Propagation of partially coherent beams carrying an edge dislocation through atmospheric turbulence along a slant path*

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This paper derives the explicit expressions for the average intensity, beam width and angular spread of Gaussian Schell-model (GSM) beams with edge dislocation propagating through atmospheric turbulence along a slant path. The propagation of GSM beams with edge dislocation through horizontal atmospheric turbulence can be treated as a special case through a slant one. The propagation properties of GSM beams with edge dislocation through slant atmospheric turbulence are studied, where the influence of edge dislocation parameters including the slope p and off-axis distance d on the spreading of GSM beams with edge dislocation in atmospheric turbulence is stressed. It shows that the spreading of the intensity profile of GSM beams with edge dislocation along a slant path is smaller than that along a horizontal path in the long-distance atmospheric propagation. The larger the slope $|p|$ and the smaller the off-axis distance $|d|$ are, the less the beam-width spreading and angular spread of GSM beams with edge dislocation are affected by turbulence. The GSM beams with edge dislocation is less affected by turbulence than that of GSM beams without edge dislocation. The results are illustrated numerically and their validity is interpreted physically.

Keywords: partially coherent beams with edge dislocation, atmospheric turbulence, slant path
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1. Introduction

The propagation of laser beams through atmospheric turbulence is a topic that has been of considerable theoretical and practical interest for a long time because of its connection with many important applications including remote sensing, optical communications, imaging and targeting systems etc.[1] It was examined both theoretically and experimentally that partially coherent beams are less sensitive to the effect of turbulence than fully coherent ones.[2−5] For some practical applications the propagation of laser beams through atmosphere along a slant path is required.[6−10] Zhang and Wang calculated the average intensity of finite laser beams in slant atmospheric turbulence.[6] By using the average structure constant $\bar{C}_n^2$, the propagation of multi-Gaussian beams through slant atmospheric turbulence was studied by Chu *et al.*[7] The effect of atmospheric turbulence on the spatial coherence property of Gaussian Schell-model (GSM) beams propagating along a slant path was dealt with by Wang *et al.*[8]

On the other hand, particular attention has been paid recently to optical beams carrying phase singularities (wavefront dislocations). There exist different kinds of wavefront dislocations, i.e., the screw dislocation (often called the optical vortex) with spiral phase, edge dislocation with π-phase jump located along a line in the transverse plane, and hybrid dislocation.[11] The vortex generation and propagation dynamics of optical vortex beams have been studied extensively.[11−14] An edge dislocation can be produced experimentally by using a phase plate to embed a π-phase jump in the host beam. The vortex-edge dislocation interaction in a nonlinear defocusing medium, a linear medium and in the presence of an astigmatic lens is found in Refs. [15]–[17]. The dynamic evolution of an edge dislocation through aligned and misaligned paraxial optical ABCD systems was analysed in Ref. [18].

One of the important subjects is the propagation of optical beams carrying phase of the singularities through atmospheric turbulence and the potential applications. More recently, Gbur and Tyson dealt with the propagation of vortex beams through weak-to-strong atmospheric turbulence and suggested that the topological charge of vortex beams could be used as in-

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formation carrier in optical communications.[19] Wang *et al.* demonstrated that the spreading of partially coherent vortex beams is less influenced by atmospheric turbulence than that of partially coherent vortex-free beams.[20] It is interesting to ask what will happen if partially coherent beams carrying an edge dislocation propagate through atmospheric turbulence? How does the edge-dislocation affect the spreading of partially coherent beams in atmospheric turbulence? This is just the purpose of the present paper. In Section 2 the theoretical formulation of the propagation of partially coherent beams carrying an edge dislocation through atmospheric turbulence along a slant path is presented, where GSM beam is taken as a typical example of partially coherent beams. Section 3 describes the evolution behaviour of GSM beams with edge dislocation in slant atmospheric turbulence. The beam width and angular spread of GSM beams with edge dislocation in slant atmospheric turbulence are illustrated in Section 4. Finally, Section 5 summarised the main results obtained in this paper.

2. Theoretical formulation

Consider a Gaussian beam with edge dislocation whose field at the source plane $L = 0$ is[11,13,18]

$$U(\boldsymbol{\rho}, L = 0) = (p\rho_x - \rho_y + d) \exp\left(-\frac{\rho_x^2 + \rho_y^2}{w_0^2}\right), \quad (1)$$

where p and d denote the slope and off-axis distance of the edge dislocation, respectively, $\boldsymbol{\rho} \equiv (\rho_x, \rho_y)$ is the two-dimensional (2D) position vector at the source plane $L = 0$ and w_0 specifies the waist width of Gaussian beam.

Introducing the Schell-model correlator,[21] we can express the cross-spectral density of a GSM beam with edge dislocation at the plane $L = 0$ as

$$\begin{aligned} & W^{(0)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, 0) \\ = & [p^2\rho_{1x}\rho_{2x} - p(\rho_{1x}\rho_{2y} + \rho_{2x}\rho_{1y}) + dp(\rho_{1x} + \rho_{2x}) \\ & -d(\rho_{1y} + \rho_{2y}) + \rho_{1y}\rho_{2y} + d^2] \\ & \times \exp\left[-\left(\frac{\rho_1^2 + \rho_2^2}{w_0^2}\right)\right] \exp\left[-\frac{(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2)^2}{2\sigma_0^2}\right], \end{aligned} \quad (2)$$

where σ_0 denotes the spatial correlation length and $\boldsymbol{\rho}_i \equiv (\rho_{ix}, \rho_{iy})(i = 1, 2$ unless otherwise stated).

According to the extended Huygens–Fresnel principle,[1] the cross-spectral density of GSM beams with edge dislocation propagating through atmospheric turbulence along a slant path is given by

$$\begin{aligned} & W(\boldsymbol{\rho}'_1, \boldsymbol{\rho}'_2, L) \\ = & \left(\frac{k}{2\pi L}\right)^2 \iint \mathrm{d}^2\rho_1 \iint \mathrm{d}^2\rho_2 W^{(0)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, 0) \\ & \times \exp\left\{-\frac{\mathrm{i}k}{2L}[(\boldsymbol{\rho}'_1 - \boldsymbol{\rho}_1)^2 - (\boldsymbol{\rho}'_2 - \boldsymbol{\rho}_2)^2]\right\} \\ & \times \langle \exp[\psi^*(\boldsymbol{\rho}_1, \boldsymbol{\rho}'_1) + \psi(\boldsymbol{\rho}_2, \boldsymbol{\rho}'_2)]\rangle, \end{aligned} \quad (3)$$

where k is the wave number related to the wave length λ by $k = 2\pi/\lambda$, $\boldsymbol{\rho}'_i \equiv (\rho'_{ix}, \rho'_{iy})$ is the 2D position vector at the L plane, $*$ is the complex conjugate, $\langle\cdot\rangle$ denotes the ensemble average, and $\psi(\boldsymbol{\rho}, \boldsymbol{\rho}')$ represents the random part of the complex phase of a spherical wave due to the turbulence. We can write[22]

$$\begin{aligned} & \langle \exp[\psi^*(\boldsymbol{\rho}_1, \boldsymbol{\rho}'_1) + \psi(\boldsymbol{\rho}_2, \boldsymbol{\rho}'_2)]\rangle \\ = & \exp\left[-\frac{1}{2}D_\psi(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2, \boldsymbol{\rho}'_1 - \boldsymbol{\rho}'_2)\right], \end{aligned} \quad (4)$$

where $D_\psi(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2, \boldsymbol{\rho}'_1 - \boldsymbol{\rho}'_2)$ is the phase structure function:[1,8,23]

$$\begin{aligned} & D_\psi(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2, \boldsymbol{\rho}'_1 - \boldsymbol{\rho}'_2) \\ = & 2.914k^2 \sec(\xi) \int_{h_0}^{H} C_n^2(h) |(1-\eta)(\boldsymbol{\rho}'_1 - \boldsymbol{\rho}'_2) \\ & + \eta(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2)|^{5/3} \mathrm{d}h. \end{aligned} \quad (5)$$

In the quadratic approximation, the following phase structure function is used in place of Eq. (5), and is shown to be a good approximation for practical situations, namely[22,24]

$$\begin{aligned} & D_\psi(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2, \boldsymbol{\rho}'_1 - \boldsymbol{\rho}'_2) \\ = & B_1(\boldsymbol{\rho}'_1 - \boldsymbol{\rho}'_2)^2 + B_2(\boldsymbol{\rho}'_1 - \boldsymbol{\rho}'_2) \cdot (\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2) \\ & + B_3(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2)^2, \end{aligned} \quad (6)$$

where

$$B_1 = 2.914k^2 \sec(\xi) \int_{h_0}^{H} C_n^2(h)(1-\eta)^2 \mathrm{d}h, \quad (7)$$

$$B_2 = 2.914k^2 \sec(\xi) \int_{h_0}^{H} 2C_n^2(h)\eta(1-\eta) \mathrm{d}h, \quad (8)$$

$$B_3 = 2.914k^2 \sec(\xi) \int_{h_0}^{H} C_n^2(h)\eta^2 \mathrm{d}h. \quad (9)$$

In Eqs. (7)–(9), $\eta = (h - h_0)/(H - h_0)$ for the downlink propagation, h_0 is the altitude between the ground level and the receiver, H is the height above ground level of the transmitter, $L = (H - h_0)\sec(\xi)$ is propagation distance along a slant path, and ξ is the zenith angle, $C_n^2(h)$ describes the variation of the structure constant versus the altitude h. If $C_n^2(h)$ is given, Eqs. (7)–(9) can be integrated analytically, one of the

most widely used models is the Hufnagel–Valley (H–V) model which is given by[1]

$$C_n^2(h) = 0.00594(v/27)^2(10^{-5}h)^{10}\exp(-h/1000) + 2.7\times10^{-16}\exp(-h/1500) + C_n^2(0)\exp(-h/100), \quad (10)$$

where v is the wind speed, $C_n^2(0)$ is a nominal value at ground level. In our calculations, we use the H–$V_{5/7}$ model with $v = 21$ m/s and $C_n^2(0) = 1.7\times10^{-14}$ m$^{-2/3}$.

Equations (5)–(9) are also applicable to the uplink propagation, for such a case $\eta = 1-(h-h_0)/(H-h_0)$, where h_0 is the altitude between the ground level and the transmitter, H is the height above ground level of the receiver.

To simplify the calculation of the two new variables of integration, we introduce

$$\boldsymbol{u} = \frac{\boldsymbol{\rho}_1+\boldsymbol{\rho}_2}{2}, \quad \boldsymbol{v} = \boldsymbol{\rho}_1-\boldsymbol{\rho}_2. \quad (11)$$

Substituting Eqs. (2), (4) and (6) into Eq. (3), we have

$$\begin{aligned} W(\boldsymbol{\rho}'_1,\boldsymbol{\rho}'_2,L) = {} & \left(\frac{k}{2\pi L}\right)^2\exp\left[-\frac{\mathrm{i}k}{2L}(\rho'^2_1-\rho'^2_2)\right]\exp\left[-\frac{B_1}{2}(\boldsymbol{\rho}'_1-\boldsymbol{\rho}'_2)^2\right]\iint \mathrm{d}^2u\iint\mathrm{d}^2v \\ & \times\left(p^2u_x^2-\frac{p^2v_x^2}{4}-2pu_xu_y+\frac{pv_xv_y}{2}+2dpu_x-2du_y+u_y^2-\frac{v_y^2}{4}+d^2\right) \\ & \times\exp\left(-\frac{2}{w_0^2}u^2\right)\exp\left[\frac{\mathrm{i}k}{L}(\boldsymbol{\rho}'_1-\boldsymbol{\rho}'_2)\cdot\boldsymbol{u}\right]\exp\left(-\frac{\mathrm{i}k}{L}\boldsymbol{u}\cdot\boldsymbol{v}\right) \\ & \times\exp(-Av^2)\exp\left[\frac{\mathrm{i}k}{2L}(\boldsymbol{\rho}'_1+\boldsymbol{\rho}'_2)\cdot\boldsymbol{v}\right]\exp\left[-\frac{B_2}{2}(\boldsymbol{\rho}'_1-\boldsymbol{\rho}'_2)\cdot\boldsymbol{v}\right], \end{aligned} \quad (12)$$

with

$$A = \frac{1}{2w_0^2}+\frac{1}{2\sigma_0^2}+\frac{B_3}{2}. \quad (13)$$

Equation (12) is the cross-spectral density of GSM beams with edge dislocation propagating through slant atmospheric turbulence. For the case that C_n^2 is independent of the altitude h, i.e., $B_1 = B_2 = B_3$, Eq. (12) reduces to the cross-spectral density of GSM beams with edge dislocation through horizontal atmospheric turbulence. If no turbulence is presented ($C_n^2 = 0$), i.e., $B_1 = B_2 = B_3 = 0$, Eq. (12) can be simplified to the cross-spectral density of GSM beams with edge dislocation in the free-space propagation.

3. Evolution of GSM beams with edge dislocation in atmospheric turbulence along a slant path

Substituting $\boldsymbol{\rho}'_1 = \boldsymbol{\rho}'_2 = \boldsymbol{\rho}'$ into Eq. (12), we obtain the average intensity of GSM beams with edge dislocation at the L plane as

$$\begin{aligned} I(\boldsymbol{\rho}',L) &= W(\boldsymbol{\rho}',\boldsymbol{\rho}',L) \\ &= \left(\frac{k}{2\pi L}\right)^2\iint\mathrm{d}^2u\iint\mathrm{d}^2v\left(p^2u_x^2-\frac{p^2v_x^2}{4}-2pu_xu_y+\frac{pv_xv_y}{2}+2dpu_x-2du_y+u_y^2-\frac{v_y^2}{4}+d^2\right) \\ &\quad\times\exp\left(-\frac{2}{w_0^2}u^2\right)\exp\left(-\frac{\mathrm{i}k}{L}\boldsymbol{u}\cdot\boldsymbol{v}\right)\exp(-Av^2)\exp\left[\frac{\mathrm{i}k}{L}\boldsymbol{\rho}'\cdot\boldsymbol{v}\right]. \end{aligned} \quad (14)$$

Recalling the integral formula[25]

$$\int x^n\exp(-px^2+2qx)\,\mathrm{d}x = n!\exp\left(\frac{q^2}{p}\right)\sqrt{\frac{\pi}{p}}\left(\frac{q}{p}\right)^n\sum_{k=0}^{E\left[\frac{n}{2}\right]}\frac{1}{(n-2k)!k!}\left(\frac{p}{4q^2}\right)^k, \quad (15)$$

and integrating Eq. (14), we obtain

$$I(\rho'_x, \rho'_y, L) = \frac{Dw_0^2}{2F} \exp\left[-D\left(\frac{\rho'^2_x}{F} + \frac{\rho'^2_y}{F}\right)\right]\left[\left(\frac{pD\rho'_x}{CA} - \frac{D\rho'_y}{CA} + d\right)^2 + D\left(\frac{p\rho'_x}{2F} - \frac{\rho'_y}{2F}\right)^2 + \frac{p^2+1}{2C} - \frac{p^2+1}{8F}\right], \quad (16)$$

where

$$C = \frac{2}{w_0^2} + \frac{D}{A}, \quad (17)$$

$$D = \frac{k^2}{4L^2}, \quad (18)$$

$$F = A + \frac{Dw_0^2}{2}. \quad (19)$$

Equation (16) provides the closed-form expression for the average intensity of GSM beams with edge dislocation through slant atmospheric turbulence, which depends on the altitude between the ground level and the receiver H, the height above ground level of the transmitter h_0, the zenith angle ξ, the wavelength λ, the spatial correlation length σ_0, and the edge dislocation parameters including the slope and off-axis distance of edge dislocation p and d.

The normalised average intensity distributions $I(\rho'_x, 0, L)/I(\rho_x, 0, 0)_{\max}$ of GSM beams with edge dislocation through atmospheric turbulence along a slant path with $\xi = 45°$ versus the ρ'_x axis are plotted in Fig. 1 for selected values of the slope $p = 2$ (solid curves) and 5 (dotted curves), where $I(\rho_x, 0, 0)_{\max}$ denotes the maximum intensity in the ρ_x direction at the source plane $L = 0$, the calculation parameters are $\lambda = 1.06$ μm, $w_0 = 5$ cm, $\sigma_0 = 2$ cm, $d = 1$ cm and $h_0 = 0$. Figure 1 shows that the Gaussian intensity profile disappears and there is an intensity profile with a central deep and two maxima at the source plane $L = 0$ due to the modulation of the edge dislocation. The two intensity maxima are not equal and the second maximum of the GSM beam with $p = 5$ is larger than that of the GSM beam with $p = 2$. In the evolution process the second maximum decreases and the minimum at the central deep increases. Finally, the second maximum vanishes and the intensity profile becomes a Gaussian-like one with a slight asymmetry when the propagation distance is sufficiently long. In comparison with the evolution of GSM beams with $p = 2$ and 5 we see that the larger the slope p is, the longer the evolution distance for the evolution into a Gaussian-like profile is required. For example, the evolution distance at which the intensity minimum is equal to a half of the second maximum, i.e., $I_{\min} = I_{2,\max}/2$ is 1.55 km and 1.9 km for $p = 2$ and 5, respectively, and the evolution distance at which a Gaussian-like profile appears is 4.7 km and 7 km for $p = 2$ and 5, respectively. The physical reason is evident, because the larger second intensity maximum for $p = 5$ results in the larger evolution distance at which $I_{\min} = I_{2,\max}/2$ is achieved than that for $p = 2$.

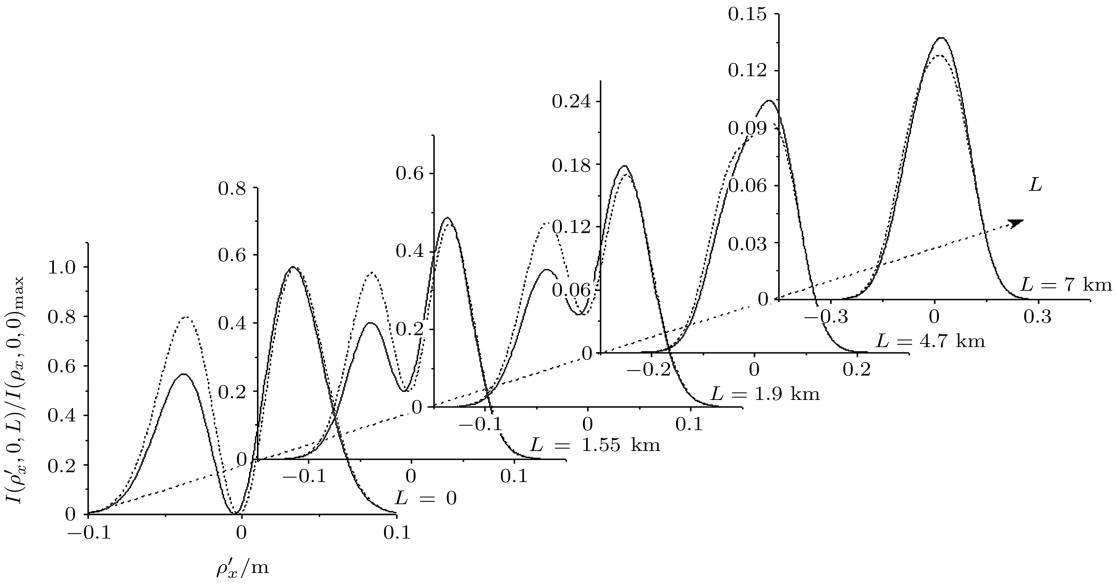


Fig. 1. The evolution of normalised average intensity distributions $I(\rho'_x, 0, L)/I(\rho_x, 0, 0)_{\max}$ of GSM beams with edge dislocation through atmospheric turbulence along a slant path for selected values of p, '—' for $p = 2$ and '- - -' for $p = 5$.

Figure 2 gives the normalised average intensity distributions $I(\rho'_x, 0, L)/I(\rho_x, 0, 0)_{\max}$ of GSM beams with edge dislocation through atmospheric turbulence along a slant path with $\xi = 45°$ versus the ρ'_x axis for selected values of the off-axis distance $d = 1$ cm (solid curves) and 3 cm (dotted curves), and $p = 3$. The other calculation parameters are the same as those in Fig. 1. As can be seen, the larger the off-axis distance d is, the shorter the evolution process is. For example, the evolution distance, at which $I_{\min} = I_{2,\max}/2$ and Gaussian-like profile takes place, is 1.75 km, 6 km for $d = 1$ cm and 1.4 km, 4.5 km for $d = 3$ cm, respectively. The reason is similar to that in Fig. 1, because the second intensity maximum for $d = 1$ cm is larger than that for $d = 3$ cm at the source plane $L = 0$.

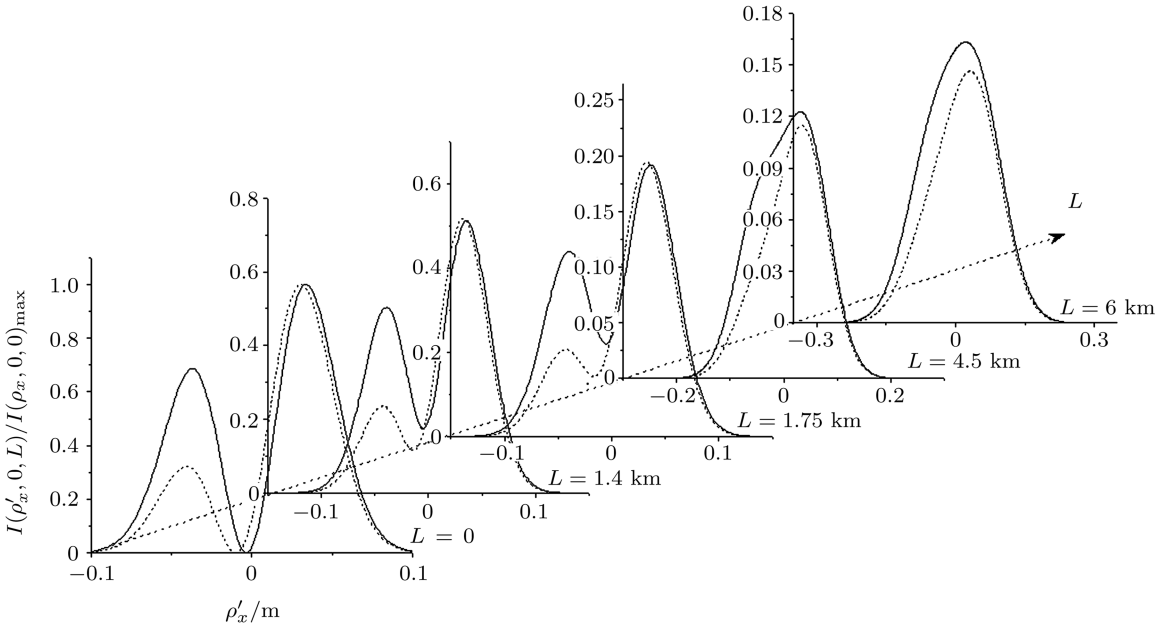


Fig. 2. The evolution of normalized average intensity distributions $I(\rho'_x, 0, L)/I(\rho_x, 0, 0)_{\max}$ of GSM beams with edge dislocation through atmospheric turbulence along a slant path for selected values of d, '—' for $d = 1$ cm and '- - -' for $d = 3$ cm.

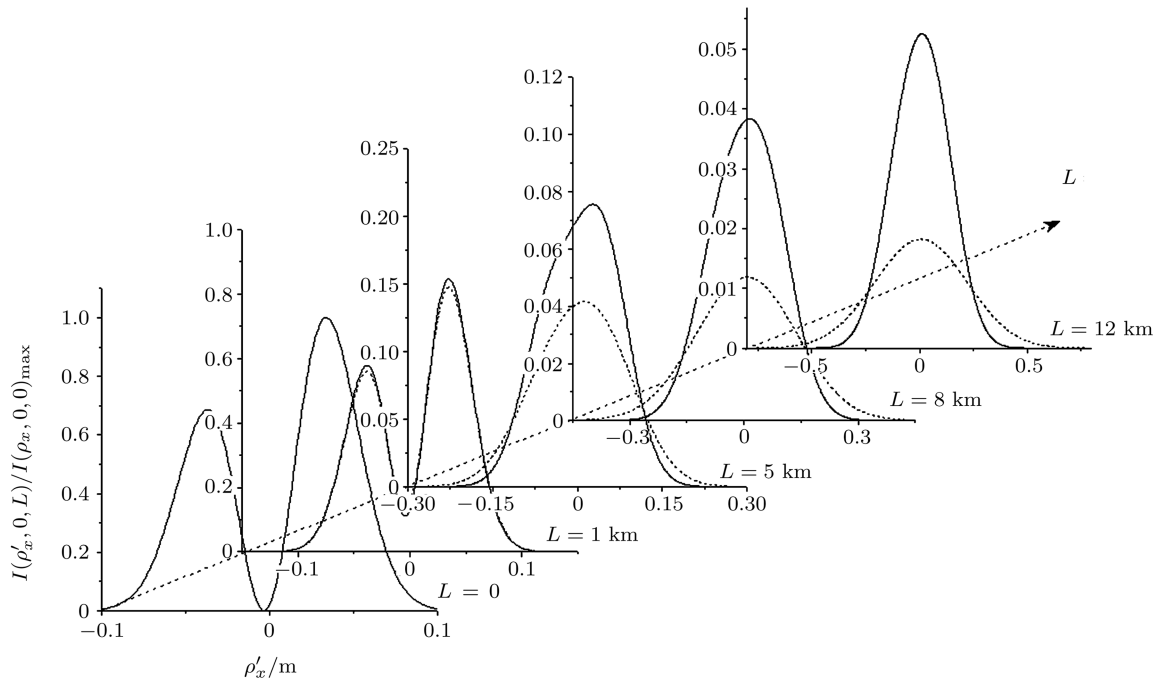


Fig. 3. The evolution of normalised average intensity distributions $I(\rho'_x, 0, L)/I(\rho_x, 0, 0)_{\max}$ of a GSM beam with edge dislocation along a slant path with $\xi = 45°$ (—), and along a horizontal path (- - -).

Figure 3 gives the normalised average intensity distributions $I(\rho'_x, 0, L)/I(\rho_x, 0, 0)_{\max}$ of a GSM beam with edge dislocation along a slant path with $\xi = 45°$ (solid curves) and along a horizontal path (dotted curves) versus the ρ'_x axis, and $p = 3$. The other calculation parameters are the same as those in Fig. 1. A comparison between the two curves shows that the intensity profile along the slant and horizontal paths is similar at the propagation distance $L \leq 1$ km. With increasing propagation distance, say $L \geq 5$ km, the spreading of the intensity profile along the horizontal path is larger than that along the slant path. In other words, the propagation of partially coherent beams with edge dislocation through atmospheric turbulence along a slant path is more beneficial than that along a horizontal path. It can be explained by means of Eq. (10), which indicates that the structure constant $C_n^2(h)$ decreases with increasing altitude h. Therefore, the influence of turbulence on the beam propagation along the slant path is less than that along the equal horizontal path.

4. Beam width and angular spread

The root mean square beam width is defined as[3]

$$w(L) = \sqrt{\frac{\int \rho'^2 I(\rho', L)\,\mathrm{d}^2\rho'}{\int I(\rho', L)\,\mathrm{d}^2\rho'}}. \tag{20}$$

Making use of the integral formula[25]

$$\int x^{2m}\exp(-\beta x^{2n})\,\mathrm{d}x = \Gamma\left(\frac{2m+1}{2n}\right)\Big/ n\beta^{\frac{2m+1}{2n}} \tag{21}$$

with $\Gamma(\bullet)$ being the Gamma function, by straightforward integral calculations we obtain

$$w(L) = \left[w_{\mathrm{a}} + w_{\mathrm{b}}L^2 + 5.828\sec(\xi)L^2\int_{h_0}^{H} C_n^2(h)\eta^2\,\mathrm{d}h\right]^{1/2}, \tag{22}$$

where

$$w_{\mathrm{a}} = \frac{w_0^2}{2} + \frac{w_0^4}{2w_0^4 + 8d^2/(p^2+1)}, \tag{23}$$

$$w_{\mathrm{b}} = \left(\frac{1}{w_0^2} + \frac{1}{\sigma_0^2} + \frac{1}{w_0^2 + 4d^2/(p^2+1)}\right)\frac{2}{k^2}. \tag{24}$$

Equation (22) indicates that the beam width of GSM beams with edge dislocation in turbulence consists of three terms. The first two terms under the square root on the right-hand side represent the beam-width spreading in free space due to diffraction. The first term w_{a} is independent of the propagation distance L, the second term $w_{\mathrm{b}}L^2$ increases with L^2, where $w_0^2/2 + (1/w_0^2 + 1/\sigma_0^2)(2/k^2)L^2$ describes the beam-width spreading of GSM beams without edge dislocation, and $w_0^4/(2w_0^4 + 8d^2/(p^2+1)) + [1/(w_0^2 + 4d^2/(p^2+1))](2/k^2)L^2$ describes the effect due to edge dislocation on the beam-width spreading in free space and increases with decreasing d and increasing p. The third term under the square root on the right-hand side $5.828\sec(\xi)L^2\int_{h_0}^{H} C_n^2(h)\eta^2\,\mathrm{d}h$ characterises the influence of turbulence on the beam-width spreading which depends on the parameters H, h_0, ξ, L, but does not depend on the beam and edge-dislocation.

Substituting $C_n^2 = 0$ into Eq. (22), the beam width of GMS beams with edge dislocation in free space turns out to be

$$w(L)_{\mathrm{free}} = \left[w_{\mathrm{b}}L^2 + 5.828\sec(\xi)L^2\int_{h_0}^{H} C_n^2(h)\eta^2\,\mathrm{d}h\right]^{1/2}. \tag{25}$$

By letting $L = 0$ in Eq. (22), the beam width of GSM beams with edge dislocation at the source place $L = 0$ is expressed as

$$w(0) = \sqrt{w_{\mathrm{a}}}. \tag{26}$$

Equation (26) implies that the beam width of GSM beams with edge dislocation is larger than that of GSM beams without edge dislocation at $L = 0$ due to the edge dislocation.

The angular spread is defined as[5]

$$\theta_{\mathrm{sp}} = \left.\frac{w(L)}{L}\right|_{L\to\infty}. \tag{27}$$

From Eqs. (22) and (27) the angular spread of GSM beams with edge dislocation in turbulence is given by

$$\theta_{\mathrm{sp_{turb}}} = \sqrt{w_{\mathrm{b}} + 5.828\sec(\xi)\int_{h_0}^{H} C_n^2(h)\eta^2\,\mathrm{d}h}. \tag{28}$$

From Eq. (28) we see that the angular spread of GSM beams with edge dislocation in turbulence is

composed of two terms. The first term $w_{\rm b}$ represents the diffraction-induced angular spread in free space, where $(1/w_0^2 + 1/\sigma_0^2)(2/k^2)$ decreases the angular spread of GSM beams without edge dislocation, and $[1/(w_0^2 + 4d^2/(p^2+1))](2/k^2)$ describes the angular spread due to the edge dislocation and decreases as d increases and p decreases. The second term $5.828\sec(\xi)\int_{h_0}^{H} C_n^2(h)\eta^2\,{\rm d}h$ characterises the turbulence-induced angular spread, which depends on the parameters H, h_0 and ξ.

If $C_n^2 = 0$, from Eq. (28) the angular spread of GSM beams with edge dislocation in free space is given by

$$\theta_{\rm sp_{free}} = \sqrt{w_{\rm b}}. \tag{29}$$

To intuitively describe the resistance of a beam to turbulence, the relative angular spread can be used, which is defined as the ratio of the angular spread $\theta_{\rm sp_{turb}}$ of a beam in turbulence to $\theta_{\rm sp_{free}}$ of the same beam in free space, i.e.,

$$\frac{\theta_{\rm sp_{turb}}}{\theta_{\rm sp_{free}}} = \sqrt{1 + \frac{5.828\sec(\xi)\int_{h_0}^{H} C_n^2(h)\eta^2\,{\rm d}h}{w_{\rm b}}}. \tag{30}$$

Physically, the smaller relative angular spread means the less sensitivity to turbulence.

The angular spreads $\theta_{\rm sp_{turb}}$, $\theta_{\rm sp_{free}}$ and relative angular spread $\theta_{\rm sp_{turb}}/\theta_{\rm sp_{free}}$ of GSM beams with edge dislocation at $L = 10$ km versus the slope p and off-axis distance d, are depicted in Figs. 4(a) and 4(b), respectively, and the other calculation parameters are the same as those in Fig. 1. As expected from Eqs. (28) and (29), both angular spreads $\theta_{\rm sp_{turb}}$ and $\theta_{\rm sp_{free}}$ of GSM beams with edge dislocation in turbulence and in free space increase with increasing $|p|$ and decreasing $|d|$. However, the relative angular spread $\theta_{\rm sp_{turb}}/\theta_{\rm sp_{free}}$ decreases as $|p|$ increases and $|d|$ decreases. It means that the larger the absolute value of the slope $|p|$ and the smaller the absolute value of the off-axis distance $|d|$ are, the less the influence of turbulence on the angular spread of GSM beams with edge dislocation. The result is physically reasonable, because the existence of any substantial original angular spread will reduce the effect of turbulence. Equations (24) and (29) indicate that the larger $|p|$ and smaller $|d|$ will result in the larger original substantial angular spread, namely, the effect of turbulence is masked by the larger angular spread in free space,[3] so that the angular spread of GSM beams with larger $|p|$ and smaller $|d|$ is less affected by turbulence than that of GSM beams with smaller $|p|$ and larger $|d|$.

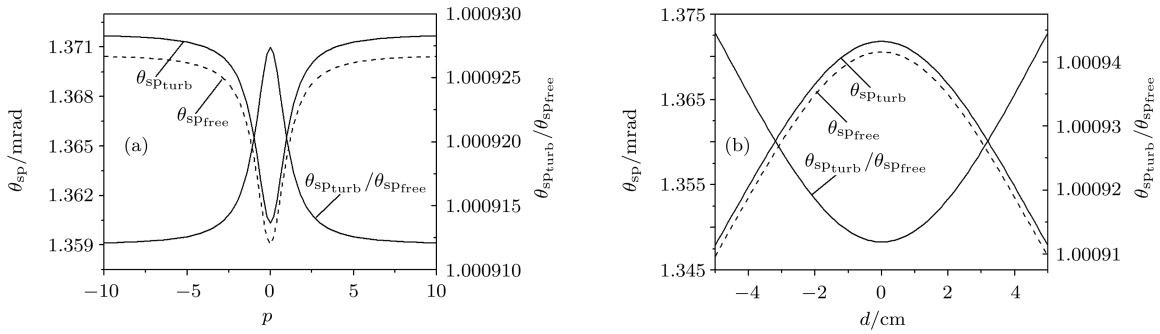


Fig. 4. The angular spreads $\theta_{\rm sp_{turb}}$, $\theta_{\rm sp_{free}}$ and relative angular spread $\theta_{\rm sp_{turb}}/\theta_{\rm sp_{free}}$ of GSM beams with edge dislocation versus (a) the slope p, and (b) the off-axis distance d.

Figure 5 gives the relative angular spreads of GSM beams with edge dislocation (solid curve) and without edge dislocation (dotted curve), versus the spatial correlation length σ_0, where $p = 5$ and the other calculation parameters are the same as those in Fig. 1. Figure 5 indicates that for the both cases $\theta_{\rm sp_{turb}}/\theta_{\rm sp_{free}}$ increases with increasing σ_0. It means that the GSM beams with smaller σ_0 (worse spatial coherence) is less affected by turbulence irrespective of the presence of the edge dislocation. In addition, a comparison of solid and dotted curves shows that the angular spread of GSM beams with edge dislocation is less affected by turbulence than that of GSM beams without edge dislocation, which can be also interpreted by using Eqs. (24) and (29) like that in Fig. 4.

The beam-width spreading of GSM beams with edge dislocation in turbulence can be analysed in a similar way by using Eqs. (22)–(26), and the same

conclusion will be drawn. The illustrative numerical examples are omitted here for saving the length.

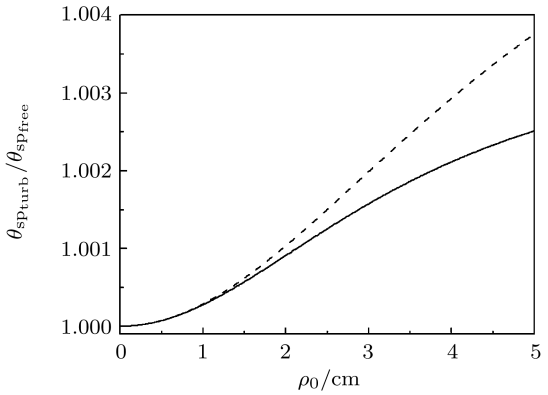


Fig. 5. The relative angular spread $\theta_{\mathrm{sp_{turb}}}/\theta_{\mathrm{sp_{free}}}$ versus the spatial correlation length σ_0. The '—' denotes GSM beam with edge dislocation, whereas '- - -' denotes GSM beam without edge dislocation.

5. Concluding remarks

In this paper, by using the extended Huygens–Fresnel principle and the quadratic approximation of the phase structure function, the explicit expressions for the average intensity, beam width and angular spread of GSM beams with edge dislocation propagating through atmospheric turbulence along a slant path have been derived, and the propagation of GSM beams with edge dislocation through horizontal atmospheric turbulence can be treated as a special case through slant one, where C_n^2 is independent of the altitude. The spreading of the intensity profile of GSM beams with edge dislocation along a slant path is smaller than that along a horizontal path in the long-distance atmospheric propagation. Therefore, the slant path is more beneficial to the long-distance atmospheric propagation of GSM beams with edge dislocation in comparison with the horizontal propagation. The larger the slope $|p|$ and the smaller the off-axis distance $|d|$ are, the less the beam-width spreading and angular spread of GSM beams with edge dislocation are affected by turbulence. The GSM beams with edge dislocation is less affected by turbulence than that of GSM beams without edge dislocation. A deep understanding of the propagation properties of partially coherent beams with phase singularities (optical vortex and/or edge dislocation) through atmospheric turbulence would be useful for some applications.

References

[1] Andrews L C and Phillips R L 1998 *Laser Beam Propagation through Random Media* (Bellingham: SPIE Press)
[2] Wu J and Boardman A D 1991 *J. Mod. Opt.* **38** 1355
[3] Gbur G and Wolf E 2002 *J. Opt. Soc. Am.* A **19** 1592
[4] Dogariu A and Amarande S 2003 *Opt. Lett.* **28** 10
[5] Shirai T, Dogariu A and Wolf E 2003 *Opt. Lett.* **28** 610
[6] Zhang Y X and Wang G G 2006 *Chin. Opt. Lett.* **4** 559 (in Chinese)
[7] Chu X X, Liu Z J and Wu Y 2008 *J. Opt. Soc. Am.* A **25** 74
[8] Wang H, Wang X Z, Zeng A J and Yang K 2008 *Acta Phys. Sin.* **57** 634 (in Chinese)
[9] Wei H Y, Wu Z S and Peng H 2008 *Acta Phys. Sin.* **57** 6666 (in Chinese)
[10] Cang J and Zhang Y X 2009 *Acta Phys. Sin.* **58** 2444 (in Chinese)
[11] Indebetouw G 1993 *J. Mod. Opt.* **40** 73
[12] Heckenberg N R, McDuff R, Smith C P and White A G 1992 *Opt. Lett.* **17** 221
[13] Soskin M S and Vasnetsov M V 2001 *Prog. Opt.* **42** 219
[14] Flossmann F, Schwarz U T and Maier M 2005 *Opt. Commun.* **250** 218
[15] Kivshar Y S, Nepomnyashchy A, Tikhonenko V, Christou J and Luther-Davies B 2000 *Opt. Lett.* **25** 123
[16] Petrov D V 2001 *Opt. Commun.* **188** 307
[17] Yan H W and Lü B D 2009 *Opt. Commun.* **282** 717
[18] Yan H W and Lü B D 2009 *J. Opt. Soc. Am.* A **26** 985
[19] Gbur G and Tyson R K 2008 *J. Opt. Soc. Am.* A **25** 225
[20] Wang T, Pu J and Chen Z 2008 *Opt. Eng.* **47** 036002
[21] Zahid M and Zubairy M S 1989 *Opt. Commun.* **70** 361
[22] Wang S C H and Plonus M A 1979 *J. Opt. Soc. Am.* **69** 1297
[23] Yura H T 1972 *Appl. Opt.* **11** 1399
[24] Leader J C 1978 *J. Opt. Soc. Am.* **68** 175
[25] Gradshteyn I S and Ryzhik I M 2007 *Table of Integrals, Series and Products* (New York: Academic)