Localisation and phase transition of acoustic waves in a soft medium containing air bubbles*

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We study via numerical experiments the localisation property of an acoustic wave in a viscoelastic soft medium containing randomly-distributed air bubbles. The behaviours of the oscillation phases of bubbles are particularly investigated in various cases for distinguishing efficiently the acoustic localisation from the effects of acoustic absorption caused by the viscosity of medium. The numerical results reveal the phenomenon of 'phase transition' characterized by an unusual collective oscillation of bubbles, which is an effective criterion to unambiguously identify the acoustic localisation in the presence of viscosity. Within the localisation region, the phenomenon of phase transition persists, and a remarkable decrease in the fluctuation of the oscillation phases of bubbles is observed. The localisation phenomenon will be impaired by the enhancement of the viscosity factors, and the extent to which the acoustic wave is localised may be determined by appropriately analyzing the values of the oscillation phases or the amount of reduction of the phase fluctuation. The results are particularly significant for the practical experiments in an attempt to observe the acoustic localisation in such a medium, which is in general subjected to the interference of the great ambiguity resulting from the effect of acoustic absorption.

Keywords: acoustic, localisation, phase transition, viscoelastic soft medium, air bubble
PACC: 4335, 0340K, 4660B

1. Introduction

The past few decades have witnessed considerable efforts in an attempt to investigate the localisation of classical waves in inhomogeneities. This was initiated by the original work of Anderson who had revealed the localisation of electronic waves for the first time.[1] Later on, the phenomena of localisation have been proved to occur for microwaves in inhomogeneities[2,3] and for light in disordered and dispersive media.[4,5] For acoustic waves, Ye and collaborators have studied the sound propagation in a bubbly liquid via numerical experiments since 1998 and have theoretically identified the acoustic localisation which could be achieved in such media under proper conditions.[6,7] Although bubbly liquid media might be simple toy models in a theoretical sense for demonstrating the significant phenomenon of acoustic localisation,[6] they should not be ideal samples employed in practical experiments due to the high instability and flowability of bubbles.[8] As compared with a bubbly liquid, a bubbly soft medium has the unique advantages for the experimental study of acoustic localisation: the extremely low shear stiffness of a soft medium enables the bubbles to become effective acoustical scatterers in contrast to the common solids with high values of shear stiffness,[9,10] while the solid nature can yield sufficiently large stress to compensate the buoyancy of bubbles for keeping the stability of the practical sample.[11] Liang and Cheng have numerically investigated the propagation of acoustic waves in a bubbly soft medium since 2006 and obtained the theoretical proof that the acoustic wave could be localised in a qualitatively similar manner as in the case of a bubbly liquid.[12−14] It is worth pointing out that, however, in most of the previous works the soft medium is in general assumed totally elastic to exclude the effect of acoustic absorption, and the phenomenon of localisation is identified only by inspecting the spatial reduction of wave fields. Note that the practical sample of a soft medium is in general assumed viscoelastic[15] and the existence of viscosity inevitably causes ambiguity in differentiating the localisation effect from the acoustic absorption which might result in the spatial decrease of wave fields as well.[7] In fact, this accounts principally for the fact that the experimental evidence of the existence of wave localisation still lacks.[6] It is therefore particularly significant to resort to an effi-

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cient alternative approach to identify the phenomenon of acoustic localisation in a bubbly soft medium unambiguously even in the presence of the viscosity effect. This might be accomplished by properly analysing the behaviours of the oscillation phases of the pulsating bubbles, as briefly reported in a recent article.[14]

In the present study, we provide detailed formulae of the methodology for solving the propagation of a longitudinal acoustic wave in a bubbly soft medium which allows the incorporation of the effect of viscosity, and make further investigation on the acoustic localisation property by particularly inspecting the oscillation phases of bubbles. The multiple scattering of the acoustic wave in this system is described by using a self-consistent approach, and the wave field is solved via numerical simulations in a rigorous manner. The soft medium is assumed viscoelastic and the values of the viscosity factors are manually adjusted in the numerical simulations to study the sensibility of the results to the viscosity effect. By using a phase diagram method the existence of an unusual collective oscillation of bubbles is revealed as the acoustic localisation takes place, which is interpreted as a significant phenomenon of 'phase transition' that is believed to be the underlying physical mechanism from which the phenomenon of localisation stems.[14] The statistical behaviours of the oscillation phases of bubbles are then displayed for a clear observation of their values that cannot be conveniently read from the phase diagrams. The result indicates that the phenomenon of phase transition persists as long as the localisation occurs, while the extent to which the acoustic wave is localised decreases monotonically as the viscosity effect is enhanced gradually. The fluctuations of the oscillation phases of bubbles are also studied as functions of frequency, which proves that the fluctuations are remarkably diminished within the localisation region and the amount of their reductions can act as effective indications of the extent of localisation. Consequently, it is reasonable to conclude that the phenomenon of phase transition is proved to be an efficient criterion to identify the existence of acoustic localisation in a bubbly soft medium. This is particularly important to the practical experiments for observing the phenomenon of acoustic localisation where the presence of viscosity effect generally leads to ambiguity in differentiating acoustic localisation from acoustic absorption.

2. Theory

In the present study, we give a theoretical analysis to the propagation of an acoustic wave of angular frequency ω in a finite-sized model of a soft medium permeated with air-filled bubbles for which the total number and the volume fraction are denoted as N and β respectively. The reason of employing a finite-size model attributes to two aspects: (i) the sizes of any sample utilised in practical experiments are inevitably finite; (ii) only for models containing finite number of scatterers could the theoretical problem be solved rigorously by using a multiple scattering self-consistent method.[6,7,11,12] Notice that in the previous works the soft medium is generally assumed totally elastic to exclude ambiguity caused by the effects of acoustic absorption, while in practice the viscosity effects should be necessarily taken into account. In the present study we thus assume the soft medium to be a viscoelastic one, in which the viscosity coefficients may be manually adjusted to inspect the influence of the acoustic absorption on the results. For simplicity while without losing generality, all the bubbles are assumed to be of uniform radius r_0. The acoustic wave is a low-frequency (i.e. $\omega r_0/c_l << 1$) longitudinal wave emitted from a unit source located at the centre of the sample. Here $c_l = \sqrt{\lambda + 2\mu/\rho}$ is the speed of the longitudinal wave in the soft elastic medium with λ and μ being the Lamé coefficients respectively, and ρ is the mass density. All bubbles are randomly distributed in a not-overlapping way inside a spatial domain chosen as a spherical shape to eliminate the influences caused by an irregular edge, so that the radius of the bubble cloud may be easily determined as $R = (N/\beta)^{1/3} r_0$. The assumption is adopted such that the scattered acoustic field is totally longitudinal owing to the fact that the mode conversion has been proved negligible in Ref. [12].

Among many useful methods suggested for describing the acoustic scattering by a finite group of random scatterers, the self-consistent method is proved to be particularly effective.[16] This method is based on a genuine self-consistent scheme proposed first by Foldy[17] and reviewed in Ref. [18]. Later Ye *et al.* employed the self-consistent method to investigate the wave propagation in a bubble liquid.[6,7] In this method, the multiple scattering of waves is represented by a set of coupled equations, and the rigorous results can be obtained by solving the equations. In the present study, the wave propagation in a bubbly

soft medium will be solved by using the self-consistent method in an exact manner, for which a modification is performed to account for the viscosity effect of the soft medium.

As the longitudinal wave emitted from the source in incident on the surrounding bubbles, the wave is subject to multiple scattering by the bubbles. Then the total acoustic wave $p(\boldsymbol{r})$ at a spatial point $\boldsymbol{r}$ is given by

$$p(\boldsymbol{r}) = p_0(\boldsymbol{r}) + \sum_{i=1}^{N} p_{\rm s}^i(\boldsymbol{r}), \tag{1}$$

where $r = |\boldsymbol{r}|$ is the distance from the source point, $p_{\rm s}^i(\boldsymbol{r})$ refers to scattered wave from the ith bubble ($i = 1, 2, \ldots, N$), and $p_0(r) = \exp(\mathrm{i}\,kr)/r$ refers to the propagating wave radiated from the source in the absence of bubbles, with k being the wave number in the soft elastic medium.

In the frequency range of interest, the plane wave approximation still stands for the incident wave upon each bubble. It has been proved against experimental data that the Rayleigh–Plesset equation may be not sufficiently accurate to describe the finite-amplitude vibration of bubble in liquid media.[19] However at present we assume that the amplitude of the incident wave is small and the radial pulsation of an individual air bubble under the action of a plane longitudinal wave in a soft medium can be appropriately described by a Rayleigh–Plesset-like equation of bubble dynamics.[12] Therefore for the ith bubble, for which the spatial location is denoted as $\boldsymbol{r}_i$, the following self-consistent equation should be satisfied:[12]

$$p_{\rm s}^i(\boldsymbol{r}) = f\left[p_0(\boldsymbol{r}_i) + \sum_{j=1, j\neq i}^{N} p_{\rm s}^j(\boldsymbol{r}_i)\right] G_0(\boldsymbol{r} - \boldsymbol{r}_i), \tag{2}$$

where $G_0(\boldsymbol{r}-\boldsymbol{r}_i) = \exp(\mathrm{i}\,k|\boldsymbol{r}-\boldsymbol{r}_i|)/|\boldsymbol{r}-\boldsymbol{r}_i|$ is the usual three-dimensional Green's function, f is the scattering function of a single bubble.

In practical situations, the existence of viscosity effect may notably affect the propagation of acoustic waves and then the localisation characteristics in a bubbly soft medium. In the presence of viscoelasticity, the Lamé coefficients of the soft medium may be rewritten as[20]

$$\lambda = \lambda_{\rm e} + \lambda_{\rm v}\frac{\partial}{\partial t}, \quad \mu = \mu_{\rm e} + \mu_{\rm v}\frac{\partial}{\partial t}, \tag{3}$$

where $\lambda_{\rm e}$ and $\mu_{\rm e}$ are the elastic Lamé coefficients, $\lambda_{\rm v}$ and $\mu_{\rm v}$ are viscosity factors given by Kelvin–Voigt viscoelastic model. In the following we shall assume $\lambda_{\rm v} = 0$, as is usually done for a soft medium.[21] The viscosity factor $\mu_{\rm v}$ may be manually adjusted in the numerical simulations to inspect the sensibility of the results to the absorption effects.

Note that the acoustic wave is a simple harmonic wave of angular frequency ω. Then the longitudinal wave number in the soft viscoelastic medium becomes a complex number as $\tilde{k} = k' + \mathrm{i}\,k'' = \omega/\tilde{c}_l$. Here the real and the imaginary parts represent the propagation and the attenuation of the longitudinal wave in a soft viscoelastic medium, respectively, and $\tilde{c}_l$ refers to the effective speed of the wave. For the acoustic wave that propagates in a soft viscoelastic medium permeated with bubbles, the influence of the viscosity effect may be ascribed to two aspects: (i) the propagation of the acoustic wave in a soft viscoelastic medium should be described by a series of complex parameters instead of the corresponding real parameters (i.e., $k \to \tilde{k}$, $c_l \to \tilde{c}_l$, etc.) to account for the absorption effects; (ii) the dynamical behaviour of an individual bubble will be greatly affected by the friction damping of pulsation that results from the viscoelastic solid wall.[20] The incorporation of the effect of acoustic absorption due to viscosity effects amounts to adding a term $\nu \cdot \mathrm{d}U/\mathrm{d}t$ in the dynamical equation of a single bubble in a soft elastic medium.[20] Here $\nu = 4\mu_{\rm v}/(\rho r_0^2)$ is a coefficient characterising the effect of acoustic absorption. By seeking the linear solution of the modified dynamical equation in the same manner as in Ref. [12], one may derive the scattering function f of a single bubble in a soft viscoelastic medium as follows:

$$f = \frac{r_0}{(\omega_0^2/\omega^2 - 1 - \mathrm{i}\,\omega r_0/c_l - \mathrm{i}\,\nu/\omega)}, \tag{4}$$

where ω_0 refers to the resonance frequency of an individual bubble in a soft medium which is defined as $\omega_0 = \sqrt{\omega_1^2 + \omega_2^2}$.[20,22] Here $\omega_1 = \sqrt{4\mu_{\rm e}/\rho r_0^2}$ and $\omega_2 = \sqrt{3\rho_{\rm a}c_{\rm a}^2/\rho r_0^2}$ correspond to the Meyer–Brendel–Tamm (MBT) resonance[21] and the Minnaert bubble resonance[22] respectively, with $c_{\rm a}$ and $\rho_{\rm a}$ being sound speed in air and the mass density of air, respectively. On condition that the soft medium is totally elastic, the expression of the scatter function f degenerates to Eq. (6) in Ref. [12] due to the vanishing of the term $-\mathrm{i}\,\nu/\omega$.

By setting $\boldsymbol{r}$ in Eq. (2) to any scatterer other than the ith, this equation becomes a set of closed self-consistent equations. For an arbitrary configuration of the bubble distribution, these equations can be rewritten in the form of $N \times N$ matrices and then

solved exactly with all multiple scattering effects properly incorporated.

To identify the phenomenon of acoustic localisation in a bubbly soft medium unambiguously, we shall investigate particularly the property of the oscillation phase of the system. This can be performed by using a phase diagram method which has been employed previously by Ye *et al.* for bubbly water.[7] Upon incidence, each bubble acts effectively as a secondary source and the scattered wave from the ith bubble can be rewritten as follows:

$$p_{\rm s}^{i}(\boldsymbol{r}) = A_i G_0(\boldsymbol{r} - \boldsymbol{r}_i), \tag{5}$$

where the complex coefficient A_i refers to the effective strength of the secondary source, which may be expressed as $A_i = |A_i| \exp(\mathrm{i}\,\theta_i)$ with the modulus and the phase physically representing the strength of secondary source and the oscillation phase, respectively.

For the ith bubble, it is convenient to assign a two-dimensional unit phase vector, $\boldsymbol{u}_i = \cos\theta_i \hat{x} + \sin\theta_i \hat{y}$ to the oscillation phase of the bubble with $\hat{x}$ and $\hat{y}$ being the unit vectors in the x and y directions, respectively. The phase of emitting source is set to be zero. Thereby the oscillation phase of every bubble is mapped to a two-dimensional plane via the introduction of the phase vectors and may be easily observed in the numerical simulations by plotting the phase vectors in a phase diagram.

In actual experiments, it is the variability of signal that is often easier to analyse.[7] Hence the behaviour of the phases of the oscillating bubbles may be readily studied by inspecting the fluctuation of the oscillation phase of bubbles, which is investigated as well. Here the fluctuation of the phase of bubbles is defined as follows:[7]

$$\delta\theta = \left[\frac{1}{N} \sum_{i=1}^{N} \theta_i^2 - \bar{\theta}^2 \right]^2,$$

where $\bar{\theta} = \frac{1}{N} \sum\limits_{i=1}^{N} \theta_i$ is the averaged phase.

3. Numerical results

A series of numerical simulations have been carried out for various structural parameters in attempt to study the propagation of the longitudinal wave in a bubbly soft medium. In this section, we shall demonstrate the typical results that help to reveal the physical picture of the significant phenomena of acoustic localisation in the presence of the effects of viscosity.

We have previously proved that the acoustical localisation is a ubiquitous phenomenon in bubbly soft elastic media; and it is worth emphasizing that the localisation property is affected by the structural parameters.[12] For different kinds of soft media, the localisation properties are qualitatively the same except that the locations of the localisation regions depend on the natural frequency of an individual bubble that is primarily determined by the shear modulus of the soft media. Although the parameters of β and N have negligible influence on the frequency range within which the localisation takes place, they are substantially crucial for the existence of localisation phenomenon and should be large enough (e.g., $N > 50$ and $\beta > 10^{-5}$) for the multiple scattering process to establish that leads to the localisation.[12] The increase in the bubble cloud size and the bubble density enhances the extent of localisation and broadens the frequency region where the most severe localisation occurs, but dramatically increases the computational complexity as well. In the present study, therefore, we assume an appropriate set of structural parameters that are sufficient for studying the significant phenomenon of localisation without resulting in much difficulties in the numerical simulations. (See Ref. [12] for more detailed discussions regarding the parameter-dependence of the localisation property in bubbly soft media.)

Following Refs. [12]–[14], we employ the material of gelatin as the host medium for which the mechanical parameters are $\rho = 1000$ kg/m^3, $\lambda = 2.3$ GPa, and $\mu = 40$ kPa, and choose the parameters of bubbles as $\beta = 0.1\%$, $r_0 = 1$ mm, and $N = 200$. And the frequency of incident wave is normalised to exclude the influence of the absolute values of bubble sizes. By focusing our attention on this particular case of a bubbly gelatin with all the structural parameters fixed except the viscosity factors, it is then convenient to investigate the localisation property in viscoelastic bubbly soft media and to make an unambiguous comparison with previous results, while without losing the physical generality. It has been manifested in Refs. [12]–[14] where the theoretical model was assumed completely elastic, that the phenomenon of acoustic localisation takes place in such a bubbly gelatin within a particular frequency range slightly above the natural frequency of a single bubble, approximately lying between the normalised frequencies of $\omega r_0/c_l = 0.017$ and 0.077.

Figure 1 displays the typical results of the phase diagrams for a bubbly gelatin at different driving fre-

quencies, with the values of viscosity factors manually adjusted to study the influence of the effect of acoustic absorption. Three particular frequencies are employed:[11] $\omega r_0/c_l = 0.01$ (Fig. 1(a), below the localisation region), $\omega r_0/c_l = 0.1$ (Fig. 1(b), above the localisation region), and $\omega r_0/c_l = 0.02$ (Figs. 1(c) and 1(d), within the localisation region). In a phase diagram, each circle and the corresponding arrow refer to the three-dimensional position and the phase vector of an individual bubble, respectively. In Figs. 1(a)–1(c) we choose the viscosity factor as μ_ν=0 P, i.e., the soft medium that serves as the host medium is assumed totally elastic; while in Fig. 1(d) the value of viscosity factor is set to be $\mu_\nu = 50$ P (1 P=0.1 Pa·s). For comparison we also examine the spatial distribution of the wave fields and plot the transmissions as a function of the distance from the source in Fig. 2 in cases corresponding to Fig. 1. In Fig. 2 the transmission is received at the distance away from the bubble cloud and the total transmission is defined as $I = |T|^2$ with $T(r) = \langle p(r)\rangle/p_0(r)$ being the normalised wave field at the receive point. Here r refers to the distance from the receiving point to the emitting source. Note that by writing the acoustic wave field as $|p|\exp(\mathrm{i}\,\theta_i)$, the energy flow of an acoustic wave is conventionally $\boldsymbol{J} \sim |p|^2\,\nabla\theta_i$. This mathematical relationship reveals the fact that the gradient of the oscillation phase of bubbles is crucial for the occurrence of localisation. Apparently, when the oscillation phases of different bubbles exhibit a coherent behaviour (i.e. θ_i is a constant) while $|p|$ is nonzero, the acoustic energy flow will stop and the acoustic wave will thereby be localised within a spatial domain.[7] Moreover, such coherence in oscillation phases of bubbles is a unique feature of the phenomenon of localisation that results from the multiple scattering of waves, but lacks when other mechanism such as absorption effect dominates, as will be discussed later. Consequently, it should be promising to effectively identify the localisation phenomenon by taking analysis of the oscillation phases of bubbles and seeking their ordering behaviours.

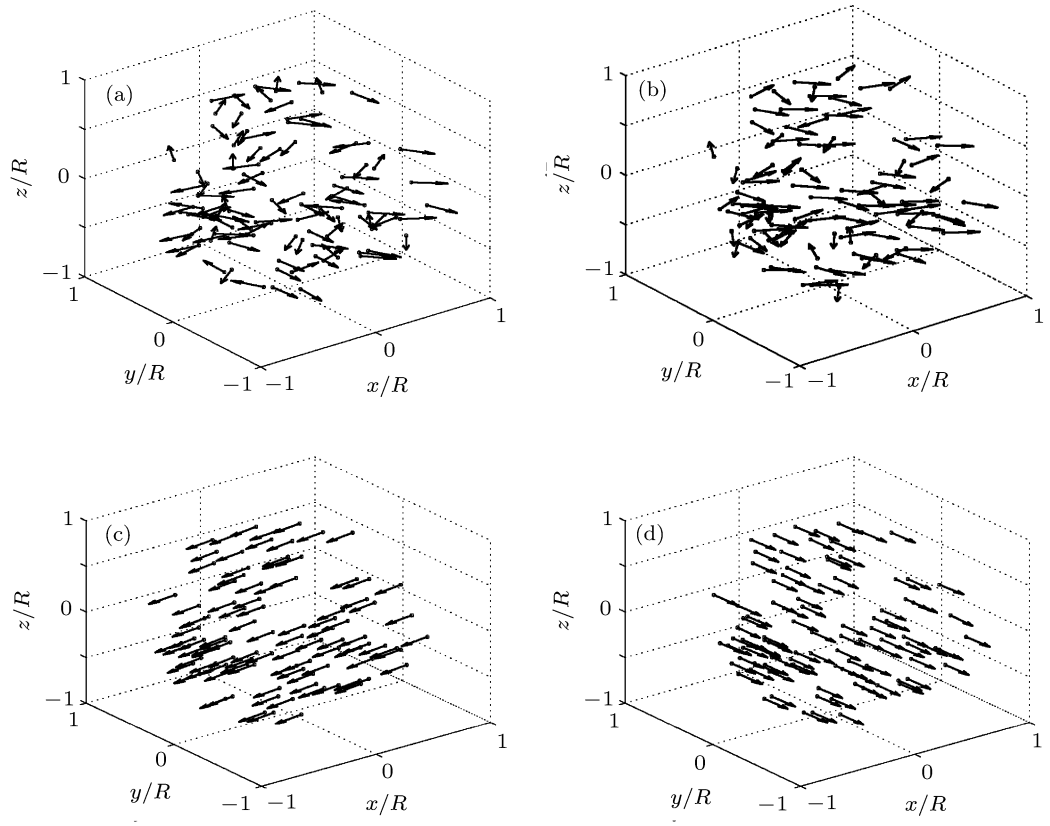


Fig. 1. The phase diagrams for the oscillating bubbles in a bubbly gelatin with different structural parameters. (a) $\omega r_0/c_l = 0.01$, $\mu_\nu = 0$ P; (b) $\omega r_0/c_l = 0.1$, $\mu_\nu = 0$ P; (c) $\omega r_0/c_l = 0.02$, $\mu_\nu = 0$ P; (d) $\omega r_0/c_l = 0.02$, $\mu_\nu = 50$ P.

It is apparent from Figs. 1(a) and 1(b) that the phase vectors pertinent to different bubbles point to various directions as the driving frequency of the source lies outside the localisation region. In other words, the oscillation phases of the bubbles located at different positions in a bubbly soft medium are random in non-localised states. Correspondingly, the curves 1 (thin solid line) and 2 (thin dashed line) in Fig. 2 shows that the non-localised waves remain extended and can propagate through the bubble cloud. As observed in Fig. 1(c), however, the phase vectors located at different spatial positions point to the same direction when localisation occurs, which indicates that the oscillation phases of all bubbles remain constant and the energy flow of the wave stops.

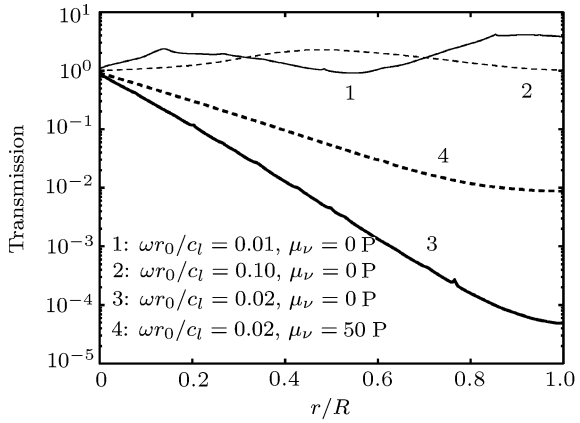


Fig. 2. Transmissions versus the distance from the source in a bubbly gelatin with different structural parameters.

The transition from the non-localised state to the localised state of the wave can be interpreted as a kind of 'phase transition', which is characterized by the unusual phenomenon that all the bubbles pulsate collectively to efficiently prohibit the acoustic wave from propagating.[14] Such a concept of phase transition is physically consistent with the order–disorder phase transition in a ferromagnet.[23] Note that the phase of emitting source is assumed to be zero in the numerical simulations, i.e., the phase vector at the source points to positive $\hat{x}$ direction, while all the phase vectors in Fig. 2(c) point to the negative $\hat{x}$-axis. This means that as the localisation occurs, almost all bubbles tend to oscillate completely in phase but exactly out of phase with the source, which leads to the fact that the localised acoustic energies are trapped within a small spatial domain adjacent to the source as shown by the curve 3 (thick solid line) in Fig. 2. These numerical results are consistent with the previous conclusions obtained for bubbly water and bubbly soft elastic media.[7,14] Therefore it is reasonable to conclude that such a phenomenon of phase transition is the intrinsic physical mechanism from which the acoustic localisation stems.[7,14]

Note that the effect of acoustic absorption has been completely excluded in Figs. 2(a)–2(c) which may cause ambiguity in identifying the phenomenon of localisation. It is thus of much more practical significance to investigate the localisation properties in the case where soft medium is assumed viscoelastic, and the corresponding results are shown in the phase diagram given by Fig. 1(d) as well as the comparison between transmission versus r in Fig. 2. As the viscosity factors of the soft medium are manually increased, the phenomena of phase transition can be identified in a bubbly soft viscoelastic medium, provided that the driving frequency of acoustic wave falls within the localisation region. Meanwhile, exponential decay of the wave fields with respect to the distance from the source is shown by the curve 4 (thick dashed line) in Fig. 2. Observation of Fig. 1(d) and Fig. 2 apparently manifests that, however, the adjustment of the values of the viscosity factors leads to changes of the direction to which all the phase vectors point collectively, and the decay rates of the transmissions versus r. For a bubbly soft viscoelastic medium, it is still possible to achieve the acoustic localisation since the condition can be satisfied that the oscillation phases of bubbles at any spatial points remain constant, but the extents of localisation are necessarily affected by the presence of viscosity effect. In such cases, the reduction of the transmissions results from the effect of acoustic localisation as well as that of acoustic absorption, and the localisation length could not be directly determined from the slope of curve 4 in Fig. 2 representing the transmissions with respect to the propagating distances, as done for a totally inviscid host medium.[24] It is thus difficult to differentiate the phenomenon of acoustic localisation from that of the acoustic absorption without referring to the analysis of the behaviour of the phases of bubbles. Notice that in Figs. 1 and 2, as the viscosity factors are gradually enhanced, the angles between the directions of the phase vectors and the negative x-axis increase while the decay rates of the transmissions versus r decrease. This means that the phase-opposition states between the oscillations of all the bubbles and the source as well as the extents to which the acoustic wave is localised are weakened due to the enhancement of the

viscosity. Therefore it may be inferred that the occurrence of phase transition in a bubbly soft medium is a criterion for identifying the phenomenon of localisation, while the localisation extents can be predicted by accurately analyzing the relationship between the oscillation phases of the bubbles and the source.

It is convenient to employ a phase diagram method for observing the collective phase properties of the bubbles and thereby seeking the existence of the phenomenon of phase transition, however the values of the oscillation phase of each bubble could not be directly read via the phase diagrams in a precise manner. We then illustrate the statistical properties of the parameters of $\langle\theta\rangle$ for all the bubbles in Fig. 3 for a more explicit observation of the values of oscillation phases of the bubbles. Here $\langle\cdot\rangle$ denotes the ensemble average over random configurations of bubble clouds, $p_\theta(\theta)$ is defined as the probability that the values of $\langle\theta\rangle$ fall between θ and $\theta+\Delta\theta$, i.e., $\theta \le \langle\theta\rangle < \theta+\Delta\theta$, with $\Delta\theta$ referring to the difference between the two neighbouring discrete values of θ. And the values of $p_\theta(\theta)$ have been normalised such that the total probability equals 1. In Fig. 3 three particular values of viscosity factors are considered: $\mu_\nu = 0$ P (curve 3, thick solid line), 50 P (curve 4, thick dashed line), 200 P (curve 5, thick dotted line). It is obvious in Fig. 3 that: (i) Outside the localisation region, as shown by the thin curves 1 (solid line) and 2 (dashed line), the values of oscillation phases $\langle\theta\rangle$ exhibit large extents of randomness, which indicates a lack of the above-mentioned collective behaviour of the bubble oscillation crucial for the existence of localisation, in accordance with the results shown in Figs. 1(a) and 1(b). (ii) When the phenomenon of localisation occurs, the oscillation phases almost remain constant for bubbles located at different spatial points, which is illustrated by the delta-function shapes of the thick curves 3–5. It is also noteworthy that the oscillation phase of each bubble approximates $-\pi$ in an elastic medium, and that the presence of the viscosity effect does not change such a phenomenon of phase transition but leads to a larger average value of oscillation phases $\langle\theta\rangle$. A monotonic increase in the values of the oscillation phases of bubbles is clearly observed as the viscosity factors are gradually enhanced. In the soft medium with viscosity factor $\mu_\nu = 50$ P, the values of $\langle\theta\rangle$ nearly equal -0.45π for all the bubbles, and $\langle\theta\rangle$ approximate -0.15π for the case of $\mu_\nu = 200$ P.

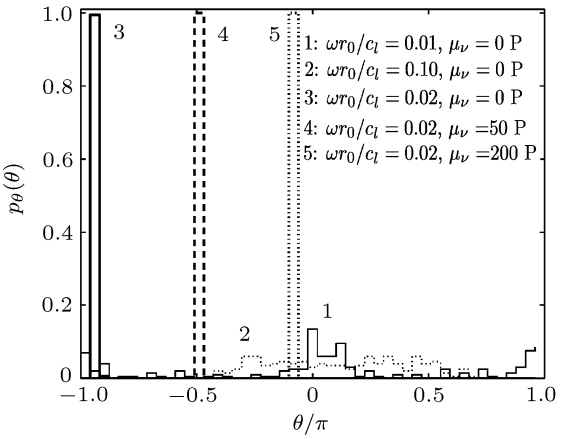


Fig. 3. The comparison between the statistical behaviours of the oscillation phases of bubbles in a bubbly gelatin with different structural parameters.

The principal influence of the viscosity effect on the localisation property in a bubbly soft medium is attributed intrinsically to two aspects of physical mechanism. The localisation phenomenon in inhomogeneities had been extensively proved to stem from the important multiple scattering processes between scatterers. In a viscoelastic medium the recursive process of multiple scattering could not be well established due to the effect of acoustic absorption caused by the viscosity, which will impair the extent to which the acoustic wave can be localised. For an individual bubble pulsating in a viscoelastic medium, on the other hand, the oscillation will be hindered by the friction damping caused by the viscoelastic solid wall. While the bubble in an elastic soft medium can behave like a high quality factor oscillator,[10] the increase of viscosity factors will definitely reduce the quality factor that is defined as $Q = \omega/\upsilon$[20] and then the strength of the resonance response of bubble to the incident wave. This prevents the bubbles from becoming effective acoustic scatterers, which is crucial for the localisation to take place.[6] As a result, it is perceivable that the increase of the viscosity effects diminishes the extent to which all bubbles pulsate out of phase with the source, and a complete prohibition of acoustic wave could not be attained.

Figure 4 displays the fluctuations of the oscillation phases of bubbles $\delta\theta$ as a function of the normalised frequency $\omega r_0/c_l$ in a bubbly gelatin for four particular values of viscosity factors: $\mu_\nu = 0$ P, 5 P, 50 P, and 500P. Note also that the fluctuations of the phases approach zero in the zero frequency limit due to the negligibility of the scattering effect of bubbles.

The phenomena of phase transitions can be clearly observed characterised by significant reduction of the fluctuations within particular ranges of frequencies whose locations are in good agreement with the corresponding frequency regions where the localisation occurs. This is consistent with the previous results obtained for bubbly water.[11] Moreover, it is apparent that the amounts by which the fluctuations $\delta\theta$ decrease can act as reflections of the extents of the acoustic localisations. In a bubbly viscoelastic soft medium, such a phenomenon of phase transition persists within the localisation region, while the increase of the value of viscosity factor leads to a weaker reduction of the fluctuation of phases. In the particular case where the viscosity effects are extremely strong, i.e., $\mu_\nu = 500$ P, the localisation is absent due to the fact that the effects of multiple scattering and the bubble resonance are severely destroyed, and the phenomenon of phase transition could not be identified. The comparison of Figs. 1–4

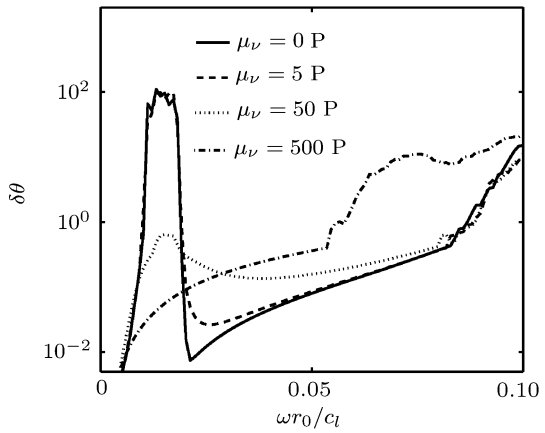


Fig. 4. The comparison between the fluctuations of the oscillation phases of bubbles and the frequency in a bubbly gelatin with different values of viscosity factors.

proved that the phenomenon of phase transition is a valid criterion of the existence of acoustic localisation in such a medium, and the values of the oscillation phases of the bubbles help to determine the extent to which the acoustic waves are localised. Consequently it is reasonable to conclude that the proper analysis of the oscillation phases of bubbles can indeed act as an efficient approach to identify the phenomenon of acoustic localisation in the actual samples of bubbly soft media in which the viscosity effects are generally nontrivial. The important phenomenon of phase transition is an effective criterion to determine the existence of localisation, while the extent to which the acoustic wave is localised may be estimated by inspecting the values of the oscillation phases or the reduction amount of the phase fluctuation.

4. Conclusion

In the present paper, the propagation of longitudinal acoustic waves in a bubbly soft medium has been studied in a numerical manner by inspecting the oscillation phases of the bubble. Numerical simulations are performed to investigate the behaviours of the oscillation phases of the bubbles as well as the spatial distributions of acoustic energy, to which the viscosity effects are manually adjusted. The proper analysis of the oscillation phases of bubbles is proved to be a valid approach to identify the existence of acoustic localisation in such a medium, which reveals the existence of the significant phenomenon of phase transition characterized by an unusual collective behaviour of the phases. As manifested by the results, the phenomenon of phase transition can serve as an efficient criterion to determine whether the localisation occurs, while the values of the oscillation phases and the amount of reduction of the phase fluctuations help to examine the extent to which the acoustic wave is localised.

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