fuzzy controller are presented to implement the synchronisation of the response chaotic system and drive chaotic system. In Section 3, less conservative LMI-based synchronisation conditions are derived. Also, some remarks are given that show the advantages of the proposed method. In Section 4, an illustrative example is provided to demonstrate the theoretical results. Finally, in Section 5, concluding remarks are given.

Notations: Throughout the paper, $\mathbb{R}$ and $\mathbb{R}^n$ denote the real number and n-dimensional Euclidean space, respectively. $\mathbb{R}^{n\times m}$ is the set of all $n\times m$ real matrices. $\boldsymbol{I}$ denotes the identity matrix with an appropriate dimension. $\|\cdot\|$ refers to the Euclidean vector norm or the induced matrix 2-norm.

2. System description and synchronisation problem

2.1. Chaotic fuzzy model

Let p be the number of fuzzy rules describing the response chaotic system with a control input term. The ith rule has the following format:

Rule i: if $f_1(\boldsymbol{x}(t))$ is M_1^i and ... and $f_\Psi(\boldsymbol{x}(t))$ is M_Ψ^i, then

$$\dot{\boldsymbol{x}}(t)=\boldsymbol{A}_i\boldsymbol{x}(t)+\boldsymbol{B}\boldsymbol{u}(t),\ i=1,2,\ldots,p, \tag{1}$$

where M_α^i is a fuzzy term of rule i corresponding to the function $f_\alpha(\boldsymbol{x}(t))$ with known form, $\alpha=1,2,\ldots,p$, Ψ is a positive integer, $\boldsymbol{x}(t)\in\mathbb{R}^{n\times 1}$ is the system state vector, $\boldsymbol{A}_i\in\mathbb{R}^{n\times n}$ and $\boldsymbol{B}\in\mathbb{R}^{n\times m}$ are known constant system and input matrices, respectively, and $\boldsymbol{u}(t)\in\mathbb{R}^{m\times 1}$ is the input vector. The overall system dynamics is described by

$$\dot{\boldsymbol{x}}(t)=\sum_{i=1}^{p}w_i(\boldsymbol{x}(t))(\boldsymbol{A}_i\boldsymbol{x}(t)+\boldsymbol{B}\boldsymbol{u}(t)), \tag{2}$$

where $\sum_{i=1}^{p}w_i(\boldsymbol{x}(t))=1$, $w_i(\boldsymbol{x}(t))\in[0,1]$ for all i. And

$$w_i(\boldsymbol{x}(t))=\frac{\mu_{M_1^i}(f_1(\boldsymbol{x}(t)))\times\ldots\times\mu_{M_\Psi^i}(f_\Psi(\boldsymbol{x}(t)))}{\sum_{i=1}^{p}(\mu_{M_1^i}(f_1(\boldsymbol{x}(t)))\times\ldots\times\mu_{M_\Psi^i}(f_\Psi(\boldsymbol{x}(t))))}$$

is a nonlinear function of $\boldsymbol{x}(t)$ and $\mu_{M_\alpha^i}(f_\alpha(\boldsymbol{x}(t)))$ is the grade of membership corresponding to the fuzzy term M_α^i.

Similarly, the drive chaotic system can be represented by a fuzzy model with q fuzzy rules in the form of Eq. (1). Consequently, the dynamics of the drive chaotic system can be represented as

$$\dot{\hat{\boldsymbol{x}}}(t)=\sum_{j=1}^{q}\hat{w}_j(\hat{\boldsymbol{x}}(t))\hat{\boldsymbol{A}}_j\hat{\boldsymbol{x}}(t), \tag{3}$$

where $\hat{\boldsymbol{x}}(t)\in\mathbb{R}^{n\times 1}$ is the system state vector, $\hat{\boldsymbol{A}}_j\in\mathbb{R}^{n\times n}$ is a known constant system matrix, $\hat{w}_j(\hat{\boldsymbol{x}}(t))\in[0,1]$ is the grade of the membership, and $\sum_{j=1}^{q}\hat{w}_j(\hat{\boldsymbol{x}}(t))=1$. It should be noted that there is no control input term for the drive chaotic system. The dynamics of the drive Rössler's system[27] is given in Fig. 1 for visual comparison.

Assumption 1 For underlying systems (2) and (3), system states $\boldsymbol{x}(t)$ and $\hat{\boldsymbol{x}}(t)$ are bounded in a bounded compact set $\mathcal{W}$.

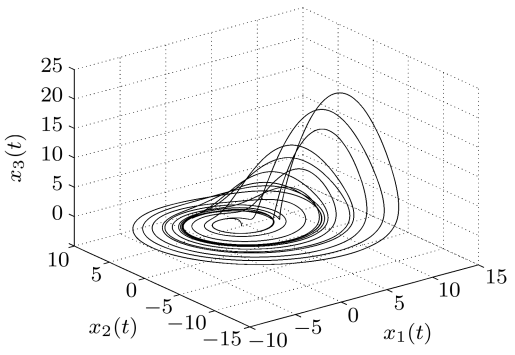


Fig. 1. Dynamics of the drive Rössler's system.[27]

In most of the existing work, the following assumption is also given for facilitating stability analysis using the parameter-dependent fuzzy Lyapunov function.

Assumption 2[28] The time derivatives of fuzzy membership functions satisfy $|\dot{w}_m(\boldsymbol{x}(t))| \leq \phi_m (m = 1, 2, \ldots, r)$, where ϕ_m is a given as a positive scalar.

2.2. Sampled-data fuzzy controller

With the rapid development of digital circuit techniques, powerful microcontroller and digital computers are now available at low cost. The sampled-data fuzzy controller can be implemented using the microcontroller of a digital computer to reduce implementation cost and time.

For the purpose of implementing the synchronisation task of systems (2) and (3), a sampled-data fuzzy controller with p fuzzy rules is designed based on the fuzzy model of the nonlinear plant. The ith rule of the fuzzy controller is as follows:

Rule i: if $f_1(\boldsymbol{x}(t_\gamma))$ is M_1^i and ... and $f_\Psi(\boldsymbol{x}(t_\gamma))$ is M_Ψ^i, then

$$\boldsymbol{u}(t) = \boldsymbol{G}_i \boldsymbol{e}(t_\gamma), i = 1, 2, \ldots, p; \quad t_\gamma < t \leq t_{\gamma+1}, \quad (4)$$

where $\boldsymbol{G}_i \in \mathbb{R}^{m \times n}$ is the feedback gain of rule j to be designed, $\boldsymbol{e}(t_\gamma) = \boldsymbol{x}(t_\gamma) - \hat{\boldsymbol{x}}(t_\gamma)$, $t_\gamma = \gamma h$, $\gamma = 0, 1, 2, \ldots, +\infty$, denotes a time instant, and $h = t_{\gamma+1} - t_\gamma$ represents the constant sampling period. The inferred output of the sampled-data fuzzy controller is given by

$$\boldsymbol{u}(t) = \sum_{i=1}^{p} w_i(\boldsymbol{x}(t_\gamma)) \boldsymbol{G}_i \boldsymbol{e}(t_\gamma), \; t_\gamma < t \leq t_{\gamma+1}, \quad (5)$$

where $\sum_{i=1}^{p} w_i(\boldsymbol{x}(t_\gamma)) = 1$, and $w_i(\boldsymbol{x}(t_\gamma)) \in [0, 1]$ for all i's. It can be seen from Eq. (5) that $\boldsymbol{u}(t) = \boldsymbol{u}(t_\gamma)$ which holds constant vector value for $t_\gamma < t \leq t_{\gamma+1}$. We have

$$\begin{aligned} \boldsymbol{u}(t) &= \sum_{j=1}^{p} w_j(\boldsymbol{x}(t_\gamma)) \boldsymbol{G}_j \boldsymbol{e}(t - (t - t_\gamma)) \\ &= \sum_{j=1}^{p} w_j(\boldsymbol{x}(t_\gamma)) \boldsymbol{G}_j \boldsymbol{e}(t - \tau(t)), \end{aligned} \quad (6)$$

where $\tau(t) = t - t_\gamma \leq h$ for $t_\gamma < t \leq t_{\gamma+1}$.

2.3. Error system

The error system is formed by the response and drive chaotic systems (2) and (3). From systems (2) and (3), with the property that $\sum_{i=1}^{p} w_i(\boldsymbol{x}(t)) = 1$, we have

$$\begin{aligned} \dot{\boldsymbol{e}}(t) &= \dot{\boldsymbol{x}}(t) - \dot{\hat{\boldsymbol{x}}}(t) \\ &= \sum_{i=1}^{p} w_i(\boldsymbol{x}(t))(\boldsymbol{A}_i \boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t)) \\ &\quad - \sum_{j=1}^{q} \hat{w}_j(\hat{\boldsymbol{x}}(t)) \hat{\boldsymbol{A}}_j \hat{\boldsymbol{x}}(t) \\ &= \sum_{i=1}^{p} w_i(\boldsymbol{x}(t)) \boldsymbol{A}_i \boldsymbol{x}(t) - \sum_{i=1}^{p} w_i(\boldsymbol{x}(t)) \boldsymbol{A}_i \hat{\boldsymbol{x}}(t) \\ &\quad + \sum_{i=1}^{p} w_i(\boldsymbol{x}(t))(\boldsymbol{A}_i \hat{\boldsymbol{x}}(t) + \boldsymbol{B}\boldsymbol{u}(t)) \\ &\quad - \sum_{j=1}^{q} \hat{w}_j(\hat{\boldsymbol{x}}(t)) \hat{\boldsymbol{A}}_j \hat{\boldsymbol{x}}(t) \\ &= \sum_{i=1}^{p} w_i(\boldsymbol{x}(t))(\boldsymbol{A}_i \boldsymbol{e}(t) + \boldsymbol{B}\boldsymbol{u}(t)) + \boldsymbol{m}_e(\boldsymbol{x}(t), \hat{\boldsymbol{x}}(t)) \\ &= \sum_{i=1}^{p} \sum_{j=1}^{p} w_i(\boldsymbol{x}(t)) w_j(\boldsymbol{x}(t_\gamma))(\boldsymbol{A}_i \boldsymbol{e}(t) \\ &\quad + \boldsymbol{B}\boldsymbol{G}_j \boldsymbol{e}(t - \tau(t))) + \boldsymbol{m}_e(\boldsymbol{x}(t), \hat{\boldsymbol{x}}(t)) \\ &= \boldsymbol{A}(t)\boldsymbol{e}(t) + \boldsymbol{B}\boldsymbol{G}(t_\gamma)\boldsymbol{e}(t_\gamma) + \boldsymbol{m}_e, \end{aligned} \quad (7)$$

where

$$\begin{aligned} &\boldsymbol{m}_e(\boldsymbol{x}(t), \hat{\boldsymbol{x}}(t)) \\ &= \sum_{i=1}^{p} w_i(\boldsymbol{x}(t)) \boldsymbol{A}_i \hat{\boldsymbol{x}}(t) - \sum_{j=1}^{q} \hat{w}_j(\hat{\boldsymbol{x}}(t)) \hat{\boldsymbol{A}}_j \hat{\boldsymbol{x}}(t), \end{aligned}$$

which is bounded as $w_i(\boldsymbol{x}(t))$, $\hat{w}_j(\hat{\boldsymbol{x}}(t))$ and $\hat{\boldsymbol{x}}(t)$ are bounded, $\boldsymbol{A}(t) = \sum_{i=1}^{p} w_i(\boldsymbol{x}(t)) \boldsymbol{A}_i$ and $\boldsymbol{G}(t_\gamma) = \sum_{j=1}^{p} w_j(\boldsymbol{x}(t_\gamma)) \boldsymbol{G}_j$.

Remark 1 Using the above technique, the problem of the synchronisation of two different chaotic systems is converted into the stabilisation of the underlying fuzzy error system with a time-varying input delay and bounded model error signals. To

further reduce conservatism, a parameter-dependent Lyapunov–Krasovskii functional and a new relaxed quadratic stabilisation technique for the underlying fuzzy error system are proposed during the process of system synthesis.

3. Main results

Referring to error system (7), synchronisation is achieved if $\boldsymbol{e}(t) \to 0$ as time goes by. Here, LMI-based synchronisation conditions are derived to guarantee synchronisation by using a new parameter-dependent Lyapunov–Krasovskii functional. For an appropriate dimensional matrix $\boldsymbol{X}$, we first introduce the following matrix functions:

$$\boldsymbol{X}(t) = \sum_{i=1}^{p} w_i(\boldsymbol{x}(t))\boldsymbol{X}_i. \tag{8}$$

Theorem 1 The sample-data fuzzy-model-based control error system (7) is guaranteed to be stable if there are constant positive scalars ρ, $d_{mn}(m = 1,2,3,4; n = 1,2,\ldots,p)$ and appropriately dimensional matrices $\tilde{\boldsymbol{P}}_i > 0$, $\tilde{\boldsymbol{S}}_i > 0$, $\tilde{\boldsymbol{Z}}_i > 0$, $\tilde{\boldsymbol{M}}_{ij}$, $\boldsymbol{F}_i$, $\tilde{\boldsymbol{R}}_{ijk}$ with $\tilde{\boldsymbol{R}}_{ijk} = \tilde{\boldsymbol{R}}_{jik}^{\mathrm{T}}$, symmetrical matrix $\tilde{\boldsymbol{X}}$, symmetrical non-singular matrix $\boldsymbol{\Omega}$, such that the following LMIs hold:

$$\tilde{\boldsymbol{\Gamma}}_{iik} < \tilde{\boldsymbol{R}}_{iik}, \tag{9}$$

$$\tilde{\boldsymbol{\Gamma}}_{ijk} + \tilde{\boldsymbol{\Gamma}}_{jik} < \tilde{\boldsymbol{R}}_{ijk} + \tilde{\boldsymbol{R}}_{jik}, \tag{10}$$

$$\begin{bmatrix} \tilde{R}_{11k} & \tilde{R}_{12k} & \ldots & \tilde{R}_{1pk} \\ \tilde{R}_{21k} & \tilde{R}_{22k} & \ldots & \tilde{R}_{2pk} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{R}_{p1k} & \tilde{R}_{p2k} & \ldots & \tilde{R}_{ppk} \end{bmatrix} < 0, \tag{11}$$

$$\tilde{\boldsymbol{Z}}_i < \tilde{\boldsymbol{S}}_j, \quad i,j,k = 1,2,\ldots,p, \tag{12}$$

where $\tilde{\boldsymbol{\Gamma}}_{ijk} = \begin{bmatrix} \tilde{\boldsymbol{\Gamma}}_{ijk}^{11} & \tilde{\boldsymbol{\Gamma}}_{ijk}^{12} & \tilde{\boldsymbol{\Gamma}}_{ijk}^{13} & \tilde{\boldsymbol{\Gamma}}_{ijk}^{14} & h\tilde{\boldsymbol{M}}_{1i} \\ * & \tilde{\boldsymbol{\Gamma}}_{ijk}^{22} & \tilde{\boldsymbol{\Gamma}}_{ijk}^{23} & \tilde{\boldsymbol{\Gamma}}_{ijk}^{24} & h\tilde{\boldsymbol{M}}_{2i} \\ * & * & \tilde{\boldsymbol{\Gamma}}_{ijk}^{33} & \tilde{\boldsymbol{\Gamma}}_{ijk}^{34} & h\tilde{\boldsymbol{M}}_{3i} \\ * & * & * & \tilde{\boldsymbol{\Gamma}}_{ijk}^{44} & h\tilde{\boldsymbol{M}}_{4i} \\ * & * & * & * & -h\tilde{\boldsymbol{Z}}_i \end{bmatrix}$, and

$$\begin{aligned}
\tilde{\boldsymbol{\Gamma}}_{ijk}^{11} &= \tilde{\boldsymbol{Q}}_i + \sum_{m=1}^{p} \phi_m(\tilde{\boldsymbol{P}}_m + \tilde{X}) + \tilde{\boldsymbol{M}}_{1i} + \tilde{\boldsymbol{M}}_{1i}^{\mathrm{T}} \\
&\quad + d_{1j}\boldsymbol{A}_i\boldsymbol{\Omega} + (d_{1j}\boldsymbol{A}_i\boldsymbol{\Omega})^{\mathrm{T}}, \\
\tilde{\boldsymbol{\Gamma}}_{ijk}^{12} &= \tilde{\boldsymbol{M}}_{2i}^{\mathrm{T}} - \tilde{\boldsymbol{M}}_{1i} + (d_{2j}\boldsymbol{A}_i\boldsymbol{\Omega})^{\mathrm{T}} + d_{1j}\boldsymbol{B}\boldsymbol{F}_k, \\
\tilde{\boldsymbol{\Gamma}}_{ijk}^{13} &= \tilde{\boldsymbol{P}}_i + \tilde{\boldsymbol{M}}_{3i}^{\mathrm{T}} + (d_{3j}\boldsymbol{A}_i\boldsymbol{\Omega})^{\mathrm{T}} - d_{1j}^{\mathrm{T}}\boldsymbol{\Omega}, \\
\tilde{\boldsymbol{\Gamma}}_{ijk}^{14} &= \tilde{\boldsymbol{M}}_{4i}^{\mathrm{T}} + (d_{4j}^{\mathrm{T}}\boldsymbol{A}_i\boldsymbol{\Omega})^{\mathrm{T}} + d_{1j}^{\mathrm{T}}\boldsymbol{\Omega}, \\
\tilde{\boldsymbol{\Gamma}}_{ijk}^{22} &= -\tilde{\boldsymbol{M}}_{2i} - \tilde{\boldsymbol{M}}_{2i}^{\mathrm{T}} + d_{2j}\boldsymbol{B}\boldsymbol{F}_k + (d_{2j}\boldsymbol{B}\boldsymbol{F}_k)^{\mathrm{T}}, \\
\tilde{\boldsymbol{\Gamma}}_{ijk}^{23} &= -\tilde{\boldsymbol{M}}_{3i}^{\mathrm{T}} + (d_{3j}\boldsymbol{B}\boldsymbol{F}_k)^{\mathrm{T}} - d_{2j}^{\mathrm{T}}\boldsymbol{\Omega}, \\
\tilde{\boldsymbol{\Gamma}}_{ijk}^{24} &= -\tilde{\boldsymbol{M}}_{4i}^{\mathrm{T}} + (d_{4j}\boldsymbol{B}\boldsymbol{F}_k)^{\mathrm{T}} + d_{2j}^{\mathrm{T}}\boldsymbol{\Omega}, \\
\tilde{\boldsymbol{\Gamma}}_{ijk}^{33} &= h\tilde{\boldsymbol{S}}_i - 2d_{3j}\boldsymbol{\Omega}, \\
\tilde{\boldsymbol{\Gamma}}_{ijk}^{34} &= d_{3j}\boldsymbol{\Omega} - d_{4j}\boldsymbol{\Omega}, \\
\tilde{\boldsymbol{\Gamma}}_{ijk}^{44} &= -\rho\tilde{\boldsymbol{Q}}_i + 2d_{4j}\boldsymbol{\Omega}.
\end{aligned}$$

Moreover, the feedback gains matrices could be attained by $\boldsymbol{G}_j = \boldsymbol{F}_j\boldsymbol{\Omega}^{-1}, j = 1,2,\ldots,q$.

Proof Choose a new parameter-dependent Lyapunov–Krasovskii functional as follows:

$$V(t) = V_1(t) + V_2(t), \tag{13}$$

where $V_1(t) = \boldsymbol{e}(t)^{\mathrm{T}}P(t)\boldsymbol{e}(t), V_2(t) = \int_{-(t-t_\gamma)}^{0}\int_{t+s}^{t} \dot{\boldsymbol{e}}(m)^{\mathrm{T}}\boldsymbol{S}(m)\dot{\boldsymbol{e}}(m)\mathrm{d}m\mathrm{d}s$, and $\boldsymbol{P}(t)$ and $\boldsymbol{S}(t)$ have been defined in Eq. (8).

Then, taking the time derivative of $V(t)$ along the trajectory of system (7), we have that

$$\dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t), \tag{14}$$

where

$$\dot{V}_1(t) = 2\boldsymbol{e}(t)^{\mathrm{T}}\boldsymbol{P}(t)\dot{\boldsymbol{e}}(t) + \boldsymbol{e}(t)^{\mathrm{T}}\dot{\boldsymbol{P}}(t)\boldsymbol{e}(t)$$

and

$$\begin{aligned}
\dot{V}_2(t) &= (t - t_\gamma)\dot{\boldsymbol{e}}(t)^{\mathrm{T}}\boldsymbol{S}(t)\dot{\boldsymbol{e}}(t) - \int_{t_\gamma}^{t} \dot{\boldsymbol{e}}(s)^{\mathrm{T}}\boldsymbol{S}(s)\dot{\boldsymbol{e}}(s)\mathrm{d}s \\
&= \tau(t)\dot{\boldsymbol{e}}(t)^{\mathrm{T}}\boldsymbol{S}(t)\dot{\boldsymbol{e}}(t) - \int_{t-\tau(t)}^{t} \dot{\boldsymbol{e}}(s)^{\mathrm{T}}\boldsymbol{S}(s)\dot{\boldsymbol{e}}(s)\mathrm{d}s.
\end{aligned}$$

According to the Newton–Leibniz formula and the error system in system (7), with appropriately dimensional matrices $\boldsymbol{M}(t)$, $\boldsymbol{N}(t)$ defined in Eq. (8), we have

$$\begin{aligned}
&2\xi^{\mathrm{T}}(t)\boldsymbol{M}(t)\left[\boldsymbol{e}(t) - \boldsymbol{e}(t-\tau(t)) - \int_{t-\tau(t)}^{t} \dot{\boldsymbol{e}}(s)\mathrm{d}s\right] \\
&= 0,
\end{aligned} \tag{15}$$

$$\begin{aligned}
&2\boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{N}(t)[\boldsymbol{A}(t)\boldsymbol{e}(t) \\
&+ \boldsymbol{B}\boldsymbol{G}(t-\tau(t))\boldsymbol{e}(t-\tau(t)) + \boldsymbol{m}_e - \dot{\boldsymbol{e}}(t)] = 0,
\end{aligned} \tag{16}$$

where $\boldsymbol{\xi}^{\mathrm{T}}(t) = [\boldsymbol{e}^{\mathrm{T}}(t), \boldsymbol{e}^{\mathrm{T}}(t_\gamma), \dot{\boldsymbol{e}}^{\mathrm{T}}(t), \boldsymbol{m}_e^{\mathrm{T}}(t)]^{\mathrm{T}}$, $\boldsymbol{M}_i = [\boldsymbol{M}_{1i}^{\mathrm{T}}, \boldsymbol{M}_{2i}^{\mathrm{T}}, \boldsymbol{M}_{3i}^{\mathrm{T}}, \boldsymbol{M}_{4i}^{\mathrm{T}}]^{\mathrm{T}}$, $\boldsymbol{N}_i = [\boldsymbol{N}_{1i}^{\mathrm{T}}, \boldsymbol{N}_{2i}^{\mathrm{T}}, \boldsymbol{N}_{3i}^{\mathrm{T}}, \boldsymbol{N}_{4i}^{\mathrm{T}}]^{\mathrm{T}}$.

Noting the fact that

$$\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{Q}(t)\boldsymbol{e}(t) - \rho\boldsymbol{m}_e^{\mathrm{T}}\boldsymbol{Q}(t)\boldsymbol{m}_e > 0, \tag{17}$$

where $\boldsymbol{Q}(t) > 0$ has also been defined in Eq. (8) and ρ is a nonzero positive scalar which could be calculated for known membership functions $w(\boldsymbol{x}(t))$ and $\hat{w}(\hat{\boldsymbol{x}}(t))$.

Based on $\sum_{i=1}^{p} \dot{w}_i(\boldsymbol{x}(t)) = 0$, it follows that $\sum_{i=1}^{p} \dot{w}_i(\boldsymbol{x}(t))\boldsymbol{X} = 0$, where $\boldsymbol{X}$ is any symmetric matrix of proper dimension. Then, combining expressions (14)–(17) and Assumption 2, we can obtain

$$\begin{aligned}
\dot{V}(t) \leq\ & 2\boldsymbol{e}(t)^{\mathrm{T}} P(t)\dot{\boldsymbol{e}}(t) + \boldsymbol{e}(t)^{\mathrm{T}} \sum_{m=1}^{p} \phi_m(\boldsymbol{P}_m + \boldsymbol{X})\boldsymbol{e}(t) + h\dot{\boldsymbol{e}}(t)^{\mathrm{T}} \boldsymbol{S}(t)\dot{\boldsymbol{e}}(t) - \int_{t-\tau(t)}^{t} \dot{\boldsymbol{e}}(s)^{\mathrm{T}} \boldsymbol{S}(s)\dot{\boldsymbol{e}}(s)\mathrm{d}s \\
& + 2\boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{M}(t)\left[\boldsymbol{e}(t) - \boldsymbol{e}(t_\gamma) - \int_{t_\gamma}^{t} \dot{\boldsymbol{e}}(s)\mathrm{d}s\right] + 2\boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{N}(t)\left[\boldsymbol{A}(t)\boldsymbol{e}(t) + \boldsymbol{B}\boldsymbol{G}(t_\gamma)\boldsymbol{e}(t_\gamma) + \boldsymbol{m}_e - \dot{\boldsymbol{e}}(t)\right] \\
& + \boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{Q}(t)\boldsymbol{e}(t) - \rho\boldsymbol{m}_e^{\mathrm{T}}\boldsymbol{Q}(t)\boldsymbol{m}_e + \tau(t)\boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{M}(t)\boldsymbol{Z}(t)^{-1}\boldsymbol{M}(t)^{\mathrm{T}}\boldsymbol{\xi}(t) \\
& - \int_{t-\tau(t)}^{t} \boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{M}(t)\boldsymbol{Z}(t)^{-1}\boldsymbol{M}(t)^{\mathrm{T}}\boldsymbol{\xi}(t)\mathrm{d}s \\
\leq\ & \boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{\Xi}\boldsymbol{\xi}(t) + h\boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{M}(t)\boldsymbol{Z}(t)^{-1}\boldsymbol{M}(t)^{\mathrm{T}}\boldsymbol{\xi}(t) \\
& - \int_{t-\tau(t)}^{t} [\boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{M}(t) + \dot{\boldsymbol{e}}(s)^{\mathrm{T}}\boldsymbol{S}(s)]\boldsymbol{S}^{-1}(s)[\boldsymbol{M}(t)^{\mathrm{T}}\boldsymbol{\xi}(t) + \boldsymbol{S}(s)\dot{\boldsymbol{e}}(s)]\mathrm{d}s \\
& - \int_{t-\tau(t)}^{t} \boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{M}(t)\boldsymbol{Z}(t)^{-1}\boldsymbol{M}(t)^{\mathrm{T}}\boldsymbol{\xi}(t)\mathrm{d}s + \int_{t-\tau(t)}^{t} \boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{M}(t)\boldsymbol{S}(s)^{-1}\boldsymbol{M}(t)^{\mathrm{T}}\boldsymbol{\xi}(t)\mathrm{d}s, \qquad (18)
\end{aligned}$$

where $\boldsymbol{\Xi} = \begin{bmatrix} \boldsymbol{\Xi}_{11} & \boldsymbol{\Xi}_{12} & \boldsymbol{\Xi}_{13} & \boldsymbol{\Xi}_{14} \\ * & \boldsymbol{\Xi}_{22} & \boldsymbol{\Xi}_{23} & \boldsymbol{\Xi}_{24} \\ * & * & \boldsymbol{\Xi}_{33} & \boldsymbol{\Xi}_{34} \\ * & * & * & \boldsymbol{\Xi}_{44} \end{bmatrix}$, with

$$\begin{aligned}
\boldsymbol{\Xi}_{11} &= \boldsymbol{Q}(t) + \sum_{m=1}^{p} \phi_m(\boldsymbol{P}_m + \boldsymbol{X}) + \boldsymbol{M}_1(t) + \boldsymbol{M}_1^{\mathrm{T}}(t) + \boldsymbol{N}_1(t)\boldsymbol{A}(t) + (\boldsymbol{N}_1(t)\boldsymbol{A}(t))^{\mathrm{T}}, \\
\boldsymbol{\Xi}_{12} &= \boldsymbol{M}_2^{\mathrm{T}}(t) - \boldsymbol{M}_1(t) + (\boldsymbol{N}_2(t)\boldsymbol{A}(t))^{\mathrm{T}} + \boldsymbol{N}_1(t)\boldsymbol{B}\boldsymbol{G}(t_\gamma), \\
\boldsymbol{\Xi}_{13} &= \boldsymbol{P}(t) + \boldsymbol{M}_3^{\mathrm{T}}(t) + (\boldsymbol{N}_3(t)\boldsymbol{A}(t))^{\mathrm{T}} - \boldsymbol{N}_1^{\mathrm{T}}(t), \\
\boldsymbol{\Xi}_{14} &= \boldsymbol{M}_4^{\mathrm{T}}(t) + (\boldsymbol{N}_4^{\mathrm{T}}\boldsymbol{A}(t))^{\mathrm{T}} + \boldsymbol{N}_1^{\mathrm{T}}(t), \\
\boldsymbol{\Xi}_{22} &= -\boldsymbol{M}_2(t) - \boldsymbol{M}_2^{\mathrm{T}}(t) + \boldsymbol{N}_2(t)\boldsymbol{B}\boldsymbol{G}(t_\gamma) + (\boldsymbol{N}_2(t)\boldsymbol{B}\boldsymbol{G}(t_\gamma))^{\mathrm{T}}, \\
\boldsymbol{\Xi}_{23} &= -\boldsymbol{M}_3^{\mathrm{T}}(t) + (\boldsymbol{N}_3(t)\boldsymbol{B}\boldsymbol{G}(t_\gamma))^{\mathrm{T}} - \boldsymbol{N}_2^{\mathrm{T}}(t), \\
\boldsymbol{\Xi}_{24} &= -\boldsymbol{M}_4^{\mathrm{T}}(t) + (\boldsymbol{N}_4(t)\boldsymbol{B}\boldsymbol{G}(t_\gamma))^{\mathrm{T}} + \boldsymbol{N}_2^{\mathrm{T}}(t), \\
\boldsymbol{\Xi}_{33} &= h\boldsymbol{S}(t) - \boldsymbol{N}_3(t) - \boldsymbol{N}_3^{\mathrm{T}}(t), \\
\boldsymbol{\Xi}_{34} &= \boldsymbol{N}_3^{\mathrm{T}}(t) - \boldsymbol{N}_4^{\mathrm{T}}(t), \\
\boldsymbol{\Xi}_{44} &= -\rho\boldsymbol{Q}(t) + \boldsymbol{N}_4(t) + \boldsymbol{N}_4^{\mathrm{T}}(t).
\end{aligned}$$

If $\boldsymbol{Z}(t) < \boldsymbol{S}(s)$, we have

$$\begin{aligned}
\dot{V}(t) \leq\ & \boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{\Xi}\boldsymbol{\xi}(t) + h\boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{M}(t)\boldsymbol{Z}(t)^{-1}\boldsymbol{M}(t)^{\mathrm{T}}\boldsymbol{\xi}(t) \\
& - \int_{t-\tau(t)}^{t} [\boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{M}(t) + \dot{\boldsymbol{e}}(s)^{\mathrm{T}}\boldsymbol{S}(s)]\boldsymbol{S}^{-1}(s)[\boldsymbol{M}(t)^{\mathrm{T}}\boldsymbol{\xi}(t) + \boldsymbol{S}(s)\dot{\boldsymbol{e}}(s)]\mathrm{d}s. \qquad (19)
\end{aligned}$$

Thus, if we have $\boldsymbol{\Xi} + h\boldsymbol{M}(t)\boldsymbol{Z}(t)^{-1}\boldsymbol{M}(t)^{\mathrm{T}} < 0$, one can always find a sufficiently small $\sigma > 0$, for $\boldsymbol{e}(t) \neq 0$, satisfying

$$\dot{V}(t) < -\sigma\|\boldsymbol{x}(t)\|^2, \qquad (20)$$

which indicates that the closed-loop error system (7) is asymptotically stable.

Using the Schur complement lemma, $\boldsymbol{\Xi} + h\boldsymbol{M}(t)\boldsymbol{Z}(t)^{-1}\boldsymbol{M}(t)^{\mathrm{T}} < 0$ is equivalent to the following matrix inequality:

$$\boldsymbol{\Theta} = \begin{bmatrix} \boldsymbol{\Xi} & h\boldsymbol{M}(t) \\ * & -h\boldsymbol{Z}(t) \end{bmatrix} < 0. \qquad (21)$$

On the other hand, reordering the expression of

$\boldsymbol{\Theta}$, we can obtain

$$\boldsymbol{\Theta} = \sum_{k=1}^{p} w_k(\boldsymbol{x}(t_\gamma)) \left(\sum_{i=1}^{p} \sum_{j=1}^{p} w_i(\boldsymbol{x}(t)) w_j(\boldsymbol{x}(t)) \boldsymbol{\Gamma}_{ijk} \right), \tag{22}$$

where $\boldsymbol{\Gamma}_{ijk} = \begin{bmatrix} \boldsymbol{\Gamma}_{ijk}^{11} & \boldsymbol{\Gamma}_{ijk}^{12} & \boldsymbol{\Gamma}_{ijk}^{13} & \boldsymbol{\Gamma}_{ijk}^{14} & h\boldsymbol{M}_{1i} \\ * & \boldsymbol{\Gamma}_{ijk}^{22} & \boldsymbol{\Gamma}_{ijk}^{23} & \boldsymbol{\Gamma}_{ijk}^{24} & h\boldsymbol{M}_{2i} \\ * & * & \boldsymbol{\Gamma}_{ijk}^{33} & \boldsymbol{\Gamma}_{ijk}^{34} & h\boldsymbol{M}_{3i} \\ * & * & * & \boldsymbol{\Gamma}_{ijk}^{44} & h\boldsymbol{M}_{4i} \\ * & * & * & * & -h\boldsymbol{Z}_i \end{bmatrix}$, with

$$\begin{aligned}
\boldsymbol{\Gamma}_{ijk}^{11} &= \boldsymbol{Q}_i + \sum_{m=1}^{p} \phi_m(\boldsymbol{P}_m + \boldsymbol{X}) + \boldsymbol{M}_{1i} + \boldsymbol{M}_{1i}^{\mathrm{T}} \\
&\quad + \boldsymbol{N}_{1j}\boldsymbol{A}_i + (\boldsymbol{N}_{1j}\boldsymbol{A}_i)^{\mathrm{T}}, \\
\boldsymbol{\Gamma}_{ijk}^{12} &= \boldsymbol{M}_{2i}^{\mathrm{T}} - \boldsymbol{M}_{1i} + (\boldsymbol{N}_{2j}\boldsymbol{A}_i)^{\mathrm{T}} + \boldsymbol{N}_{1j}\boldsymbol{B}\boldsymbol{G}_k, \\
\boldsymbol{\Gamma}_{ijk}^{13} &= \boldsymbol{P}_i + \boldsymbol{M}_{3i}^{\mathrm{T}} + (\boldsymbol{N}_{3j}\boldsymbol{A}_i)^{\mathrm{T}} - \boldsymbol{N}_{1j}^{\mathrm{T}}, \\
\boldsymbol{\Gamma}_{ijk}^{14} &= \boldsymbol{M}_{4i}^{\mathrm{T}} + (\boldsymbol{N}_{4j}^{\mathrm{T}}\boldsymbol{A}_i)^{\mathrm{T}} + \boldsymbol{N}_{1j}^{\mathrm{T}}, \\
\boldsymbol{\Gamma}_{ijk}^{22} &= -\boldsymbol{M}_{2i} - \boldsymbol{M}_{2i}^{\mathrm{T}} + \boldsymbol{N}_{2j}\boldsymbol{B}\boldsymbol{G}_k + (\boldsymbol{N}_{2j}\boldsymbol{B}\boldsymbol{G}_k)^{\mathrm{T}} \\
\boldsymbol{\Gamma}_{ijk}^{23} &= -\boldsymbol{M}_{3i}^{\mathrm{T}} + (\boldsymbol{N}_{3j}\boldsymbol{B}\boldsymbol{G}_k)^{\mathrm{T}} - \boldsymbol{N}_{2j}^{\mathrm{T}}, \\
\boldsymbol{\Gamma}_{ijk}^{24} &= -\boldsymbol{M}_{4i}^{\mathrm{T}} + (\boldsymbol{N}_{4j}\boldsymbol{B}\boldsymbol{G}_k)^{\mathrm{T}} + \boldsymbol{N}_{2j}^{\mathrm{T}}, \\
\boldsymbol{\Gamma}_{ijk}^{33} &= h\boldsymbol{S}_i - \boldsymbol{N}_{3j} - \boldsymbol{N}_{3j}^{\mathrm{T}}, \\
\boldsymbol{\Gamma}_{ijk}^{34} &= \boldsymbol{N}_{3j}^{\mathrm{T}} - \boldsymbol{N}_{4j}^{\mathrm{T}}, \\
\boldsymbol{\Gamma}_{ijk}^{44} &= -\rho\boldsymbol{Q}_i + \boldsymbol{N}_{4j} + \boldsymbol{N}_{4j}^{\mathrm{T}}.
\end{aligned}$$

Then, inequality (21) holds if we have $\sum_{i=1}^{p}\sum_{j=1}^{p} w_i(\boldsymbol{x}(t))w_j(\boldsymbol{x}(t))\boldsymbol{\Gamma}_{ijk} < 0$. Moreover, if there exist appropriately dimensional matrices $\boldsymbol{R}_{ijk} = \boldsymbol{R}_{jik}^{\mathrm{T}}$ such that the following matrix inequalities hold:

$$\boldsymbol{\Gamma}_{iik} < R_{iik}, \tag{23}$$

$$\boldsymbol{\Gamma}_{ijk} + \boldsymbol{\Gamma}_{jik} < R_{ijk} + R_{jik}, \tag{24}$$

$$\begin{bmatrix} R_{11k} & R_{12k} & \dots & R_{1pk} \\ R_{21k} & R_{22k} & \dots & R_{2pk} \\ \vdots & \vdots & \ddots & \vdots \\ R_{p1k} & R_{p2k} & \dots & R_{ppk} \end{bmatrix} < 0. \tag{25}$$

$i, j, k = 1, 2, \dots, p;$

Thus we have the following fact:

$$\begin{aligned}
&\sum_{i=1}^{p}\sum_{j=1}^{p} w_i(\boldsymbol{x}(t))w_j(\boldsymbol{x}(t))\boldsymbol{\Gamma}_{ijk} \\
&= \sum_{i=1}^{p} w_i(\boldsymbol{x}(t))^2\boldsymbol{\Gamma}_{iik} + \sum_{i=1}^{p-1}\sum_{j=i+1}^{p} w_i(\boldsymbol{x}(t))w_j(\boldsymbol{x}(t))\boldsymbol{\Gamma}_{ijk} \\
&\le \sum_{i=1}^{p} w_i(\boldsymbol{x}(t))^2 R_{iik} + \sum_{i=1}^{p-1}\sum_{j=i+1}^{p} w_i(\boldsymbol{x}(t))w_j(\boldsymbol{x}(t))(R_{ijk} + R_{jik}) \\
&= \begin{bmatrix} w_1(\boldsymbol{x}(t))I \\ w_2(\boldsymbol{x}(t))I \\ \vdots \\ w_p(\boldsymbol{x}(t))I \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} R_{11k} & R_{12k} & \dots & R_{1pk} \\ R_{21k} & R_{22k} & \dots & R_{2pk} \\ \vdots & \vdots & \ddots & \vdots \\ R_{p1k} & R_{p2k} & \dots & R_{ppk} \end{bmatrix} \begin{bmatrix} w_1(\boldsymbol{x}(t))I \\ w_2(\boldsymbol{x}(t))I \\ \vdots \\ w_p(\boldsymbol{x}(t))I \end{bmatrix} \\
&< 0.
\end{aligned} \tag{26}$$

Let $\boldsymbol{N}_{mn} = d_{mn}\boldsymbol{\Omega}(m = 1, 2, 3, 4; n = 1, 2, \dots, p)$ where $\boldsymbol{\Omega}$ is a symmetrical non-singular matrix. Pre- and post-multiply both sides of (23), (24) and (25) by $\mathtt{diag}(\underbrace{\boldsymbol{\Omega}, \dots, \boldsymbol{\Omega}}_{5})$ and $\mathtt{diag}(\underbrace{\boldsymbol{\Omega}, \dots, \boldsymbol{\Omega}}_{5p})$ respectively, then applying the following of variables:

$$\begin{aligned}
&\tilde{\boldsymbol{P}}_i = \boldsymbol{\Omega}\boldsymbol{P}_i\boldsymbol{\Omega}, \quad \tilde{\boldsymbol{X}} = \boldsymbol{\Omega}\boldsymbol{X}\boldsymbol{\Omega}, \quad \tilde{\boldsymbol{Q}}_i = \boldsymbol{\Omega}\boldsymbol{Q}_i\boldsymbol{\Omega}, \quad \tilde{\boldsymbol{M}}_{mi} = \boldsymbol{\Omega}\boldsymbol{M}_{mi}\boldsymbol{\Omega}(m = 1, 2, 3, 4), \\
&\tilde{\boldsymbol{S}}_i = \boldsymbol{\Omega}\boldsymbol{S}_i\boldsymbol{\Omega}, \quad \tilde{\boldsymbol{R}}_{ijk} = \boldsymbol{\Omega}\boldsymbol{R}_{ijk}\boldsymbol{\Omega}, \quad \tilde{\boldsymbol{Z}}_i(t) = \boldsymbol{\Omega}\boldsymbol{Z}_i\boldsymbol{\Omega}, \quad \boldsymbol{F}_i = \boldsymbol{G}_i\boldsymbol{\Omega}.
\end{aligned} \tag{27}$$

Those LMIs (9)–(11) are derived. In other words, LMIs (9)–(12) are the stability conditions to guarantee error system (7). Moreover, the feedback gains matrices could be attained by $\boldsymbol{G}_j = \boldsymbol{F}_j\boldsymbol{\Omega}^{-1}, j = 1, 2, \dots, q$. This completes the proof.

Remark 2 As stated system (7), the term $\boldsymbol{m}_e(\boldsymbol{x}(t)$ is bounded as $w_i(\boldsymbol{x}(t)), \hat{w}_j(\hat{\boldsymbol{x}}(t))$ and $\hat{\boldsymbol{x}}(t)$ are bounded. For most actual settings, the membership functions $w_i(\boldsymbol{x}(t)), \hat{w}_j(\hat{\boldsymbol{x}}(t))$ are known for the underlying chaotic fuzzy systems and hence the value of ρ in inequality (17) can be calculated as follows:

$$\rho = \min_{\boldsymbol{x}(t),\hat{\boldsymbol{x}}(t)\in\mathcal{W}} \frac{\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{e}(t)}{\boldsymbol{m}_e^{\mathrm{T}}\boldsymbol{m}_e},$$

where $\mathcal{W}$ denotes a bounded compact set. However, in practical applications, the value of ρ can be estimated with the knowledge of known membership functions.

Remark 3 To drive the synchronisation conditions in terms of LMIs, the tuning parameter approach is used to convert the nonlinear matrix inequalities (23)–(25) into linear matrix inequalities (9)–(11). After slight modification, the numerical optimisation algorithm in Ref. [29] is also applicable for finding appropriate values of parameter d_{mn} in Theorem 1. Furthermore, the parameter-dependent bounded inequalities (17) is also introduced to add the freedom of the resulting optimisation problems.

Remark 4 In the process of controller design, a new parameter-dependent Lyapunov–Krasovskii functional (13), parameter-dependent type free-weighted matrices method (15), (16) and extra matrices introducing technique (23)–(25) for the underlying error system are used to further reduce conservatism. Under the LMI-based framework of Theorem 1, relationships between different fuzzy sub-systems are more efficiently considered while the computational burden increases a little as a trade-off.

4. Simulation results

In the simulation, the synchronisation of two Rössler systems using the proposed sampled-data fuzzy controller is presented to illustrate the effectiveness of the proposed results.

The dynamics of the response Rössler system[27] with input term is described as follows:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}(\boldsymbol{x}(t))\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t), \tag{28}$$

$$\boldsymbol{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix},$$

$$\boldsymbol{A}(\boldsymbol{x}(t)) = \begin{bmatrix} 0 & -1 & -1 \\ 1 & a & 0 \\ b & 0 & -(c(t) - x_1(t)) \end{bmatrix},$$

$$\boldsymbol{B} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},$$

where $a = 0.34$, $b = 0.4$, and $c(t) = [(c_2 + c_1)/2 + (c_2 - c_1)/2]\sin(t) \in [c_{\min}, c_{\max}] = [c_1, c_2] > 0$ is an uncertain value, with $c_1 = 4.5$ and $c_2 = 7.7$. It is assumed that $x_1(t) \in [c_{\min} - d, c_{\max} + d]$ and $d = 25$. The exact fuzzy model of the response Rössler system is represented by the following fuzzy rules:

Rules i: if $x_1(t)$ is M^i
then $\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_i\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t),\ i = 1, 2.$

The inferred response Rössler system is defined as

$$\dot{\boldsymbol{x}}(t) = \sum_{i=1}^{2} w_i(x_1(t))(\boldsymbol{A}_i\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t)), \tag{29}$$

where $A_1 = \begin{bmatrix} 0 & -1 & -1 \\ 1 & a & 0 \\ b & 0 & -d \end{bmatrix}, A_2 = \begin{bmatrix} 0 & -1 & -1 \\ 1 & a & 0 \\ b & 0 & d \end{bmatrix}$, $w_1(x_1(t)) = (1/2)(1 + (c(t) - x_1(t))/d)$, and $w_2(x_1(t)) = 1 - w_1(x_1(t))$.

On the other hand, the dynamics of the drive Rössler system is given as follows:

$$\dot{\hat{\boldsymbol{x}}}(t) = \hat{\boldsymbol{A}}(\hat{\boldsymbol{x}}(t))\hat{\boldsymbol{x}}(t), \tag{30}$$

where

$$\hat{\boldsymbol{x}}(t) = \begin{bmatrix} \hat{x}_1(t) \\ \hat{x}_2(t) \\ \hat{x}_3(t) \end{bmatrix},$$

$$\hat{\boldsymbol{A}}(\hat{\boldsymbol{x}}(t)) = \begin{bmatrix} 0 & -1 & -1 \\ 1 & a & 0 \\ b & 0 & -(\hat{c}(t) - \hat{x}_1(t)) \end{bmatrix},$$

and $\hat{c}(t) = (c_2 + c_1)/2 + (c_2 - c_1)/2\cos(t)/2$.

With $\hat{x}_1(t) \in [c_{\min} - d, c_{\max} + d]$, the drive Rössler system can be represented by a fuzzy model with the following rules:

Rules i: if $\hat{x}_1(t)$ is $\bar{\boldsymbol{M}}^i$,
then $\dot{\hat{\boldsymbol{x}}}(t) = \boldsymbol{A}_i\hat{\boldsymbol{x}}(t), i = 1, 2.$

Thus, the drive Rössler's system is defined as

$$\dot{\hat{\boldsymbol{x}}}(t) = \sum_{i=1}^{2} \hat{w}_i(\hat{x}_1(t))\boldsymbol{A}_i\hat{\boldsymbol{x}}(t), \tag{31}$$

where $\hat{w}_1(\hat{x}_1(t)) = (1/2)(1 + (\hat{c}(t) - \hat{x}_1(t))/d)$ and $\hat{w}_2(\hat{x}_1(t)) = 1 - \hat{w}_1(\hat{x}_1(t))$.

A two-rule sampled-data fuzzy controller is employed to achieve synchronisation. The rules of the sampled-data fuzzy controller are defined as follows.

Rules i: if $x_1(t_\gamma)$ is N^j,
then $\boldsymbol{u}(t) = \boldsymbol{G}_j \boldsymbol{e}(t_\gamma), t_\gamma < t \leq t_{\gamma+1}$.

The control output of the sample-data fuzzy controller is defined as follows:

$$\boldsymbol{u}(t) = \sum_{j=1}^{2} w_j(x_1(t_\gamma)) \boldsymbol{G}_j \boldsymbol{e}(t_\gamma). \tag{32}$$

Due to these assumptions $x_1(t) \in [c_{\min} - d, c_{\max} + d]$ and $\hat{x}_1(t) \in [c_{\min} - d, c_{\max} + d]$, the error signal $e_1(t) \in [c_{\min} - c_{\max} - 2d, c_{\max} - c_{\min} + 2d]$. Thus, using $\rho = \min_{\boldsymbol{x}(t), \hat{\boldsymbol{x}}(t) \in \mathcal{W}} \boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{e}(t)/\boldsymbol{m}_e^{\mathrm{T}}\boldsymbol{m}_e$, we ccan select $\rho = 0.06$.

Based on Theorem 1, with those parameters selected as $h = 0.01$ s, $d_{11} = 0.2$, $d_{12} = 0.2$, $d_{21} = 1.2$, $d_{22} = 1.2$, $d_{31} = 0.6$, $d_{32} = 0.6$, $d_{41} = 0.3$, $d_{42} = 0.3$, the control gain matrices $\boldsymbol{G}_j$ is calculated as follows:

$$\boldsymbol{G}_1 = \begin{bmatrix} 765.3652 & 3466.3652 & -61.2365 \end{bmatrix},$$

$$\boldsymbol{G}_2 = \begin{bmatrix} 841.5698 & 3692.2643 & -56.6532 \end{bmatrix}.$$

The proposed sampled-data fuzzy controller is employed to synchronise the two chaotic systems. Figure 2 shows the system state responses of the two chaotic systems with the initial conditions $\boldsymbol{x}(0) = [\,1 \;\; 0.5 \;\; -1\,]$ and $\hat{\boldsymbol{x}}(0) = [\,0.5 \;\; 1 \;\; -0.5\,]$ respectively. The control signal is applied when $t \geq 40$ s. After the control signal is applied at $t = 40$ s, the synchronisation is achieved within a short period of time. Figure 3 shows the system state responses of the error system. Referring to this figure, it can be seen that the errors between the system states of both drive and response chaotic systems become sufficiently small for $t \geq 40$ s, which indicates that synchronisation has been successfully achieved.

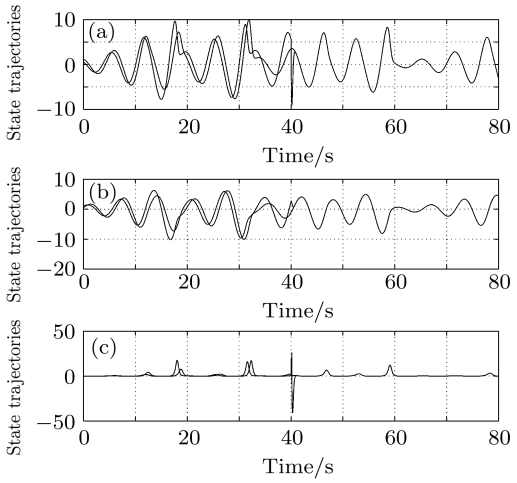


Fig. 2. State trajectories of $\boldsymbol{x}(t)$ and $\hat{\boldsymbol{x}}(t)$ with $\boldsymbol{u}(t) = 0$ for $0 \leq t < 40$ s and the sampled-data fuzzy controller for $t \geq 40$ s. (a) $x_1(t)$ and $\hat{x}_1(t)$; (b) $x_2(t)$ and $\hat{x}_2(t)$; (c) $x_3(t)$ and $\hat{x}_3(t)$.

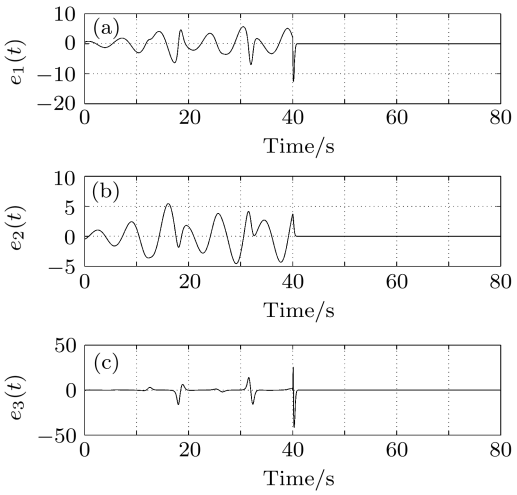


Fig. 3. Tracking errors between the response and drive Rössler's systems.

5. Conclusions

In this paper, we have investigated the problem of synchronising two chaotic systems using a sampled-data fuzzy controller. This class of sampled-data fuzzy controller can be implemented efficiently using microcontrollers or digital computers to lower the implementation cost and time. LMI-based synchronising conditions have been derived by using a new parameter-dependent Lyapunov–Krasovskii functional and relaxed quadratic stabilisation techniques for the closed-loop sampled-data error system. An application example has been given to illustrate the effectiveness and the design procedure of the proposed approach. Searching for more relaxed LMI-based synchronizing conditions and novel modeling methods of nonlinear chaotic systems would be an interesting future topic.

References

[1] Boccaletti S, Kurths J, Osipov G, Valladares D L and Zhou C S 2002 *Phys. Rep.* **366** 1

[2] Pecora L M and Carroll T L 1990 *Phys. Rev. Lett.* **64** 821

[3] Lü L, Guo Z A and Zhang C 2007 *Chin. Phys.* **16** 1603

[4] Huang H, Li H X and Zhong J 2006 *Int. J. Bifurc. Chaos* **16** 281

[5] Chen M Y and Han Z Z 2003 *Chaos, Solitons and Fractals* **17** 709

[6] Lü L, Zhang Q L and Guo Z A 2008 *Chin. Phys.* B **17** 498

[7] Lü L, Guo Z A, Li Y and Xia X L 2007 *Acta Phys. Sin.* **56** 95 (in Chinese)

[8] Yassen M T 2007 *Phys. Lett.* A **360** 582

[9] Liu Y F, Yang X G, Miao D and Yuan R P 2007 *Acta Phys. Sin.* **56** 6250 (in Chinese)

[10] Huang G Y, Jiang C S and Wang Y H 2007 *Acta Phys. Sin.* **56** 6224 (in Chinese)

[11] Zhang H, Ma X K and Liu W Z 2004 *Chaos, Solitons and Fractals* **21** 1249

[12] Lou X Y and Cui B T 2008 *Chin. Phys.* B **17** 520

[13] Hu J and Zhang Q J 2008 *Chin. Phys.* B **17** 503

[14] Zhang H G, Huang W, Wang Z L and Chai T Y 2006 *Phys. Lett.* A **350** 363

[15] Zhang H G,, Xie Y H, Wang Z L and Zheng C D 2007 *IEEE Trans. Neural Netw.* **18** 1841

[16] Ma T D, Zhang H G and Wang Z L 2007 *Acta Phys. Sin.* **56** 3796 (in Chinese)

[17] Chen S H, Yang Q and Wang C P 2004 *Chaos, Solitons and Fractals* **20** 751

[18] Li Y, Liao X F, Li C D and Chen G 2006 *Chin. Phys.* **15** 2890

[19] Haeri M and Dehghani M 2008 *Chaos, Solitons and Fractals* **38** 120

[20] Zhang R, Xu Z Y and He X M 2007 *Chin. Phys.* **16** 1912

[21] Njah A N and Vincent U E 2008 *Chaos, Solitons and Fractals* **37** 1356

[22] Fridman E, Seuret A and Richard J P 2004 *Automatica* **40** 1441

[23] Zhang H G, Zhao Y, Yu W and Yang D S 2008 *Chin. Phys.* B **17** 4056

[24] Zhang L L, Huang L R, Zhang Z Z and Wang Z Y 2008 *Phys. Lett.* A **372** 6082

[25] Lam H K, Ling W K, Herbert H C and Steve S H 2008 *IEEE Trans. on Circuits and Systems-I: Regular Papers* **55** 893

[26] Ma D Z, Zhang H G, Wang Z S and Feng J 2010 *Chin. Phys.* B **19** 050506

[27] Tanaka K, Ikeda T and Wang H O 1998 *IEEE Trans. on Circuits and Systems-I: Fundamental Applications* **45** 1021

[28] Tanaka K, Hori T and Wang H O 2003 *IEEE Trans. on Fuzzy Systems* **11** 582

[29] Zhang X M, Wu M, She J H and He Y 2005 *Automatica* **41** 1405