Effect of strong magnetic field on chemical potential and electron capture in magnetar*

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The chemical potential of electrons in a strong magnetic field is investigated. It is shown that the magnetic field has only a slight effect on electron chemical potential when $B < 10^{11}$ T, but electron chemical potential will decrease greatly when $B > 10^{11}$ T. The effects of a strong magnetic field on electron capture rates for ^{60}Fe are discussed, and the result shows that the electron capture sharply decreases because of the strong magnetic field.

Keywords: strong magnetic field, electron chemical potential, electron capture, magnetar
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1. Introduction

In the past decades the investigation of electron capture has played a major role in understanding weak interactions and their structure. Despite being a little bit out of fashion, there are still interesting problems to be investigated. Weak interactions play an important role in a variety of astrophysical phenomena. In particular, the evolution of massive stars in their pre-supernova stage is strongly affected by electron capture rate.[1−5] Electron capture leads to the deleptonization of the stellar environment. Isotopic composition of the star is changed and the dynamics of the collapse process is modified.[6,7] Moreover, the process of electron capture produces lots of neutrinos, which can be used for the neutrino-related research.[8−10] Electron capture is very important in the astrophysical evolution, and sensitive to astrophysical environment, so it is necessary to investigate the influence of astrophysical environment on electron capture.

The existence of strong magnetic is one of the main factors affecting electron capture, and magnetic field is one of primary features in astrophysical environment. So, it is meaningful to investigate the electron capture in a strong magnetic field. At present, there is compelling evidence that many compact astronomical objects (soft gamma-ray repeaters, anomalous x-ray pulsars, and some radio pulsars) identified with neutron stars have surface magnetic fields as high as 10^{10}–10^{11} T.[11−13] Recent observations of the explosive phenomena like magnetars and gamma ray bursts indicate that the inferred field strengths around the compact objects are larger than 10^{11} T.[14−16] Some magnetars are observed to have magnetic fields of 10^{10}–10^{11} T.

In recent years a lot of interesting studies have been performed on the properties of dense astrophysical and cosmological matters in the presence of strong magnetic fields.[17−23] The decreasing of electron chemical potential and electron capture rate in strong magnetic field is one of these phenomena. In the present work, we first investigate the influence of a strong magnetic on electron chemical potential, then further study the effects on electron capture rates (take ^{60}Fe for example), with considering the stellar environment of strong and sometimes super-strong magnetic field.

2. Electron gas and electron chemical potential in strong magnetic field

The energy spectrum of the electron in a strong magnetic field is obtained by solving the Dirac equation for a homogeneous magnetic field. The positive energy levels are given by[24]

$$\varepsilon_n = \left(p_z^2c^2 + m_\mathrm{e}^2c^4 + 2n\hbar\omega_\mathrm{B}m_\mathrm{e}c^2\right)^{1/2}, \quad (n = 0, 1, 2, 3\ldots). \tag{1}$$

When electron energy is in units of $m_\mathrm{e}c^2$ and the momentum in units of $m_\mathrm{e}c$, Eq. (1) can be substituted

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by[25]

$$\varepsilon_{\rm n} = \left(p_z^2 + 1 + 2n\Theta\right)^{1/2}, \quad (n = 0, 1, 2, 3\ldots), \quad (2)$$

where $\Theta = heB/(m_{\rm e}^2c^3) = 0.0227B_{12}$, $B = 10^{12}B_{12}$, and P_z is the electron momentum along the field, thus the electron number density is

$$n_{\rm e} = \frac{\rho}{\mu_{\rm e}} = \frac{\Theta}{2\pi^2\lambda_{\rm e}^3}\sum_{n=0}^{\infty} q_{n0}\int_0^{\infty}(f_{-e} - f_{+e})\mathrm{d}p_z, \quad (3)$$

where ρ is the density in g/cm^3, $\mu_{\rm e}$ is the average molecular weight, $\lambda_{\rm e} = h/(m_{\rm e}c)$ is Compton wavelength, and $q_{n0} = 2 - \delta_{n0}$ is the electron degenerate number. The electron and the positron distribution functions are f_{-e} and f_{+e} respectively.[25]

Figures 1 and 2 show the variations of the Fermi energy with strong magnetic field at several temperatures, $n_{\rm e} = 10^7$ and 10^8 g·cm^{-3}.

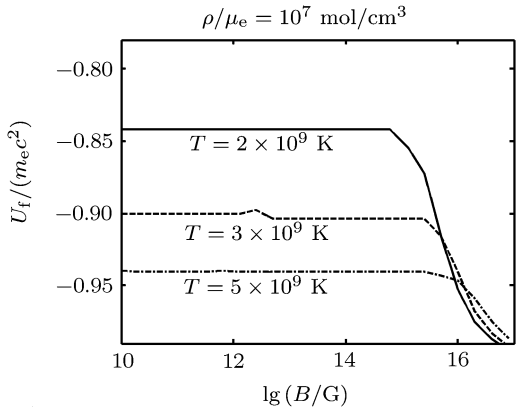


Fig. 1. Curves for chemical potential, $U_{\rm f}(m_{\rm e}c^2)$, as a function of magnetic field, with $\rho/\mu_{\rm e} = 10^7$ mol·cm^{-3} , and $T = 2\times10^9$ (solid line), 3×10^9 (dash line), 5×10^9 (dot and dash line) K (1 G=10^{-4} T).

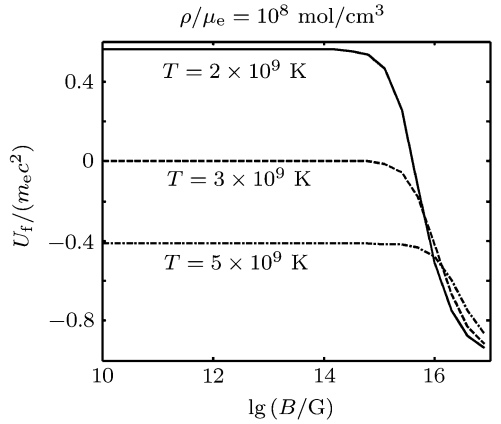


Fig. 2. Curves for chemical potential, $U_{\rm f}/(m_{\rm e}c^2)$, as a function of magnetic field, with $\rho/\mu_{\rm e} = 10^8$ mol·cm^{-3} , and $T = 2\times10^9$ (solid line), 3×10^9 (dash line), 5×10^9 (dot and dash line) K.

Numerical simulations show that for magnetic fields below 10^{11} T, the Fermi energy is basically constant, but it will decrease greatly when $B > 10^{11}$ T, approaching to $-m_{\rm e}c^2$ in the end. This is because that the greater the magnetic field is, the more obvious the quantum effect will be, and that the larger the interval between the Landau levels is, the more imhomogeneous the distribution, and thus the greater the variation of electron chemical potential will be. When the magnetic field becomes strong enough so as to bound electron in ground state, electron chemical potential will approach to $-m_{\rm e}c^2$.

3. Effect on electron capture of ^{60}Fe in strong magnetic field

The electron quantum effect is obvious, due to the existence of strong magnetic fields. Magnetic field will change the distribution of the electronic gas, and the stronger the magnetic field is, the more nonuniform the electronic distribution of gas will become. That is to say, the existence of the magnetic field will change the electronic distribution of the space phase factor, thus changing the electronic gas and nuclear chemical potential rate of electron capture.

In the absence of magnetic field, the phase space factor $f(Q_{\rm if})$ in the energy space can be expressed as[26,27]

$$f\left(Q_{\rm if}\right) = \int_{\varepsilon_0}^{\infty} \varepsilon^2\left(Q_{\rm if} + \varepsilon\right)^2 G\left(z, \varepsilon\right) f_{\varepsilon}\mathrm{d}\varepsilon, \quad (4)$$

where $G(z, \varepsilon)$ is the Coulomb wave correction.[27]

In the presence of magnetic field, the phase space factor can be expressed as

$$f_{\rm if}^B = \frac{\Theta}{2}\sum_{n=0}^{\infty} q_{n0}\int_{\varepsilon_0}^{\infty}\left(Q_{\rm if} + \varepsilon_n\right)^2\varepsilon_n^2 G\left(z, \varepsilon_n\right) \times\frac{f_{\varepsilon_n}}{\varepsilon_n^2 - 2n\Theta - 1}\mathrm{d}\varepsilon_n, \quad (5)$$

where ε_0 is defined as

$$\varepsilon_0 = \begin{cases} |Q_{\rm if}| & \text{when } Q_{\rm if} < -\sqrt{1+2n\Theta}, \\ \sqrt{1+2n\Theta} & \text{when } Q_{\rm if} \geq -\sqrt{1+2n\Theta}. \end{cases} \quad (6)$$

The electron capture rate can be expressed as[24,26,28]

$$\lambda = \lambda_0 + \lambda_{\rm GT}. \quad (7)$$

Here $\lambda_0 = \ln 2 \times B_{\rm eff} f_{00}^B$ is the electron capture rate of the magnetic field of the transition zone of the

low energy; $\lambda_{\rm GT} = \ln 2 B_{\rm GT} f(Q_{00} - E_{\rm GT})$ is the electron capture rate of resonance transition zone. $E_{\rm GT}$ and $M_{\rm GT}$ are the location and the matrix element of GT resonance transition respectively. The detailed calculations of them can be found from relevant references.[26,29]

In the presence of magnetic field, electron capture rate can be expressed as

$$\lambda = \ln 2 \times \frac{\Theta}{2} \sum_{n=0}^{\infty} q_{n0} \int_{\varepsilon_0}^{\infty} \frac{[B_{\rm eff}\,(Q_{00}+\varepsilon_n)^2 + B_{\rm GT}(Q_{00} - E_{\rm GT} + \varepsilon_n)^2]\varepsilon_n^2 G\,(z,\varepsilon_n)\, f_{\varepsilon_n}}{\varepsilon_n^2 - 2n\Theta - 1} {\rm d}\varepsilon_n \tag{8}$$

where ε_0 is defined as that in Eq. (6).

Figures 3 and 4 show the variations of the electron capture rate of ^{60}Fe with strong magnetic field for several temperatures, $n_{\rm e} = 10^7$, and 10^8 g$\cdot$ cm^{-3}.

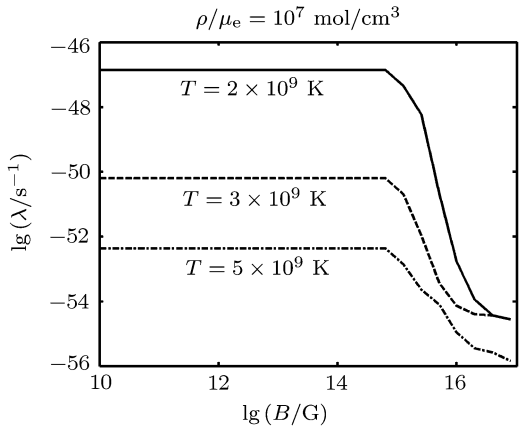


Fig. 3. Curves for electron capture rate, λ (s^{-1}), as a function of magnetic field, with $\rho/\mu_{\rm e} = 10^7$ mol$\cdot$cm^{-3}, $T = 2 \times 10^9$ (solid line), 3×10^9 (dash line), and 5×10^9 dot and dash line) K.

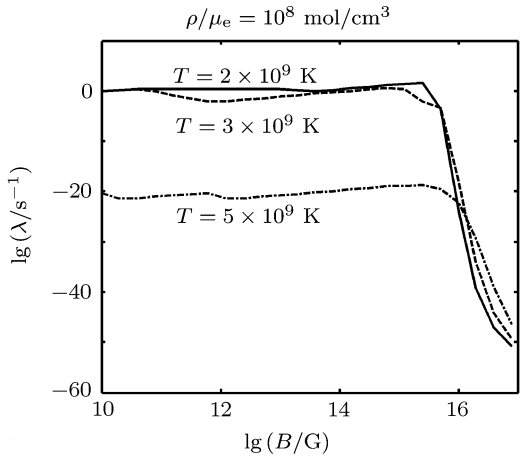


Fig. 4. curves for electron capture rate, λ (s^{-1}), as a function of magnetic field with $\rho/\mu_{\rm e} = 10^8$ mol$\cdot$cm^{-3}, $T = 2 \times 10^9$ (solid line), 3×10^9 (dash line), and 5×10^9 (dot and dash line) K.

Numerical simulations show that for magnetic fields below 10^{11} T, the electron capture rate of ^{60}Fe is basically constant; but it will decrease greatly when $B > 10^{11}$ T, approaching to zero in the end.

4. Conclusion

The results show that the magnetic fields have only a slight effect on electron chemical potential when $B < 10^{11}$ T, but electron chemical potential will decrease greatly when $B > 10^{11}$ T, approaching to $-m_{\rm e}c^2$; there is a similar conclusion to electron capture rates except that it will approach zero. This will greatly affect the outcome of stellar evolution in the process of neutrino loss rate, etc., and have a great influence on the outbreak of the process of supernovae, which we are studying now.

5. Summary

In this paper, we discuss the influence of super-strong magnetic fields on electron capture in magnetar. The theory of this paper comes from well-known classic textbook Quantum Mechanism, written by Landau and Lifshitz. Landau energy levels and the state number of energy levels are often cited by international scholars. Recently, we found that the electron–cyclotron kinetic energy is greater than electron-static energy in super-strong magnetic fields in magnetar. The microscopic state number of electron momentum along the field in the interval of $P_z - P_z + {\rm d}P_z$, is obtained via using the non-relativistic motion equation. So it is not applicable for the case of super-strong magnetic fields and needs corrections. We will study this subject further in another paper.

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