Effect of superstrong magnetic field on electron screening at the crusts of neutron stars*

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The influences of electron screening (ES) and electron energy correction (EEC) are investigated by superstrong magnetic field (SMF). We also discuss in detail the discrepant factor between our results and those of Fushiki, Gudmundsson and Pethick (FGP) in SMF. The results show that SMF has only a slight effect on ES when $B < 10^9$ T on the surfaces of most neutron stars. Whereas for some magnetars, SMF influence ES greatly when $B > 10^9$ T . For instance, due to SMF the ES potential may be increased about 23.6% and the EEC may be increased about 4 orders of magnitude at $\rho/\mu_e = 1.0 \times 10^6$ mol/cm^3 and $T_9 = 1$. On the other hand, the discrepant factor shows that our results are in good agreement with FGP's when $B < 10^9$ T . But the difference will be increased with increasing SMF.

Keywords: electron screening, superstrong magnetic field, neutron star
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1. Introduction

In the last decades, electron screening (ES) has attracted strong interest in the field of nuclear astrophysics due to its effect on the calculation of reaction rates at energies around the Gamow peak in low-energy astrophysical relevant reactions between light nuclei. The ES has also been an interesting and challenging problem due to the dependence on the details of the stellar weak-interaction rates in presupernova stellar evolution and stellar nucleosynthesis. What is the status of and the role played by ES at the late stages of stellar evolution? How to affect the presupernova explosion and make influence on some weak interaction rates by ES? These are extremely interesting, as well as important problems for us to understand the ES and screening corrections in dense stars under the condition of the relativistic degenerate electron liquid. The ES has been addressed by many authors, such as Luo and Peng,[1−3] Liu and Luo[4−7] and Itoh *et al.*[8] However in their work they discussed the ES to neglect the influence of superstrong magnetic field (SMF).

As is well known, the strength of the surface magnetic fields of most neutron stars is in the range from 10^4 to 10^9 T.[9] Some magnetars have magnetic fields ranging from 10^9 to 10^{11} T.[10] Under the condition of so large an SMF, the momentum space of the electron gas will be changed very asymmetrically due to SMF. It will affect the transformation of ES and nuclear reaction rates in a great many of nuclides. Therefore, it is very interesting to discuss the influence of SMF on ES. According to linear response theory, as an example, we investigate the influence of SMF on ES of nuclide ^{56}Fe at the crust of neutron star.

2. The ES in an SMF

The SMF B is considered along the z-axis. The Dirac equation can be solved exactly. The positive energy levels are given by

$$\varepsilon_n = (p_z^2 + 1 + 2n\Theta)^{1/2}, \quad (n = 0, 1, 2, 3\ldots), \quad (1)$$

where $\Theta = h_e B/m_e^2 c = 0.0225 B_{12}$, $B = 10^{12} B_{12}$, and p_z is the electron momentum along the field. Note that in this paper all of the energies and the moments are respectively in units of $m_e c^2$ and $m_e c$, where m_e is the electron mass and c is the light speed. The electron chemical potential is found by inverting the expression for the lepton number density

$$n_e = \frac{\Theta}{2\pi^2 \lambda_e^3} \sum_{n=0}^{\infty} q_{n0} \int_0^{\infty} (f_{-e} - f_{+e}) \mathrm{d}p_z, \quad (2)$$

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where $\lambda_{\rm e} = h/m_{\rm e}c$ is the Compton wavelength, $q_{n0} = 2 - \delta_{n0}$ is the electron degenerate number,

$$f_{-\rm e} = \left[1 + \exp\left(\frac{\varepsilon_n - U_{\rm F} - 1}{kT}\right)\right]^{-1}$$

and

$$f_{+\rm e} = \left[1 + \exp\left(\frac{\varepsilon_n + U_{\rm F} + 1}{kT}\right)\right]^{-1}$$

are the electron and positron distribution functions respectively, k is the Boltzmann constant, T is the electron temperature, and $U_{\rm F}$ is the electron chemical potential.

At high density, the total energy of Wigner–Seitz cell in an atom is given by the sum of two parts. The first is the energy of matter with a uniform electron distribution and the second is the energy due to nonuniformity of the electron gas. Thus the total energy of atom is written as[11]

$$E_j = z_j\varepsilon_o(n_{\rm e}) + U_{\rm latt}(z_j) + \Delta E, \tag{3}$$

where ε_0 is kinetic energy and exchange energy, $U_{\rm latt} = -\frac{9}{10}z^{5/3}e^2/r_{\rm e}$ is the lattice energy, $r_{\rm e} = (3/4\pi n_{\rm e})^{1/3}$ is the typical electron spacing, ΔE is the electron energy correction (EEC) of nonuniformity electron gas and given by[12]

$$\Delta E = -\frac{18}{175}z_j^2e^2x^2a_j, \tag{4}$$

where $x^2 = 4\pi e^2(\partial n_{\rm e}/\partial U_{\rm F})$. Due to the SMF the distribution of the electron gas is broken up. Therefore the electron screening potential (ESP) can be written as

$$U_{\rm s} = U_{\rm latt} + \Delta E, \tag{5}$$

in the condition of high density and temperature, according to linear response theory, the EEC due to SMF is written as

$$\Delta E^B_{\rm LJ} = -\frac{72}{175}\pi e^4 z^{7/3}\left(\frac{3}{4\pi}\right)^{1/3}(\rho/\mu_{\rm e})^{2/3} \times\left(\frac{1}{n_{\rm e}}\frac{\partial n_{\rm e}}{\partial U_{\rm F}}\right)N_{\rm A}^{2/3}, \tag{6}$$

where $N_{\rm A}$ is the Avogadro constant; we define $D = (1/n_{\rm e})(\partial n_{\rm e}/\partial U_{\rm F})$, according to Eq. (2) we have

$$\begin{aligned} D &= \frac{1}{n_{\rm e}}\frac{\partial n_{\rm e}}{\partial U_{\rm F}} = \frac{1}{n_{\rm e}}\frac{\Theta}{2\pi^2\lambda_{\rm e}^3}\sum_{n=0}^{\infty}q_{n0}\int_0^{\infty}\frac{\partial}{\partial U_{\rm F}}\left(f_{-\rm e} - f_{+e}\right){\rm d}p_z \\ &= \frac{1}{n_{\rm e}}\frac{\Theta}{2\pi^2\lambda_{\rm e}^3}\sum_{n=0}^{\infty}q_{n0}\int_{\sqrt{\varepsilon_n - 1 - 2n\Theta}}^{\infty}\frac{\varepsilon_n}{kT\sqrt{\varepsilon_n - 1 - 2n\Theta}}\left[\left(f_{-\rm e} - 1\right)f_{-\rm e}^{(-2)} - \left(f_{+\rm e} - 1\right)f_{+\rm e}^{(-2)}\right]{\rm d}\varepsilon_n. \end{aligned} \tag{7}$$

Thus the ESP at high density can be expressed as

$$\begin{aligned} U^B_{\rm s_{LJ}} &= U_{\rm latt} + \Delta E^B_{\rm LJ} \\ &= -\frac{9}{10}z^{5/3}e^2/r_{\rm e} - \frac{72}{175}\pi e^4 z^{7/3}\left(\frac{3}{4\pi}\right)^{1/3}(\rho/\mu_{\rm e})^{2/3}DN_{\rm A}^{2/3}. \end{aligned} \tag{8}$$

The influence of SMF on ES was discussed in detail by Fushiki, Gudmundsson and Pethick (FGP),[13] the EEC due to SMF was given by

$$\Delta E^B_{\rm FGP} = -\frac{864}{1575}\frac{e^2}{a_0}z^{7/3}\left(\frac{r_{\rm e}}{r_{\rm cyc}}\right)^4 \approx -254z^{7/3}\left(\frac{A}{z}\right)^{4/3}B_{12}^2, \tag{9}$$

where $r_{\rm cyc} = (2hc/eB)^{1/2} \approx 3.63 \times 10^{-10}B_{12}^{-1/2}$ is the cyclotron radius in the lowest Landau orbial, and $a_0 = h^2/m_{\rm e}e^2 \approx 5.29 \times 10^{-9}$ cm is the Bohr radius. Thus the ESP at high density can be expressed as

$$U^B_{\rm s_{FGP}} = U_{\rm latt} + \Delta E^B_{\rm FGP} = -\frac{9}{10}z^{5/3}e^2/r_{\rm e} - 254z^{7/3}\left(\frac{A}{z}\right)^{4/3}B_{12}^2. \tag{10}$$

In order to compare the results of ours with those of FGP's, the factors C is defined as follows:

$$C = \frac{U^B_{\rm s_{LJ}} - U^B_{\rm s_{FGP}}}{U^B_{\rm s_{LJ}}}. \tag{11}$$

3. Some numerical results and discussion

Figures 1 and 2 show respectively the factor D as a function of B at densities of 1.0×10^6 mol/cm^3 and 1.0×10^8 mol/cm^{3}and different temperatures. It can be seen from Fig. 1 that SMF has only a slight effect on the factor D when $B < 10^9$ T. However, the influences of SMF are increased when $B > 10^9$ T . The factor D may be decreased almost about 4 orders of magnitude. One can see that the SMF has a minor effect on the factor D except for at lower temperature $T_9 = 3$ (T_9 is the temperature in units of 10^9 K) when $B < 10^9$ T, and the factor D may be decreased almost about 3 orders of magnitude when $B > 10^{11}$ T in Fig. 2.

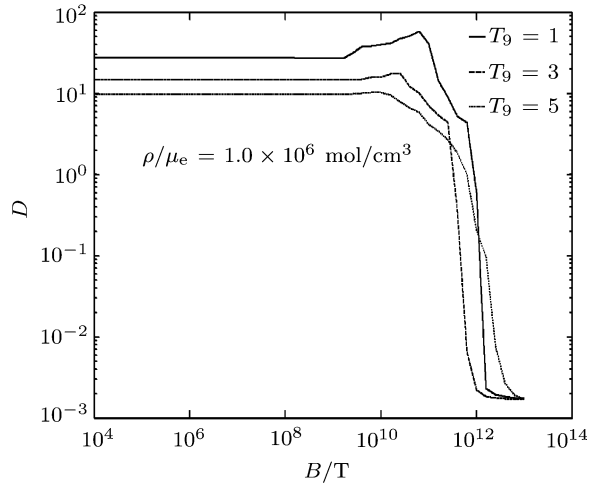


Fig. 1. The factor D as a function of B for $\rho/\mu_e = 1.0 \times 10^6$ mol/cm^3 at $T_9 = 1, 3, 5$.

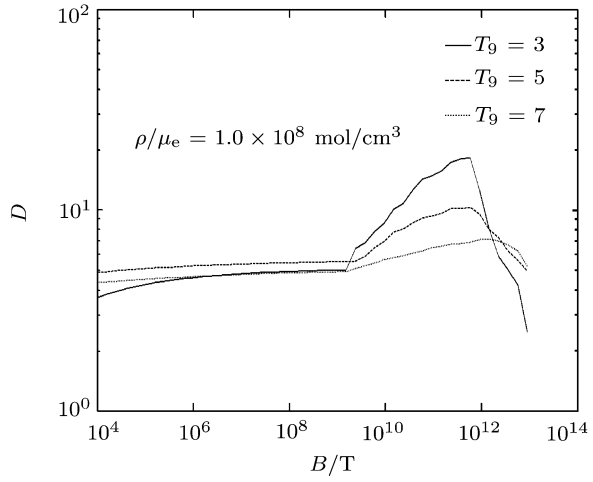


Fig. 2. The factor D as a function of B for $\rho/\mu_e = 1.0 \times 10^8$ mol/cm^3 at $T_9 = 3, 5, 7$.

Figures 3 and 4 show respectively the EEC due to SMF as a function of B at densities of 1.0×10^6 mol/cm^3 and 1.0×10^8 mol/cm^3 and temperatures of $T_9 = 1, 3, 5, 7$. The SMF has only a slight influence on $\Delta E^B_{\rm LJ}$ of nuclide ^{56}Fe when $B < 10^9$ T. But the EEC will be decreased to some extent by SMF when 10^9 T $< B < 10^{11}$ T. For instance, it decreases about 30% at $T_9 = 1$ in Fig. 3 and about 70% at $T_9 = 3$ in Fig. 4. Numerical simulations showed that the influence on $\Delta E^B_{\rm LJ}$ may be increased quickly to a large extent when $B > 10^{11}$ T. For example the $\Delta E^B_{\rm LJ}$ due to SMF will be increased about 4 orders of magnitude at $\rho/\mu_e = 1.0 \times 10^6$ mol/cm^3 and $T_9 = 1$.

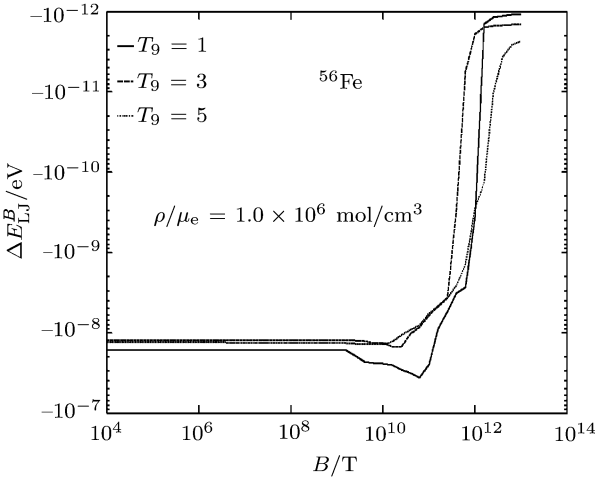


Fig. 3. The EEC $\Delta E^B_{\rm LJ}$ as a function of B for $\rho/\mu_e = 1.0 \times 10^6$ mol/cm^3 at $T_9 = 1, 3, 5$.

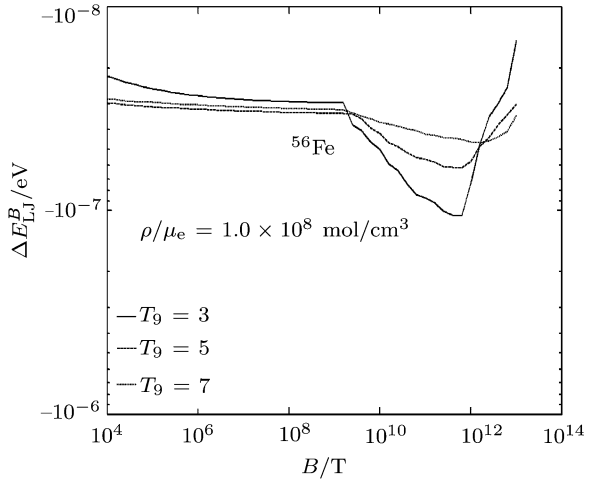


Fig. 4. The EEC $\Delta E^B_{\rm LJ}$ as a function of B for $\rho/\mu_e = 1.0 \times 10^8$ mol/cm^3 at $T_9 = 3, 5, 7$.

Figures 5 and 6 show respectively the influences of SMF on the factor C for ^{56}Fe for densities of 1.0×10^6 mol/cm^3 and 1.0×10^8 mol/cm^3 at different temperatures. The SMF has only a slight effect on the factor C at lower density when $B < 10^{10}$ T, whereas at higher density (such as $\rho/\mu_e = 1.0 \times 10^8$ mol/cm^3)

it may be influenced greatly by SMF. For instance the factor C will be decreased quickly, almost to about 4 orders of magnitude when $B > 10^{10}$ T in Fig. 5, and about one order of magnitude when $B > 10^{12}$ T in Fig. 6.

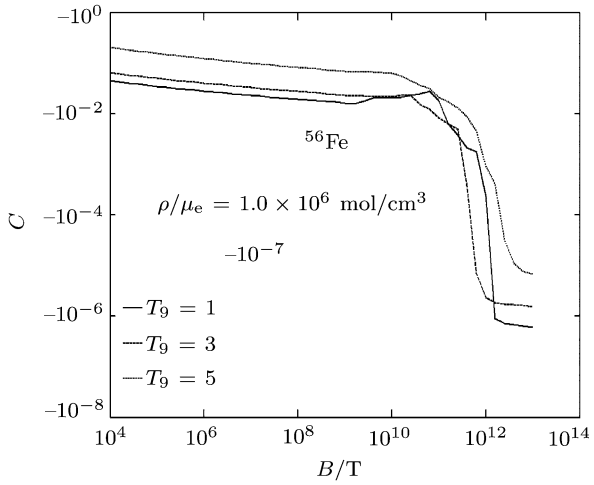


Fig. 5. The factor C as a function of B for $\rho/\mu_e = 1.0 \times 10^6$ mol/cm^3 at $T_9 = 1, 3, 5$

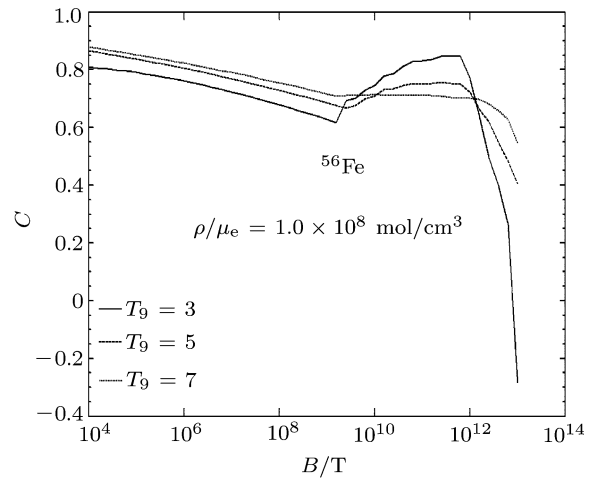


Fig. 6. The factor C as a function of B for $\rho/\mu_e = 1.0 \times 10^8$ mol/cm^3 at $T_9 = 3, 5, 7$.

We analyse the Fermi energy of electron gas in an SMF, taking into account the quantum effects, as an example, and discuss the effect of SMF on ESP of nuclide ^{56}Fe at different densities and temperatures in Figs. 7 and 8. The two figures show that the SMF has only a slight effect on the ESP when $B < 10^9$ T. Whereas the ESP would be influenced greatly by SMF when $B > 10^9$ T. The lower the temperature the larger the effect is. For instance, in Fig. 7 the ESP decreases about 14.8% and then increases about 23.6% for $\rho/\mu_e = 1.0 \times 10^6$ mol/cm^3 at $T_9 = 1$ when 10^9 T $< B < 10^{12}$ T; it decreases about 14.89% and then increases about 20.23% for $\rho/\mu_e = 1.0 \times 10^8$ mol/cm^3 at $T_9 = 3$ when 10^9 T $< B < 10^{13}$ T in Fig. 8.

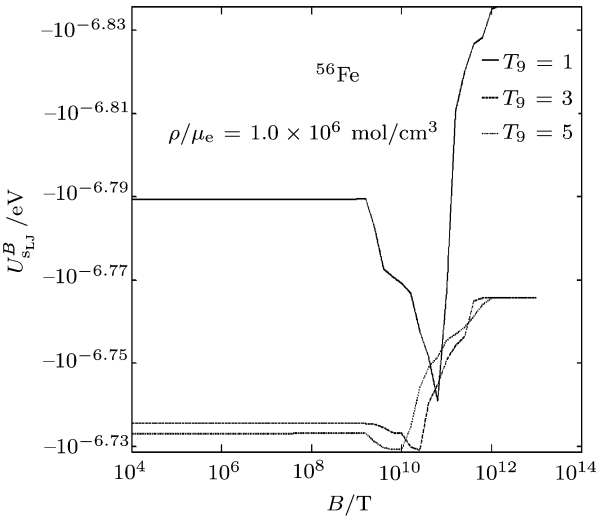


Fig. 7. The ESP as a function of B for $\rho/\mu_e = 1.0 \times 10^6$ mol/cm^3 at $T_9 = 1, 3, 5$.

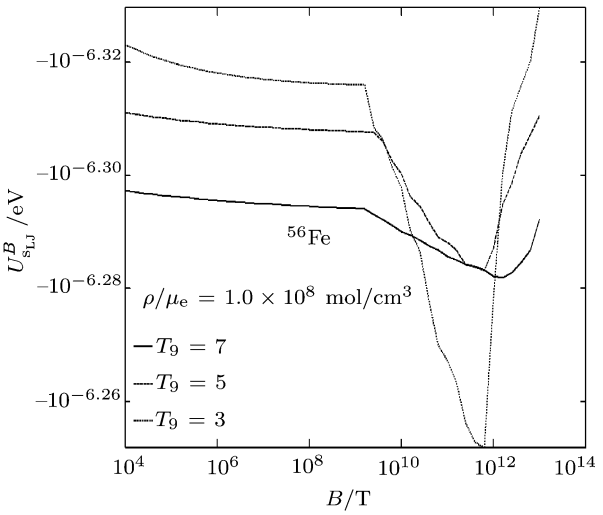


Fig. 8. The ESP as a function of B for $\rho/\mu_e = 1.0 \times 10^8$ mol/cm^3 at $T_9 = 3, 5, 7$.

According to Eq. (7) and the discussions on D in Figs. 1 and 2, one can see that the electron chemical potential is found by inverting the expression for the electron number density, and the factor D is one of the important factors roles which will determine the EEC in SMF. One can see that the effect on D is very slight for relatively lower density and $B < 10^9$ T in the SMF. But the influence will be increased when $B > 10^9$ T. This is because the electron quantum effect will become very great due to SMF which will change the electron distribution remarkably, and affect the electronic space phase factor and the electron

number density. Thus a large amounts of electrons will occupy the lowest Landau levels.

According to the discussions on EEC in Figs. 3 and 4, the results show that the EEC has a slight effect when $B < 10^9$ T, however it will be influenced greatly with increasing SMF. The higher the density the larger the EEC is, and the lower the temperature the larger the influence at the same density. Because the electron energy is so low at lower temperature that the temperature will affect the electronic nonuniform distribution greatly. At the same time, the electron space distribution will be broken up by SMF and the ES changes the Coulomb wave function. Thus the comparatively lower Landau levels will be occupied by a great many of electrons.

Some results of the discrepancy factors C are shown in Figs. 5 and 6. One finds that a minor difference will be shown when $B < 10^9$ T. But with increasing magnetic field, its effect on the factor C will be increased. From Eq. (9), we know that it displays only when the lowest Landau level is occupied in the condition of $r_{\rm e} > (3\pi/8)^{1/3} r_{\rm cyc}$ or equivalently $\rho < 7.04 \times 10^3 B_{12}^{3/2} (A/z)$ g/cm^3 in the SMF. But for the free electron gas FGP used the expression of $(\partial n_{\rm e}/\partial U_{\rm F})\, n_{\rm e} = (3/2)\,(\partial n_{\rm e}/\partial U_{\rm F})$ when they dealt with $\partial n_{\rm e}/\partial U_{\rm F}$. On the other hand, they thought at high density the exchange correction is very small, thus they neglected the exchange correction to $\partial n_{\rm e}/\partial U_{\rm F}$, and used $(\partial n_{\rm e}/\partial U_{\rm F})\, n_{\rm e} = (1/2)\,(\partial n_{\rm e}/\partial U_{\rm F})$ in the SMF.

4. Conclusion

In this paper, the influence of SMF on ES has been investigated, also discussed in detail is the influence on EEC. In order to compare the results we obtain with those of FGP's, the discrepancy factor C is investigated. The results show that the SMF has only a slight effect on the ES when $B < 10^9$ T on surfaces of most neutron stars. Whereas for some magnetars, the SMF is in the range of 10^9 to 10^{14} T, thus it influences greatly on the ES. For instance, the ESP may be increased about 23.6% for $\rho/\mu_{\rm e} = 1.0 \times 10^6$ mol/cm^3 at $T_9 = 1$. Due to the SMF the EEC may be increased about 4 orders of magnitude when $B > 10^{11}$ T. On the other hand, the difference between the present results and those of FGP is very slight when $B < 10^9$ T. But with increasing SMF, the effect on the factor C will be increased.

As is well known, for stars such as white dwarfs and neutron stars, the problem of ES has always been a very challenging subject in SMF, and the ES always plays an important role for dense stars at the late stages of stellar evolution, particularly on nuclear reaction rates and nucleosynthesis. Hence the conclusion we obtained may have significant influence on further research of nuclear astrophysics, especially the research of stellar terminal evolution and neutron astrophysics.

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