Optical transmission through basic-structural-unit superlattices*

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In this paper, by the transparent-component-decimation (TCD) method we obtain three kinds of new basic-components (BCs) through simplifying and decomposing the BCs of three-component Thue–Morse (3CTM) sequence. Based on these new BCs we propose a type of basic-structural-units (BSUs) and investigate the optical transmission of the one-dimensional (1D) superlattices composed of these BSUs. It is found that if the substrates of the 1D BSU superlattices are certain, the optical transmission at the central wavelength (CW) will be determined completely by the number and the type of BSUs and has nothing to do with the marshalling sequence. In particular, if the substrates are identical, the numbers of different types of BSUs are all the same and the middle two elements of BSUs constitute a cycle, then no matter whether the system is periodic, or quasiperiodic, or aperiodic, or unordered, or even random, it will be transparent at the CW. The conclusion is confirmed by the numerical results. Similar to the even layers of neighbourhood identical elements in TCD method, such a kind of optical BSU subsystem can also be decimated from the chain in the process of transmission investigation. There would be a potential application in the designing of some interesting optical devices.

Keywords: optical transmission, basic-structural-units, superlattices

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1. Introduction

Much attention has been paid to the investigations on the structural and the physical properties of quasicrystals and quasiperiodic superlattices since the fundamental discovery of quasicrystal.[1−3] Particularly, optical transmission through quasiperiodic multilayers has been extensively studied. Kohmoto *et al.*[4] studied the light propagation through Fibonacci (FC(1)) system, and their theoretical results were confirmed by the experiment of dielectric multilayers.[5] Following these studies, many results of optical transmission through aperiodic superlattices have been reported. The optical transmission properties for generalized Fibonacci (GF(m,n)) sequences were investigated[6−8] and it was found that their characteristics are much richer than those of FC(1) system. Fu *et al.*[9] constructed Fibonacci-class (FC(n)) models, which possess perfect self-similarity, and Yang *et al.*[10] studied the transmission of light through the superlattices following these one-dimensional (1D) sequences.

On the other hand, being a bridge between linking periodic models and quasiperiodic systems in a geometrical structure Thue–Morse (TM) model has been extensively investigated over a long period of time. Riklund *et al.*[11] studied the electronic properties of TM system and found Block-like electronic states which are different from those of FC(1) chain. Chattopadhyay and Chakrabarti[12] researched the optical transmission through the 1D TM superlattices and discovered that this aperiodic structure presents a unique kind of positional correlation between its constituents, leading to an unattenuated transmission of light as well as electrons through it. Furthermore, one has also paid much attention to the investigations on the physical properties and the optical transmissions for generalized Thue–Morse (GTM(m,n)) models, the extended sequences of TM chain.[13−16] The three-component Thue–Morse (3CTM) model, another interesting extension of TM sequence, was proposed by Tong[17] and the electronic energy spectra were obtained to be self-similar, i.e., they have a Cantor-like structure.[17,18] It can be generated by the following

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substitution rules:[17]

$$A \to AB,\ B \to BC,\ C \to CA. \tag{1}$$

Afterwards, Hu *et al.*[19] studied the 3CTM system and found that it possesses an interesting transmissive characteristic.

Based on the theory of electromagnetic propagation one can define the optical transmission coefficient (TC) as[4]

$$T = \frac{4}{{m_{11}}^2 + {m_{12}}^2 + {m_{21}}^2 + {m_{22}}^2 + 2}, \tag{2}$$

where m_{ij} $(i, j = 1, 2)$ is an element of the optical propagation matrix (PM) $\boldsymbol{M}$. Another parameter, the quasi-phase δ, which is the optical phase difference between the ends of a layer,[4] is denoted as

$$\delta_i = \frac{n_i d_i k}{\cos\theta_i}, \tag{3}$$

where n_i is the refraction index of material i, d_i is the thickness of layer i, k is the wave number in vacuum, and θ_i is the incident angle in layer i. Usually, one considers the simplest experimental settings and takes into account the cases of vertical incidence and identical optical phase difference for each layer (condition 1). On the other hand, in order to make the quasiperiodicity of the multilayer systems most effective, wavelengths should satisfy the following quasiresonance condition:[4]

$$n_j d_j = \frac{\lambda_{\rm C}}{4}, \tag{4}$$

where $\lambda_{\rm C}$ is the central wavelength (CW) (condition 2). In this paper we take into account the cases of normal incidence and calculate PMs and TCs at $\lambda_{\rm C}$. Under this condition, we choose

$$\begin{aligned} \theta &= \theta_i = 0, \\ \delta &= \delta_i = \frac{\pi}{2}, \end{aligned} \tag{5}$$

then the corresponding PMs can be written as

$$\boldsymbol{M}_{ij} = \boldsymbol{M}_{ji}^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & N_{ji} \end{pmatrix}, \tag{6}$$

$$\boldsymbol{M}_j = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \tag{7}$$

where $\boldsymbol{M}_{ij}$ denotes the PM from layer i to layer j, $\boldsymbol{M}_i$ is the PM through the single layer i, and N_{ji} is the ratio between the indices of refraction for two media j and i and can be defined as

$$N_{ji} = \frac{n_j}{n_i}. \tag{8}$$

From Eqs. (6)–(8) one can calculate the PM for light to pass through the 1D optical superlattices and obtain the corresponding TC by use of Eq. (2).

Based on the theory of electromagnetic propagation one studied optical transmissions through many kinds of 1D aperiodic superlattices by analytical derivations and numerical calculations, but the relationship between the optical transmission properties and the structural characteristics for aperiodic sequences has been rarely reported yet. Are there any tight correlations between them? Is it possible to forecast directly the optical transmission properties of 1D aperiodic superlattices only by analysing the corresponding structural characteristics, without through any conventional analytical derivation and numerical calculation? Based on this idea Yang[20] proposed a so-called transparent-component-decimation (TCD) method and demonstrated that under the two aforementioned conditions, even layers of neighbourhood identical elements are transparent and can be decimated from the chain in the process of transmission investigation. Taking advantage of this interesting characteristic the TCD method can simplify arbitrary 1D optical superlattices without through any numerical calculation and/or analytical derivation, then one can predict directly the optical stabilities, circulations and/or pseudo-circulations by analysing the symmetry and/or the circularity of the structure for the corresponding simplified sequences. As one of the series studies of our group, Hu *et al.*[19] proposed a so-called decomposition-substitution (DS) method and studied optical transmissions through 3CTM multilayers by this method. Unlike TCD method's simplifying sequences directly, the DS method analyses the structural characteristic of 3CTM chain. It is found that[19] from the 5th generation on, 3CTM sequence is composed of three kinds of basic components (BCs), α, β and γ, and further on, α, β and γ constitute three kinds of effective components (ECs), Ω_1, Ω_2 and Ω_3. Optical transmission through BC is equivalent to that of EC. Based on this structural property, without through any complicated calculation on total sequence one can predict the optical tristability of 3CTM optical superlattices by analysing the relationship between the three kinds of BCs and the total

chain. Obviously, the DS method provides a new way for investigating optical transmission, i.e., if the aperiodic optical superlattices consist of some BCs one can predict the optical transmission (or other physical) properties by analysing the relationship between the BCs and the total sequence. Simplifying arbitrary aperiodic sequence freely, the TCD method will not be limited by any sequence order and possesses general applicability. Although the DS method analyses the BCs of aperiodic sequences, different models may possess different BCs. Therefore, the expressions of BCs are not uniform. The idea of seeking for BCs of aperiodic sequences may be an effective method.

As another one of the series studies of our group, by TCD and DS methods we simplify and decompose the BCs of 3CTM sequence and obtain three kinds of new BCs. Generalizing these new BCs we propose a type of basic-structural-units (BSUs), which possess interesting optical transmission properties. It is found that at the CW, no matter what the order of BSUs is, the optical transmission through the 1D BSU superlattices will be determined completely by the number and the type of BSUs. Particularly, if the numbers of different types of BSUs are all the same and the middle two elements of BSUs constitute a cycle, then no matter whether the system is periodic, or quasiperiodic, or aperiodic, or unordered, or even random, it will be transparent at the CW. Similar to the even layers of neighbourhood identical elements in TCD method, optical BSU subsystems of this kind can also be decimated from the chain in the process of transmission investigation. There would be a potential application in the designing of some interesting optical devices.

The rest of this paper is organized as follows. In Section 2, we introduce the origin and the characteristics for BSUs. The properties of optical superlattices constructed by interesting BSUs are studied in Section 3. And a brief summary is given in Section 4.

2. Basic structural unit

From Ref.[20] if follows that under the aforesaid two conditions, the even layers of neighbourhood identical elements in TCD method are transparent and can be decimated from the chain in the process of transmission investigation. By the TCD method one can simplify the BCs of 3CTM,[19] α, β and γ, as follows:

$$\begin{aligned}
\alpha &= A\underline{BB}CB\underline{CC}AB\underline{CC}AC\underline{AA}B \\
&\Leftrightarrow \overbrace{ACBA}\overbrace{BACB}, \\
\beta &= B\underline{CC}AC\underline{AA}BC\underline{AA}BA\underline{BB}C \\
&\Leftrightarrow \overbrace{BACB}\overbrace{CBAC}, \\
\gamma &= C\underline{AA}BA\underline{BB}CA\underline{BB}CB\underline{CC}A \\
&\Leftrightarrow \overbrace{CBAC}\overbrace{ACBA}, \qquad (9)
\end{aligned}$$

where '$\Leftrightarrow$' means 'be equivalent to', '$\underline{\quad}$' denotes the even layers of neighbourhood identical elements, which can be decimated by the TCD method. '$\overbrace{\quad}$' refers to a new BC. From Eq. (9) one can see that three kinds of new BCs are obtained as follows:

$$\begin{aligned}
\mathrm{BC}_1 &= ACBA, \\
\mathrm{BC}_2 &= BACB, \\
\mathrm{BC}_3 &= CBAC. \qquad (10)
\end{aligned}$$

They are all composed of four elements, where the 1st and 4th ones are identical. Generalizing these new BCs we propose a type of so-called BSUs, which possess the following form:

$$\mathrm{BSU}_i = w_{1i}w_{2i}w_{3i}w_{4i}, \qquad (11)$$

where i is a positive integer and $w_{1i} = w_{4i}$. By Eqs. (6)–(8) one can obtain the following PM of BSU_i under the two aforementioned conditions:

$$\boldsymbol{M}_{\mathrm{BSU}_i} = \begin{pmatrix} x_i & 0 \\ 0 & x_i^{-1} \end{pmatrix}, \qquad (12)$$

where the PM element

$$x_i = \frac{n_{w_{2i}}}{n_{w_{3i}}}. \qquad (13)$$

If a BSU_i is sandwiched between two layers of medium u, the PM

$$\boldsymbol{M}_{u-\mathrm{BSU}_i-u} = \begin{pmatrix} x_i & 0 \\ 0 & x_i^{-1} \end{pmatrix}. \qquad (14)$$

If the substrates on the left- and right-hand sides are media u and u', respectively, the PM

$$\boldsymbol{M}_{u-\mathrm{BSU}_i-u'} = \begin{pmatrix} x_i & 0 \\ 0 & x_i^{-1} \end{pmatrix} \boldsymbol{M}_{uu'}, \qquad (15)$$

where the substrate PM from medium u to medium u', $\boldsymbol{M}_{uu'}$, can be obtained from Eqs. (6) and (8). From

Eqs. (12)–(15) one can see that even if the boundary conditions have been taken into account, the TC of BSU_i depends only on the relative refractive index of the 2nd and 3rd elements of the BSU_i. Similarly, the PM for 1D optical BSU superlattices arranged by following the $\mathrm{BSU}_1\ \mathrm{BSU}_2\ \ldots\mathrm{BSU}_m$ sequence without substrates can be written as

$$\boldsymbol{M}_{\mathrm{BSUs}} = \begin{pmatrix} X & 0 \\ 0 & X^{-1} \end{pmatrix}, \tag{16}$$

where the PM element

$$X = \prod_{i=1}^{m} x_i \tag{17}$$

and x_i can be obtained from Eq. (13). If the BSU system is sandwiched between two layers of medium u, the PM

$$\boldsymbol{M}_{u-\mathrm{BSUs}-u} = \begin{pmatrix} X & 0 \\ 0 & X^{-1} \end{pmatrix}. \tag{18}$$

If the substrates on the left- and right-hand sides are media u and u', respectively, the PM

$$\boldsymbol{M}_{u-\mathrm{BSUs}-u'} = \begin{pmatrix} X & 0 \\ 0 & X^{-1} \end{pmatrix} M_{uu'}. \tag{19}$$

Obviously, from Eqs. (16)–(19) one can see that even if the boundary conditions have been taken into account, the TC of a 1D optical BSU superlattices arranged in the following sequence: $\mathrm{BSU}_1\ \mathrm{BSU}_2\ \ldots\mathrm{BSU}_m$ depends mainly on the product of the PM element x_i for each BSU_i $(i = 1, 2, \ldots, m)$ while the influence of the substrate PM is independent of the optical transmission. Therefore, for 1D optical BSU superlattices, only when the number and type of BSU_i are determinate, will the optical transmission be certain no matter what the marshalling sequence of BSU_i is. In short, the wonderful BSUs are just as the colorful glasses in a kaleidoscope, arbitrary rollings form numerous and complicated magnificent patterns, but all of the combination optical superlattices possess a certain TC at the CW.

3. Optical BSU superlattices

From the aforementioned introduction it is easy to know that a BSU system with m types of BSU_i $(i = 1, 2, \ldots, m)$ possesses a variety of combination forms. In this section, we mainly study two types of 1D typical optical BSU superlattices. For the 1st type of BSU systems, we do not limit the type and the number of BSUs and investigate the optical general character. For the 2nd type of BSU systems, the numbers of all types of BSUs are required to be identical and we confirm the transparent property.

3.1. General BSU systems

By the theory of permutation and combination one knows that three kinds of elements can construct six kinds of BSU systems. In this subsection, taking arbitrary four kinds of BSU systems as an example, we study the general character of the optical transmission of the 1D ternary BSU superlattices, where the structure of the BSUs can be written as

$$\begin{aligned} \mathrm{BSU}_1 &= ABCA \longrightarrow a, \\ \mathrm{BSU}_2 &= BACB \longrightarrow b, \\ \mathrm{BSU}_3 &= BABB \longrightarrow c, \\ \mathrm{BSU}_4 &= CBAC \longrightarrow d, \end{aligned} \tag{20}$$

where in order to simplify the expression we express BSU_i $(i = 1, 2, 3, 4)$ as a, b, c, and d, respectively. We choose randomly five a's, four b's, two c's, and one d to construct the following four kinds of BSU sequences:

$$\begin{aligned} S_1 &= aaaaabbbbccd, \\ S_2 &= abcdaababcab, \\ S_3 &= dbbabaacbaac, \\ S_4 &= abacbabacabd, \end{aligned} \tag{21}$$

where S_1 is an overlapping sequence, S_2 is a quasiperiodic one arranged by following the 4CF order (which is similar to 3CF model[18] and its substitution rules are $A \to AB$, $B \to C$, $C \to D$, and $D \to A$, respectively), S_3 is an aperiodic one arranged by following the 4CTM order (which is similar to 3CTM model[17] and its substitution rules are $A \to AB$, $B \to BC$, $C \to CD$, and $D \to DA$, respectively), and S_4 is a random one generated arbitrarily by computer. Randomly we choose media A and C as the substrates. By use of the conventional optical transmission theory introduced in Section 1, one can obtain the PMs at the CW of these four kinds of 1D optical BSU superlattices as follows:

$$\begin{aligned} \boldsymbol{M}_{S_1} = \boldsymbol{M}_{S_2} = \boldsymbol{M}_{S_3} = \boldsymbol{M}_{S_4} \\ = \begin{pmatrix} \dfrac{{n_{\mathrm{A}}}^5 {n_{\mathrm{B}}}^4}{{n_{\mathrm{C}}}^9} & 0 \\ 0 & \dfrac{{n_{\mathrm{C}}}^9}{{n_{\mathrm{A}}}^5 {n_{\mathrm{B}}}^4} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & \dfrac{n_{\mathrm{C}}}{n_{\mathrm{A}}} \end{pmatrix}. \end{aligned} \tag{22}$$

Consequently, these four kinds of systems possess the following identical TCs:

$$T_{S_1} = T_{S_2} = T_{S_3} = T_{S_4} = \frac{4}{\frac{{n_{\rm A}}^{10}{n_{\rm B}}^{8}}{{n_{\rm C}}^{18}} + \frac{{n_{\rm C}}^{20}}{{n_{\rm A}}^{12}{n_{\rm B}}^{8}} + 2}. \quad (23)$$

The numerical simulations are illustrated in Fig. 1, with the indices of refraction being $n_{\rm A} = 1.2$, $n_{\rm B} = 2.0$, and $n_{\rm C} = 3.0$. At the CW the TCs are

$$T_{S_1} = T_{S_2} = T_{S_3} = T_{S_4} = 2.6185 \times 10^{-6}. \quad (24)$$

Obviously, substituting the values of the indices of refraction into Eq. (23) one can deduce that the values of TCs are in accord with the results in Eq. (24). The analytic results are confirmed by numerical simulations.

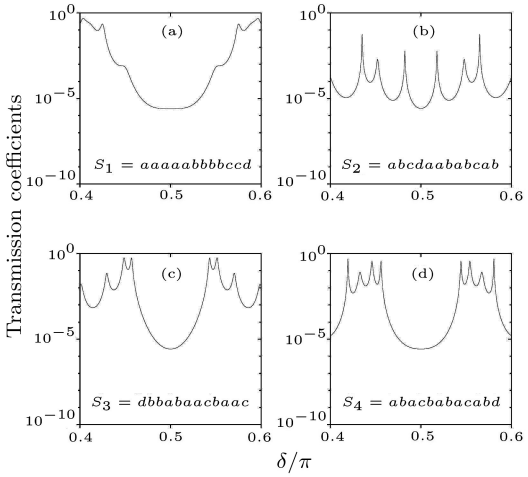


Fig. 1. Relationships between transmission coefficient and the quasi-phase δ for four kinds of optical BSU superlattices, with the indices of refraction being $n_{\rm A} = 1.2$, $n_{\rm B} = 2.0$, and $n_{\rm C} = 3.0$. At the point of $\delta/\pi = 0.5$, the values of transmission coefficients are all equal to 2.6185×10^{-6}.

From Fig. 1 one can also see that for the systems composing of four kinds of BSUs, a, b, c, and d, when their numbers are determinate, whether they are arranged by following periodic, quasiperiodic, aperiodic, unordered, or random sequences respectively, the optical TC at the CW will be uniform. This is the general character of optical BSU superlattices. In brief, if the substrates are certain, the optical transmission through 1D optical BSU superlattices will be completely determined by the number and the type of BSUs and has nothing to do with the order of BSUs.

3.2. Transparent BSU systems

By the theory of permutation and combination one knows that four kinds of elements can construct twelve kinds of BSU systems. In this subsection, taking arbitrary four kinds of BSU systems as an example, we study the transparent character of the optical transmission of the 1D quaternionic BSU superlattices, where the structure of the BSUs can be written as

$$\begin{aligned} {\rm BSU}_1 &= BABB \longrightarrow a, \\ {\rm BSU}_2 &= ABCA \longrightarrow b, \\ {\rm BSU}_3 &= CCDC \longrightarrow c, \\ {\rm BSU}_4 &= BDAB \longrightarrow d, \end{aligned} \quad (25)$$

where in order to simplify the expression we denote ${\rm BSU}_i$ $(i = 1, 2, 3, 4)$ as a, b, c, and d, respectively. Here the two middle elements of the four kinds of BSUs constitute a cycle. For example, the four 2nd elements of ${\rm BSU}_i$ $(i = 1, 2, 3, 4)$ constitute a cycle of $A - B - C - D$, and the four 3rd elements constitute a cycle of $B - C - D - A$, where the two cycles are both the same. Our research shows that if the 1D optical BSU superlattices possess transparent feature, the numbers of all types of BSUs should be equal to each other. In order to fulfill the requirement of BSU number, we choose randomly four a's, b's, c's, and d's, respectively, to construct the following four kinds of BSU sequences:

$$\begin{aligned} S_1 &= aaaabbbbccccdddd, \\ S_2 &= abcdabcdabcdabcd, \\ S_3 &= abbcbccdcddadaab, \\ S_4 &= abcdcdabcbaadbdc, \end{aligned} \quad (26)$$

where S_1 is an overlapping sequence, S_2 is a periodic one, S_3 is an aperiodic one arranged by following the 4CTM order, and S_4 is a random one generated arbitrarily by computer. Randomly we choose medium A as the substrates. By use of the conventional optical transmission theory introduced in Section 2, one can obtain the PMs at the CW of these four kinds of 1D optical BSU superlattices as follows:

$$M_{S_1} = M_{S_2} = M_{S_3} = M_{S_4} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \quad (27)$$

Therefore, these four kinds of systems possess the following identical TCs:

$$T_{S_1} = T_{S_2} = T_{S_3} = T_{S_4} = 1. \quad (28)$$

The numerical simulations are illustrated in Fig. 2, with the indices of refraction being $n_{\rm A} = 1.2$, $n_{\rm B} = 2.0$, and $n_{\rm C} = 3.0$. At the CW the TCs are

$$T_{S_1} = T_{S_2} = T_{S_3} = T_{S_4} = 1.0000. \quad (29)$$

Obviously, the values of TCs in Eq. (28) are in accordance with the numerical results in Eq. (29). The analytic results are confirmed by the numerical simulations.

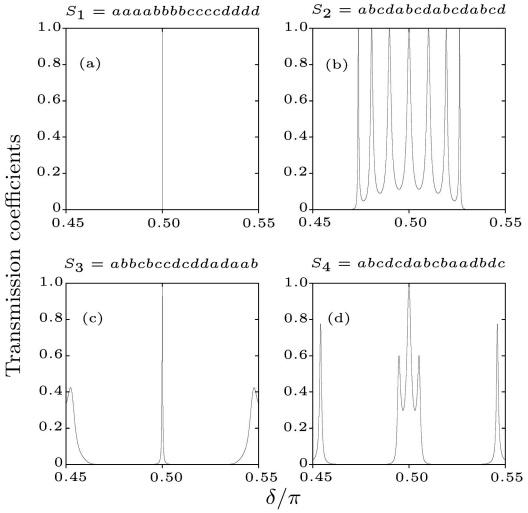


Fig. 2. Relationships between transmission coefficient and the quasi-phase δ for four kinds of optical BSU superlattices, with the indices of refraction being $n_{\rm A} = 1.2$, $n_{\rm B} = 2.0$, and $n_{\rm C} = 3.0$. At the point of $\delta/\pi = 0.5$, the values of transmission coefficients are all equal to 1.0000.

From Fig. 2 one can also see that for the systems composing of four kinds of BSUs, a, b, c, and d, if their numbers are all the same, the substrates are identical, and the middle two elements of BSU constitute a cycle, then whether they are arranged by following periodic, quasiperiodic, aperiodic, unordered, or even random sequences respectively, the optical TC at the CW will forever be equal to 1.0000 and the total BSU system can be regarded as a transparent component. Generally, for the optical BSU superlattices with identical substrates, if the numbers of different types of BSUs are all the same and the middle two elements of BSU constitute a cycle, then no matter how many kinds of BSU and BSU elements there exist and no matter whether the BSUs are arranged by following periodic, quasiperiodic, aperiodic, unordered, or even random sequences, the optical TC at the CW will always be equal to 1.0 and the total BSU system can be regarded as a transparent component. Similar to the even layers of neighbourhood identical elements in TCD method, optical BSU subsystems of this kind can be decimated from the sequence in the process of transmission investigation.

4. Conclusion

In this paper, by the TCD method we obtain three kinds of new BCs by simplifying and decomposing the BCs of the 3CTM sequence. Based on these new BCs we propose a type of interesting BSUs and investigate the optical transmission through the 1D BSU superlattices.

It is found that even if the boundary conditions have been taken into account, the TC of 1D optical BSU superlattices arranged by following the $\rm BSU_1\ BSU_2\ \ldots BSU_{\it m}$ sequence depends mainly on the product of the PM element x_i for each $\rm BSU_{\it i}$ $(i = 1, 2, \ldots, m)$ while the influence of the substrate PM is independent of the optical transmission. Therefore, for 1D optical BSU superlattices, only when the number and type of $\rm BSU_{\it i}$ are determinate will the optical transmission be certain no matter what the marshalling sequence of $\rm BSU_{\it i}$ is. As an example, we investigate arbitrary four kinds of BSU systems, where the systems are arranged by following overlapping, quasiperiodic 4CF, aperiodic 4CTM, and random sequences, respectively. The analytic results are confirmed by numerical simulations.

In particular, if the substrates are identical, the numbers of different types of BSU are all the same and the middle two elements of BSUs constitute a cycle, then no matter whether the system is periodic, or quasiperiodic, or aperiodic, or unordered, or even random, the BSU systems will be transparent at the CW. Similar to the even layers of neighbourhood identical elements in TCD method, optical BSU subsystems of this kind can also be decimated from the chain in the process of transmission investigation. As an example, we investigate arbitrary four kinds of BSU systems, where the systems are arranged by following the overlapping, periodic, aperiodic 4CTM, and random sequences, respectively. The analytic results are confirmed by numerical simulations.

For these significant optical BSU superlattices, there would be a potential application in the designing of some interesting optical devices.

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