The dispersion relations for surface plasmon in a nonlinear–metal–nonlinear dielectric structure*

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In the asymmetric and symmetric nonlinear–metal–nonlinear dielectric structures, this paper studies the analytic dispersion relation for surface plasmon in a system consisting of a thin metallic film covered on two sides media of intensity-dependent refractive indexes by applying a generalised first integral approach. Especially in the symmetric waveguide structure, two possible modes can exist: the odd mode and the even mode. The dispersion relations of the two modes are obtained. Due to the nonlinear dielectric, the squared magnitude of the electric field at the interface appears and alters the dispersion relations. Numerical results are compared to those from a certain approximate treatment.

Keywords: dispersion relation, metal, surface plasmon, nonlinear
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1. Introduction

Since the extraordinary optical transmission through a two-dimensional metallic hole array was first reported by Ebbesen *et al.*,[1] much interest in surface plasmons (SPs) has been excited in subwavelength metallic structures.[2−5] The phenomenon of enhanced transmission in patterned metallic films and advancements in experimental techniques have triggered the rapid developments in plasmonics.[6] The SP-based optical manipulations open up a new avenue for designing new types of nano-optic devices, including enhanced spectroscopy, high resolution microscopy and sensing,[7] better light sources,[8] and cloaking metamaterials.[9] Two possible geometries seem interesting for guiding plasmons, namely, insulator–metal–insulator (IMI) and metal–insulator–metal (MIM). Especially, in IMI structure, the plasmon propagation distances increase substantially due to the coupling of plasmons at the neighbouring interfaces and the field concentration in dielectric rather than metal. Recently, nonlinear optical devices based on subwavelength metallic structures have been proposed to actively control plasmonic signals by nonlinear material.[10,11] Optical bistability in nonlinear surface-plasmon polaritonic crystals has been observed.[12] Full numerical simulations of the nonlinear SP crystals in various geometries have recently been performed by using finite-different time domain method.[13,14]

Since only transverse magnetic (TM) electromagnetic waves are supported by a metal–insulator interface, the corresponding nonlinear Maxwell equations involve two components of the electric field. The equations cannot be solved analytically in a general case. Several approximations and simplifications have been employed, such as, Agranovich *et al.*[15] assumed that nonlinearity depends on the longitudinal component of the electric field; Stegeman *et al.*[16] assumed that nonlinearity is caused only by the transverse field component. But these assumptions are valid only in specific cases. The dispersion relation at a metal/Kerr-type nonlinear interface was studied by using a first integral approach.[17] Huang *et al.*[18] studied how a Kerr medium affects the SP frequency at large wave vectors. Instead of a semi-infinite metallic region, in this paper, we study the SP dispersion relations of a thin slab of metal of thickness d covered on two sides media of Kerr nonlinear dielectric. Exact analytic expressions for obtaining the SP dispersion relations are derived by using a generalised first integral approach.

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2. Theory

Figure 1 shows the system which we study in this paper, a thin slab of metal of thickness d is covered on two sides media of intensity-dependent refractive indexes, i.e. Kerr nonlinear dielectric, occupying the space of $z > d$ and $z < 0$, respectively.

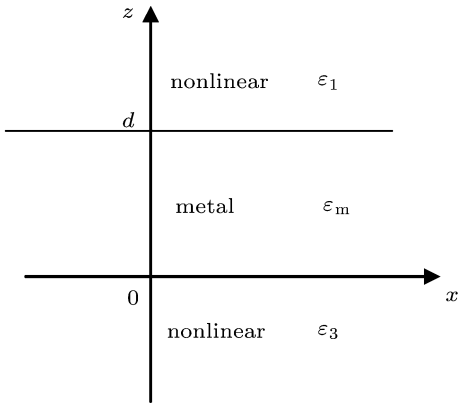


Fig. 1. The system consists of a metallic film separating two semi-infinite nonlinear dielectric regions.

The nonlinear dielectric functions ε_1 and ε_3 can be presented as follows:

$$\varepsilon_1 = \varepsilon_{1\mathrm{L}} + \alpha_1 |\boldsymbol{E}|^2, \quad \varepsilon_3 = \varepsilon_{3\mathrm{L}} + \alpha_3 |\boldsymbol{E}|^2, \tag{1}$$

where $\varepsilon_{1\mathrm{L}}$, $\varepsilon_{3\mathrm{L}}$ and α_1, α_3 are the linear and nonlinear dielectric coefficients, respectively, and are frequency dependent in general. The metal has a Drude dielectric constant:

$$\varepsilon_m(\omega) = 1 - \frac{\omega_\mathrm{p}^2}{\omega^2}, \tag{2}$$

where ω_p is the bulk plasma frequency. In this paper, all media are assumed to be lossless and nonmagnetic.

Consider a TM-polarised wave with the $\boldsymbol{H}$ field having y-component only. The electric field at a location in the system can in general be written as

$$\boldsymbol{E}(\boldsymbol{r}, t) = \frac{1}{2}\{\mathrm{i}\, E_x(z)\hat{x} + E_z(z)\hat{z}\} \exp[\mathrm{i}(kx - \omega t)] + \mathrm{c.c.}, \tag{3}$$

where k is the x-component of the wave vector.

The equations for the field amplitudes $E_x(z)$, $E_z(z)$ and $H_y(z)$ were obtained from Maxwell equations in the from:

$$\frac{\mathrm{d}E_x}{\mathrm{d}z} = \left(k - \frac{\omega^2}{kc^2}\varepsilon_i\right) E_z, \tag{4}$$

$$\frac{\mathrm{d}(\varepsilon_i E_z)}{dz} = k\varepsilon_i E_x, \tag{5}$$

$$H_y(z) = -\frac{\omega}{k}\varepsilon_0 \varepsilon_i E_z, \tag{6}$$

here, $\varepsilon_i = \varepsilon_1$, ε_m, ε_3, depend on the location. In the metal, since ε_m does not depend on z, Eq. (5) reduces to $\mathrm{d}E_z/\mathrm{d}z = kE_x$. In the nonlinear region, however, ε_i cannot be cancelled in Eq. (5).

2.1. The asymmetric structure

In this case, ε_1 and ε_3 are different. In the metal, note that for a decaying wave, k^2 should be greater than $\varepsilon_\mathrm{m}\omega^2/c^2$, $E_x(z)$ and $E_z(z)$ can be expressed as

$$E_x = \{E_{dx}\sinh(q_2 z) + E_{0x}\sinh[q_2(d-z)]\}/\sinh(q_2 d), \quad 0 < z < d, \tag{7}$$

$$E_z = -k\{E_{dx}\cosh(q_2 z) + E_{0x}\cosh[q_2(d-z)]\}/q_2\sinh(q_2 d), \quad 0 < z < d, \tag{8}$$

here $q_2 = \sqrt{k^2 - (\omega^2/c^2)\varepsilon_\mathrm{m}}$

The continuity of E_x and D_z across $z = 0$ and $z = d$ then yields

$$q_2 E_{0z}^- [\varepsilon_{3\mathrm{L}} + \alpha_3 (E_{0z}^-)^2 + \alpha_3 (E_{0x})^2]\sinh(q_2 d) = -k\varepsilon_\mathrm{m}[E_{dx} - E_{0x}\cosh(q_2 d)], \tag{9}$$

$$q_2 E_{dz}^+ [\varepsilon_{1\mathrm{L}} + \alpha_1 (E_{dz}^+)^2 + \alpha_1 (E_{dx})^2]\sinh(q_2 d) = -k\varepsilon_\mathrm{m}[E_{dx}\cosh(q_2 d) - E_{0x}], \tag{10}$$

here $E_{0z}^\pm = E_z(0^\pm)$; $E_{dz}^\pm = E_z(d^\pm)$; $E_{0x}^+ = E_{0x}^- = E_{0x}$; $E_{dx}^+ = E_{dx}^- = E_{dx}$.

A standard treatment of the nonlinear region 1 and region 3 invoke a 'first integral'[15,16] obtained at an equation for $\mathrm{d}E_x/\mathrm{d}z$. Differentiating Eq. (4) with respect to z and multiplying through by $\mathrm{d}E_x/\mathrm{d}z$, in region 1 and region 3 we obtain

$$\frac{\mathrm{d}E_{x1,3}}{\mathrm{d}z}\frac{\mathrm{d}^2 E_{x1,3}}{\mathrm{d}z^2} = k\frac{\mathrm{d}E_{x1,3}}{\mathrm{d}z}\frac{\mathrm{d}E_{z1,3}}{\mathrm{d}z} - \frac{\omega^2}{c^2}\varepsilon_{1,3}E_{x1,3}\frac{\mathrm{d}E_{x1,3}}{\mathrm{d}z}. \tag{11}$$

Integrating with respect to z, in region 3 we obtain

$$\frac{\omega^2}{c^2}(E_{0z}^-)^2[\varepsilon_{3\mathrm{L}} + \alpha_3(E_{0z}^-)^2 + \alpha_3(E_{0x})^2]^2 = k^2\left[\varepsilon_{3\mathrm{L}}(E_{0z}^-)^2 - \varepsilon_{3\mathrm{L}}(E_{0x})^2 + \frac{3}{2}\alpha_3(E_{0z}^-)^4 + \alpha_3(E_{0x})^2(E_{0z}^-)^2 - \frac{1}{2}\alpha_3(E_{0x})^4\right], \tag{12}$$

in region 1 we have

$$\frac{\omega^2}{c^2}(E_{dz}^+)^2[\varepsilon_{1\mathrm{L}}+\alpha_1(E_{dz}^+)^2+\alpha_1(E_{dx})^2]^2$$
$$=k^2\left[\varepsilon_{1\mathrm{L}}(E_{dz}^+)^2-\varepsilon_{1\mathrm{L}}(E_{dx})^2+\frac{3}{2}\alpha_1(E_{dz}^+)^4+\alpha_1(E_{dx})^2(E_{dz}^+)^2-\frac{1}{2}\alpha_1(E_{dx})^4\right]. \tag{13}$$

Where we have set the integration constant $C=0$ as $E_{x1,3}$, $E_{z1,3}$, $\mathrm{d}E_{x1,3}/\mathrm{d}z$ must vanish as $z\to\pm\infty$.

Equations (9), (10) and (12), (13) are the dispersion relations in the asymmetric structure.

2.2. The symmetric structure

In this case, region 1 and region 3 are the same Kerr dielectric, i.e. $\varepsilon_1=\varepsilon_3$, two possible modes can exist: the odd mode and the even mode.

2.2.1. The odd TM mode

In the metal, $E_x(z)$ and $E_z(z)$ can be expressed as

$$E_x=E_{dx}\sinh[q_2(z-d/2)]/\sinh(q_2d/2),\quad 0<z<d, \tag{14}$$
$$E_z=-\frac{kE_{dx}\cosh[q_2(z-d/2)]}{q_2\sinh(q_2d/2)},\quad 0<z<d. \tag{15}$$

The continuity of E_x and D_z across $z=d$ then yields

$$q_2\tanh(q_2d/2)E_{dz}^+[\varepsilon_{1\mathrm{L}}+\alpha_1(E_{dz}^+)^2+\alpha_1(E_{dx})^2]=-k\varepsilon_{\mathrm{m}}E_{dx}. \tag{16}$$

In a similar way with Eq. (13), we obtain

$$\frac{\omega^2}{c^2}(E_{dz}^+)^2[\varepsilon_{1\mathrm{L}}+\alpha_1(E_{dz}^+)^2+\alpha_1(E_{dx})^2]^2$$
$$=k^2\left[\varepsilon_{1\mathrm{L}}(E_{dz}^+)^2-\varepsilon_{1\mathrm{L}}(E_{dx})^2+\frac{3}{2}\alpha_1(E_{dz}^+)^4+\alpha_1(E_{dx})^2(E_{dz}^+)^2-\frac{1}{2}\alpha_1(E_{dx})^4\right]. \tag{17}$$

Rewrite Eq. (17) as

$$\left(\frac{\omega^2\varepsilon_1^2}{k^2c^2}-2\varepsilon_1\right)E_{dz}^{+2}+\varepsilon_{1\mathrm{L}}E_d^2+\frac{1}{2}\alpha_1E_d^4=0, \tag{18}$$

where $E_d^2=E_{dx}^2+E_{dz}^{+2}$ is the squared magnitude of the electric field at the interface between regions 1 and 2. The relation between E_{dz}^{+2} and E_d^2 is given by Eq. (16),

$$(E_{dz}^+)^2=\frac{k^2\varepsilon_{\mathrm{m}}^2}{k^2\varepsilon_{\mathrm{m}}^2+[q_2\tanh(q_2d/2)\varepsilon_1]^2}E_d^2. \tag{19}$$

Substituting Eq. (19) into Eq. (18), we obtain

$$\left(\frac{\omega^2\varepsilon_1^2}{k^2c^2}-2\varepsilon_1\right)\frac{k^2\varepsilon_{\mathrm{m}}^2}{k^2\varepsilon_{\mathrm{m}}^2+[q_2\tanh(q_2d/2)\varepsilon_1]^2}+\varepsilon_{1\mathrm{L}}+\frac{1}{2}\alpha_1E_d^2=0. \tag{20}$$

Equation (20) is the dispersion relations of the odd TM mode.

2.2.2. The even TM mode

$E_x(z)$ and $E_z(z)$ in the metal can be expressed as

$$E_x=E_{dx}\cosh[q_2(z-d/2)]/\cosh(q_2d),\quad 0<z<d, \tag{21}$$
$$E_z=\frac{-kE_{dx}\sinh[q_2(z-d/2)]}{q_2\cosh(q_2d)},\quad 0<z<d. \tag{22}$$

In a similar way with Eqs. (16) and (17), we obtain

$$-q_2 \coth(q_2 d/2) E_{dz}^{+}[\varepsilon_{1\mathrm{L}} + \alpha_1 (E_{dz}^{+})^2 + \alpha_1 (E_{dx})^2] = k\varepsilon_{\mathrm{m}} E_{dx}, \tag{23}$$

$$\frac{\omega^2}{c^2}(E_{dz}^{+})^2[\varepsilon_{1\mathrm{L}} + \alpha_1 (E_{dz}^{+})^2 + \alpha_1 (E_{dx})^2]^2 = k^2\left[\varepsilon_{1\mathrm{L}}(E_{dz}^{+})^2 - \varepsilon_{1\mathrm{L}}(E_{dx})^2 + \frac{3}{2}\alpha_1 (E_{dz}^{+})^4 + \alpha_1 (E_{dx})^2 (E_{dz}^{+})^2 - \frac{1}{2}\alpha_1 (E_{dx})^4\right]. \tag{24}$$

Substituting Eq. (23) into Eq. (24) yields the following relation:

$$\left(\frac{\omega^2 \varepsilon_1^2}{k^2 c^2} - 2\varepsilon_1\right)\frac{k^2 \varepsilon_{\mathrm{m}}^2}{k^2 \varepsilon_{\mathrm{m}}^2 + [q_2 \coth(q_2 d/2)\varepsilon_1]^2} + \varepsilon_{1\mathrm{L}} + \frac{1}{2}\alpha_1 E_d^2 = 0. \tag{25}$$

Equation (25) is the dispersion relations of the even TM mode.

3. Discussion

In the linear IMI structure, the dispersion relations of the odd mode and the even mode can be expressed as[19]

$$\tanh(q_2 d/2) = -\frac{q_1 \varepsilon_{\mathrm{m}}}{q_2 \varepsilon_1}, \tag{26}$$

$$\coth(q_2 d/2) = -\frac{q_1 \varepsilon_{\mathrm{m}}}{q_2 \varepsilon_1}, \tag{27}$$

where ε_1 is the dielectric of linear media, $q_1 = \sqrt{k^2 - \frac{\omega^2}{c^2}\varepsilon_1}$, $q_2 = \sqrt{k^2 - \frac{\omega^2}{c^2}\varepsilon_{\mathrm{m}}}$.

In order to compare with the linear case, Eqs. (20) and (25) are rewritten as

$$\tanh(q_2 d/2) = -\frac{\sqrt{k^2 - \dfrac{\omega^2}{c^2}\varepsilon_1 + k^2 \dfrac{\alpha_1 E_d^2}{2\varepsilon_1}}\varepsilon_{\mathrm{m}}}{q_2 \varepsilon_1} \times \frac{1}{\sqrt{1 - \dfrac{\alpha_1 E_d^2}{2\varepsilon_1}}}, \tag{28}$$

$$\coth(q_2 d/2) = -\frac{\sqrt{k^2 - \dfrac{\omega^2}{c^2}\varepsilon_1 + k^2 \dfrac{\alpha_1 E_d^2}{2\varepsilon_1}}\varepsilon_{\mathrm{m}}}{q_2 \varepsilon_1} \times \frac{1}{\sqrt{1 - \frac{\alpha_1 E_d^2}{2\varepsilon_1}}}. \tag{29}$$

In the limit $\alpha_1 \to 0$, the results in Eqs. (28) and (29) reduce back to Eqs. (26) and (27), respectively. When the linear media instead of nonlinear media, however, in some literatures,[20,21] the dispersion has been inadvertently applied with the dielectric function ε_1 being simply substituted by Eq. (1). This approach to the study of dispersion of the SPs in IMI structure is not suitable and can lead to appreciable errors in case of strong field intensity. To evaluate the effect of nonlinearity, in Fig. 2 we have plotted the dispersion curves according to the odd and even modes. The values for the relevant parameters involved: $\alpha_1 = 6.4\times10^{-12}$ m$^2\cdot$V^{-2}, $\omega_{\mathrm{p}} = 1.36 \times 10^{16}$ rad/s (for Ag), $d = 100$ nm, $\varepsilon_{1\mathrm{L}} = 2.405$, and a field $E_d = 2.5 \times 10^5$ V/m for our computations.[18]

Figure 2 shows the differences between our models and those in Refs. [20] and [21]. Due to the nonlinearity, the dispersion relations of the odd and even mode move little larger. The reason is that in the approximation model, the Kerr medium is taken to behave as a homogenous medium with a dielectric constant; whereas in our approach, the gradual decrease in the nonlinear term away from the interface is taken into account.

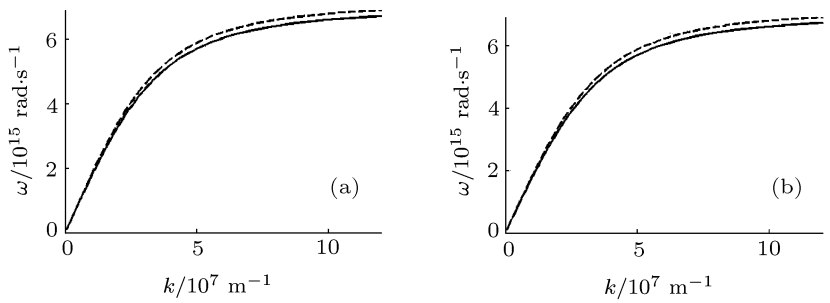


Fig. 2. The dispersion relations of SP modes in symmetric nonlinear–metal–nonlinear system: (a) the odd mode; (b) the even mode. The broken lines represent the results of Eqs. (28) and (29); the continuous lines represent the results of Eqs. (26) and (27).

4. Conclusion

In summary, we derived the SP dispersion relations analytically for a metallic thin film covered on two sides media of Kerr nonlinear dielectric, by introducing a generalised first integral approach in treating electromagnetic waves in the nonlinear dielectric. Due to the nonlinear dielectric, the strength E_d at the interface appears and it can alter the dispersion relations. The dispersion relations of the odd mode and the even mode are obtained in the symmetric structure. Especially, explicit results are compared to those from a certain approximate treatment appeared in some literatures. The discrepancy between the two approaches is especially significant for the case with a nonlinear term in the Kerr dielectric function.

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