Interference phase of mass neutrino in Reissner–Nordström–de Sitter space–time*

Huang He(黄 河) and Wang Yong-Jiu(王永久)†

Institute of Physics, Hunan Normal University, Changsha 410081, China

(Received 30 November 2009; revised manuscript received 15 April 2010)

In Reissner–Nordström–de Sitter space–time, we calculate the interference phase of mass neutrino along geodesic in the radial direction, and then investigate the effects of the cosmological constant Λ on the phase. Morever, the expression of the interference phase can be reduced to that in Reissner–Nordström space–time when Λ approaches to zero.

Keywords: interference phase, Reissner–Nordström–de Sitter space–time, mass neutrino
PACC: 9760L, 0460, 0470

1. Introduction

On account of the great success of the Super–Kamiokande atmospheric neutrino experiment to assure the neutrino mass,[1] the mass neutrino oscillation has been a hot topic recently. Due to the theoretical considerations, the desciption of neutrino oscillation in the flat space–time should be natrually extended to that in the curved spacetime as the gravitational background is taken into account. Various theoretical considerations on the gravitationally induced neutrino oscillation were proposed to deal with the above case.[2,3] More recently, the gravitational effects on the neutrino have received much attention of a number of authors to calculate the interference phase of mass neutrino in different gravitational fields,[4−10] including plane wave approximation, wave packet treatment and standard treatment.[3−5]

In 1996, Fornengo *et al.*[5] studied the interference phase of relativistic neutrinos in the cases of radial and non-radial propagations, by using plane wave analysis, in Schwarzschild space–time. In 1997, Cardall and Fuller[6] developed a simple formalism for treating neutrino oscillations in a curved space–time. In 2001, Zhang and Beesham[7] used a unified description of the phases for both high energy particle and low energy particle to calculate the gravitationally induced interference phase of mass neutrino, and they found that in the high energy limit, the phase along the null is equivalent to the half phase along the geodesic. In recent years, our group have made an effort to study the neutrino oscillation. Huang and Wang investigated the interference phase of mass neutrino in Kerr space–time in Ref. [8] and Reissner–Nordström (RN) space–time in Ref. [9]. According to the previous research, we extend our work to Reissner–Nordström–de Sitter space–time.

In this paper, we calculate the interference phase of the mass neutrinos in the Reissner–Nordström–de Sitter space–time, and analyse the effects on the interference phase because of the existence of the cosmological constant in the gravitational field. In addition, we will discuss the reduced expressions of the phase under some particular conditions. We set $G = c = \hbar = 1$ throughout this paper.

2. Geodesic phase of mass neutrino in Reissner–Nordström–de Sitter space–time

Reissner–Nordström–de Sitter space–time is written as [11]

$$\mathrm{d}s^2 = g_{00}\mathrm{d}t^2 + g_{11}\mathrm{d}r^2 + g_{22}\mathrm{d}\theta^2 + g_{33}\mathrm{d}\varphi^2, \quad (1)$$

where

$$\begin{aligned} g_{00} &= 1 - \frac{2M}{r} + \frac{Q^2}{r^2} + \frac{\Lambda r^2}{3}, \\ g_{11} &= -(1 - \frac{2M}{r} + \frac{Q^2}{r^2} + \frac{\Lambda r^2}{3})^{-1}, \\ g_{22} &= -r^2, \\ g_{33} &= -r^2 \sin^2\theta, \end{aligned} \quad (2)$$

*Project supported by the State Key Development Program for Basic Research Program of China (Grant No. 2010CB832803), the National Natural Science Foundation of China (Grant No. 10873004), the Scientific Research Fund of Hunan Provincial Education Department, China (Grant No. 08B051).

†Corresponding author. E-mail: wyj@hunnu.edu.cn

© 2010 Chinese Physical Society and IOP Publishing Ltd

where M and Q are the gravitational mass and the electric charge of the star respectively, and Λ is the cosmological constant.

We consider the phase of the mass neutrino in Reissner–Nordström–de Sitter space–time by using the following particle geodesic equations:

$$\frac{\mathrm{d}^2x^\lambda}{\mathrm{d}s^2}+\Gamma^\lambda_{\mu\nu}\frac{\mathrm{d}x^\mu}{\mathrm{d}s}\frac{\mathrm{d}x^\nu}{\mathrm{d}s}=0, \tag{3}$$

$$\Gamma^\lambda_{\mu\nu}=\frac{1}{2}g^{\lambda\tau}(g_{\mu\tau,\nu}+g_{\tau\nu,\mu}-g_{\nu\mu,\tau}). \tag{4}$$

Along the radial direction, $\mathrm{d}\theta=\mathrm{d}\varphi=0$, and the non-trivial time-like equation is

$$\frac{\mathrm{d}^2t}{\mathrm{d}s^2}+2\Gamma^0_{01}\frac{\mathrm{d}t}{\mathrm{d}s}\frac{\mathrm{d}r}{\mathrm{d}s}=0, \tag{5}$$

$$\Gamma^0_{01}=\frac{1}{2}g^{00}\frac{\mathrm{d}g_{00}}{\mathrm{d}r}. \tag{6}$$

Subsituting Eq. (6) into Eq. (5), we obtain

$$\frac{\mathrm{d}}{\mathrm{d}s}\left(g_{00}\frac{\mathrm{d}t}{\mathrm{d}s}\right)=0. \tag{7}$$

Equation (7) gives

$$g_{00}\frac{\mathrm{d}t}{\mathrm{d}s}=\text{const}, \quad \text{or}$$

$$E=P_0=mg_{00}\frac{\mathrm{d}t}{\mathrm{d}s}=\text{const}. \tag{8}$$

Equation (8) represents that the covariant energy is the motion constant along the geodesic, with which the calculation of the phase is proceeded. It is necessarily noticed that it is the covariant energy P_0, the constant of motion, and the value of this constant E is the asymptotic energy of the neutrino at $r=\infty$.[12]

The mass shell condition can be written in Reissner–Nordström–de Sitter space–time as

$$g_{\mu\nu}P^\mu P^\nu=m^2. \tag{9}$$

Considering the radial direction, $\mathrm{d}\theta=\mathrm{d}\varphi=0$, Eq. (1) can be rewritten as

$$1=g_{00}\left(\frac{\mathrm{d}t}{\mathrm{d}s}\right)^2+g_{11}\left(\frac{\mathrm{d}r}{\mathrm{d}s}\right)^2. \tag{10}$$

According to Eqs. (8) and (9), we obtain

$$\begin{aligned}\frac{\mathrm{d}s}{\mathrm{d}r}&=\left[-g_{00}g_{11}/\left(\frac{P_0^2}{m^2}-g_{00}\right)\right]^{1/2}\\&=[-g_{00}g_{11}/(k+1-g_{00})]^{1/2}\\&=\left[k+\frac{2M}{r}-\frac{Q^2}{r^2}+\frac{\Lambda r^2}{3}\right]^{1/2},\end{aligned} \tag{11}$$

where $k=(P_0/m)^2-1$.

In the curved space–time, the interference phase can be written as[13]

$$\Phi=\int m\mathrm{d}s=\int P_\nu\mathrm{d}x^\nu=\int g_{\mu\nu}P^\mu\mathrm{d}x^\nu, \tag{12}$$

where Φ is the covariant phase, and m is the mass of neutrino. Then we obtain

$$\begin{aligned}\Phi_{(\mathrm{geod})}&=\int m\frac{\mathrm{d}s}{\mathrm{d}r}\mathrm{d}r\\&=m\int\left[k+\frac{2M}{r}-\frac{Q^2}{r^2}+\frac{\Lambda r^2}{3}\right]^{1/2}\mathrm{d}r.\end{aligned} \tag{13}$$

As the cosmological constant Λ is a small quantity, expanding Eq. (9) in terms of Λ, we obtain

$$\begin{aligned}\Phi_{(\mathrm{geod})}&\simeq\int\frac{m\mathrm{d}r}{\sqrt{k-Q^2/r^2+2M/r}}-\int\frac{m\Lambda r^2\mathrm{d}r}{6(k-Q^2/r^2+2M/r)}\\&=\frac{mr}{k}\sqrt{kr^2+2Mr-Q^2}-\frac{Mm}{k^{3/2}}\ln[M+kr+\sqrt{kr^2+2Mr-Q^2}]\\&\quad-\frac{m\Lambda}{3}\left\{\frac{1}{6k\sqrt{kr^2+2Mr-Q^2}}r^4-\frac{7M}{12k^2\sqrt{kr^2+2Mr-Q^2}}r^3\right.\\&\quad+\left(\frac{2Q^2}{3k^2\sqrt{kr^2+2Mr-Q^2}}+\frac{35M^2}{12k^3\sqrt{kr^2+2Mr-Q^2}}\right)r^2\\&\quad\left.-\frac{15M(7M^4+10kM^2Q^2+3k^2Q^4)}{4k^{9/2}(M^2+kQ^2)}\ln(M+kr+\sqrt{k(kr^2+2Mr-Q^2)})\right\}.\end{aligned} \tag{14}$$

(15)

Expanding Eq. (11) in terms of electric charge Q, we obtain

$$\begin{aligned}\Phi_{(\mathrm{geod})}&\simeq\frac{m}{k}\sqrt{kr^2+2Mr}-\frac{Mm}{k^{3/2}}\ln(kr+M+\sqrt{k(kr^2+2Mr)})\\&\quad+\frac{m}{2\sqrt{k}}\frac{\sqrt{kr^2+2Mr}+\sqrt{k}}{\sqrt{kr^2+2Mr}(kr+M+\sqrt{k(kr^2+2Mr)})}\end{aligned}$$

$$-\frac{2m\Lambda}{k\sqrt{k+2M/r}}r^3+\frac{7Mm\Lambda}{k^2\sqrt{k+2M/r}}r^2-\left(\frac{35M^2m}{k^3\sqrt{k+2M/r}}-\frac{mQ^2\Lambda}{4k(k+2M/r)^{3/2}}\right)r$$
$$-\frac{5M^2mQ^2\Lambda}{k^3(k+2M/r)^{3/2}}r^{-1}-\frac{210M^2m\Lambda}{k^4\sqrt{k+2M/r}}-\frac{10MmQ^2\Lambda}{3k^2(k+2M/r)^{3/2}}$$
$$+\frac{35M^3m\Lambda}{12k^{9/2}}\ln(kr+M+\sqrt{k(kr^2+2Mr)})+\frac{5Mm\Lambda}{4k^{7/2}}\ln(kr+M+\sqrt{k(kr^2+2Mr)})+o[Q^4]. \quad (16)$$

The first three terms on the right-hand side of Eq. (14) is analogous to the expression of the phase in RN space–time. And the rest describe the contributions by the cosmological constant.

When $r\sim\infty$, the gravitational field is quite weak. Substituting the upper integral limit r and lower limit r_0 into Eq. (14) separately, the interference phase is reduced to

$$\begin{aligned}\Phi_{(\mathrm{geod})} \simeq\ & \frac{m}{\sqrt{k}}(r-r_0)-\frac{Mm}{k^{3/2}}\ln(\frac{2kr+M}{2kr_0+M})+\frac{mQ^2}{k^{3/2}}\left(\frac{1}{r}-\frac{1}{r_0}\right)\\ &-\frac{2m}{k^{3/2}}(r^3-r_0^3)\Lambda+\frac{7Mm}{k^{5/2}}(r^2-r_0^2)\Lambda\\ &-\left(\frac{35M^2m}{k^{7/2}}+\frac{mQ^2}{4k^{5/2}}\right)(r-r_0)\Lambda-\frac{5M^2m\Lambda Q^2}{k^3}\left(\frac{1}{r}-\frac{1}{r_0}\right)\Lambda\\ &+\left[\frac{35M^3m}{12k^{9/2}}\ln\left(\frac{2kr+M}{2kr_0+M}\right)+\frac{5Mm}{4k^{7/2}}\ln\left(\frac{2kr+M}{2kr_0+M}\right)\right]\Lambda+o[Q^4]. \quad (17)\end{aligned}$$

If we consider the high energy limit, which means that k is a large quantity, the phase will be approximately rewritten as

$$\begin{aligned}\Phi_{(\mathrm{geod})} \simeq\ & \frac{m}{\sqrt{k}}(r-r_0)-\frac{Mm}{k^{3/2}}\ln\left(\frac{2kr+M}{2kr_0+M}\right)\\ &+\frac{mQ^2}{k^{3/2}}\left(\frac{1}{r}-\frac{1}{r_0}\right)-\frac{2m}{k^{3/2}}(r^3-r_0^3)+o[Q^4]. \quad (18)\end{aligned}$$

Here, those terms with higher orders about k have been ignored. The first three terms play a principal role in the interference phase, and the last term can be regarded as the correction of the cosmological constant acting on the phase in RN space–time.

3. Conclusion and perspective

In this paper, we calculate the interference phase of the mass neutrino propagating in the radial direction along the geodesic in the Reissner–Nordström–de sitter space–time. On condition that the energy of the neutrino is extraordinarily high, we find that the cosmological constant Λ will, to some extent, affect the interference phase. Especially, if $\Lambda=0$, the phase in the Reissner–Nordström–de Sitter space–time will naturally reduce to that in the RN space–time. Furthermore, if $\Lambda=0$ and $Q=0$, the phase in the Reissner–Nordström–de sitter space–time will ultimately reduce to that in the Schwarzchild space–time. At present, the mass neutrino oscillation has been attracting more comprehensive attention, and is frequently related to other fields of physics.[14,15] With the further research on it, we are convinced that it will inevitably be a promissing topic in the future.

References

[1] Fukuda Y, Hayakawa T, Ichihara E, Inoue K, Ishihara K, Ishino H, Itow Y, Kajita T, Kameda J, Kasuga S, Kobayashi Y, Koshio Y, Martens K, Miura M, Nakahata M, Nakayama S, Okada A, Oketa M, Okumura K, Ota M, Sakurai N, Shiozawa M, Suzuki Y, Takeuchi Y, Totsuka Y, Yamada S, Earl M, Habig A, Hong J T, Keams E, Kim S B, Masuzawa M, Messier M D, Scholberg K, Stone J L, Sulak L R, Walter C W, Goldhaber M, Barszczak T, Gajewski W, Halverson P G, Hsu J, Kropp W R, Price L R, Reines F, Sobel H W, Vagins M R, Ganezer K S, Keig W E, Ellsworth R W, Tasaka S, Flanagan J W, Kibayashi A, Learned J G, Matsuno S, Stenger V, Takemori D, Ishii T, Kanzaki J, Kobayashi T, Nakamura K, Nishikawa K, Oyama Y, Sakai A, Saluda M, Sasaki O, Echigo S, Ko-

hama M, Suzuki A T, Haines T J, Blaufuss E, Sanford R, Svoboda R, Chen M L, Conner Z, Goodman J A, Sullivan G W, Mori M, Hill J, Jung C K, Mauger C, McGrew C, Sharkey E, Viren B, Yanagisawa C, Doki W, Ishizuka T, Kitaguchi Y, Koga H, Miyano K, Okazawa H, Saji C, Takahata M, Kusano A, Nagashima Y, Takita M, Yamaguchi T, Yoshida M, Etoh M, Fujita K, Hasegawa A, Hasegawa T, Hatakeyama S, Iwamoto T, Kinebuchi T, Koga M, Maruyama T, Qgawa H, Suzuki A, Tsushima F, Koshiba M, Nemoto M, Nishijima K, Futagami T, Hayato Y, Kanaya Y, Kaneyuki K, Watanabe Y, Kielczewska D, Doyle R, George J, Stachyra A, Wai L, Wilkes J and Young K 1998 *Phys. Rev. Lett.* **81** 1158

[2] Wudka J 1991 *Mod. Phys. Lett.* A **6** 3291

[3] Ahluwalia D V and Burgard C 1996 *Gen. Rel. Grav.* **28** 1161

[4] Kojima Y 1996 *Mod. Phys. Lett.* A **11** 2965

[5] Fornengo N, Giunit C, Kim C W and Song J 1997 *Phys. Rev.* D **56** 1895

[6] Cardall C Y and Fuller G M 1997 *Phys. Rev.* D **55** 7960

[7] Zhang C M and Beesham A 2001 *Gen. Rel. Grav.* **33** 1011

[8] Huang X J and Wang Y J 2003 *Commun. Theor. Phys.* **40** 742

[9] Huang X J and Wang Y J 2004 *Chin. Phys.* **13** 1588

[10] Chen X and Wang Y J 2009 *Chin. Phys.* B **18** 1707

[11] Lake K 1979 *Phys. Rev.* D **19** 420

[12] Bhattacharya T, Habib S and Mottola E 1996 gr-qc/9605074

[13] Stodolsky L 1979 *Gen. Rel. Grav.* **11** 391

[14] Akhmedov E K, Maltoni M and Smirnov A Y 2008 *J. High. Energy. Phys.* **0806** 072

[15] Cuesta H J M and Lambiase G 2009 *Int. Mod. Phys. Lett.* **18** 435