Generation of a four-particle entangled state via cross-Kerr nonlinearity*

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We propose a scheme for generating a genuine four-particle polarisation entangled state $|\chi^{00}\rangle$ that has many interesting entanglement properties and potential applications in quantum information processing. In our scheme, we use the weak cross-Kerr nonlinear interaction between field-modes and the non-demolition measurement method based on highly efficient homodyne detection, which is feasible under the current experiment conditions.

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It is well known that entanglement is one of the most distinct features of quantum mechanics, which plays an important role in quantum communication and quantum computation.[1−3] There are many typical entangled states, including Bell states, W states, Greenberger–Horne–Zeilinger (GHZ) states, and cluster states. All these entangled states are widely used in quantum teleportation,[4] quantum dense coding,[5,6] quantum cloning,[7,8] and quantum computation.[9] Therefore much effort has been made to generate entangled states in various physical systems over the past few years, such as entanglement between atoms,[10] spontaneous parametric down converter,[11] high-Q cavity,[12,13] ion trap,[14] superconducting quantum-interference device,[15,16] and the other systems.[17−19]

Among quantum entangled states, the bipartite entangled states are the simplest ones which have been well understood in many aspects. Interest is now focused on the multiparticle entangled states. Compared with bipartite entangled states, the multipartite entangled states are more stable and have a longer decoherence time,[20] therefore they seem more attractive as entanglement resources for quantum information processing. Recently, Yeo and Chua proposed a genuine four-particle entangled state,[21] which is given by

$$|\chi^{00}\rangle = \frac{1}{2\sqrt{2}}(|0000\rangle_{1234} - |0011\rangle_{1234} - |0101\rangle_{1234} + |0110\rangle_{1234} + |1001\rangle_{1234} + |1010\rangle_{1234} + |1100\rangle_{1234} + |1111\rangle_{1234}). \quad (1)$$

Different from the four-particle W state, GHZ state, cluster state and the product state of two pairs of Bell state, the genuine entangle state $|\chi^{00}\rangle$ has many interesting entanglement properties. It has the maximum entanglement between particles (1,2) and (3,4), and between (1,3) and (2,4), which can be used to implement the quantum teleportation of an arbitrary two-qubit state, the quantum state sharing and quantum dense coding.[21]

In the present paper, we propose a scheme to prepare the above four-particle genuine entangled state $|\chi^{00}\rangle$ by using the weak cross-Kerr nonlinear interaction and the non-demolition measurements, in which the polarisations of the single-photons serve as qubit states and the bright coherent states act as quantum bus. The weak cross-Kerr nonlinear interaction is used to conditionally generate a phase shift in the bright coherent state. By making a highly efficient homodyne detection on the auxiliary coherent state, different photonic qubits can be collapsed into the quantum entanglement. Our scheme has the following advantages. (i) The scheme involves only the cross-Kerr nonlinear interaction between coherent state and single-photon

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state. It is not necessary that the cross-Kerr nonlinearity is very large, as long as the coherent light is bright enough. (ii) It does not rely on the two-photon effect and therefore is unnecessary to require identical photonic qubits. (iii) Our approach takes advantage of the high efficiency of the homodyne detection, which can be made much more efficient than the single-photon detection in which linear optical elements are used. (iv) Compared with other systems for entanglement engineering,[17−19] our scheme is all optical, which can be extended easily to long-distance distributed quantum information processing without the need of qubit interconversion. Through a similar interaction mechanism, several schemes have also been proposed recently to generate the GHZ and W states of photons.[18,19,22−24]

First, we briefly review the cross-Kerr nonlinear interaction between a mode a and a mode c. The interaction Hamiltonian has the form[25,26]

$$H = \hbar\chi\hat{a}^{+}\hat{a}\hat{c}^{+}\hat{c}, \tag{2}$$

where χ is proportional to the third-order nonlinear susceptibility $\chi^{(3)}$ of the medium, $\hat{a}^{+}\,(\hat{c}^{+})$ and $\hat{a}\,(\hat{c})$ are the creation operator and the annihilation operator for mode $a\,(c)$, respectively. The time-evolution operator is

$$U\,(t) = \mathrm{e}^{-\,\mathrm{i}\,\theta\hat{a}^{+}\hat{a}\hat{c}^{+}\hat{c}}, \tag{3}$$

where $\theta = \chi t$, which can be named the scaled interaction time or the nonlinear phase shift. As shown in Fig. 1, by choosing $\chi t = \pi$, the action of the cross-Kerr unitary evolution operator on the coherent-state input $|\alpha\rangle$ is given by

$$U\,(t)\,|n_a\rangle\,|\alpha_c\rangle = |n_a\rangle\,\left|\alpha_c \mathrm{e}^{-\,\mathrm{i}\,n_a\theta}\right\rangle. \tag{4}$$

Based on a general homodyne detection (HD)[27,28] on the phase of the coherent-state, the Fock state will be projected into a definite number state. Because the measurement can be performed with a high efficiency, this interaction process is nearly deterministic.

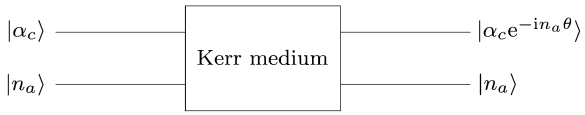


Fig. 1. Fock state and coherent-state interaction in a cross-Kerr medium.

Secondly, we will show how to generate a genuine four-photon entangled state $\left|\chi^{00}\right\rangle$ by using the cross-Kerr nonlinearity. The schematic representation of the scheme for generating such a four-photon entangled state is shown in Fig. 2.

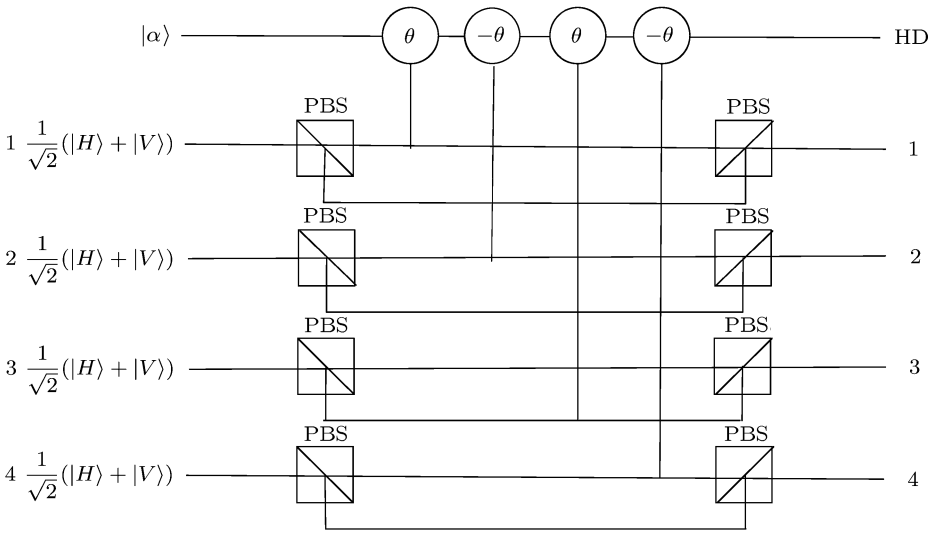


Fig. 2. Schematic setup for the generation of a genuine four-photon entangled state.

In the schematic setup, PBSs are the polarising beam splitters oriented in the HV basis, and they will always transmit H-polarised photons and reflect V-polarised photons, HD is a general homodyne detection. Now, we explain the scheme of Fig. 2 in detail. Here we use the polarisation of photons as qubit and define horizontal (vertical) polarisation $|H\rangle\,(|V\rangle)$ as qubit $|0\rangle\,(|1\rangle)$. Assume that the input state is

$$|\psi\rangle_{\rm in} = \frac{1}{\sqrt{2}}(|H_1\rangle + |V_1\rangle) \otimes \frac{1}{\sqrt{2}}(|H_2\rangle + |V_2\rangle) \otimes \frac{1}{\sqrt{2}}(|H_3\rangle + |V_3\rangle) \otimes \frac{1}{\sqrt{2}}(|H_4\rangle + |V_4\rangle) \otimes |\alpha\rangle . \quad (5)$$

After the first set of polarisation beam splitters (PBSs), for the output photons from the four input modes, the interaction mechanism in the first step enables them to evolve as

$$\begin{aligned}
&U|H_1\rangle|\alpha\rangle = |H_1\rangle\left|\alpha e^{i\theta}\right\rangle, \quad U|V_1\rangle|\alpha\rangle = |V_1\rangle|\alpha\rangle,\\
&U|H_2\rangle|\alpha\rangle = |H_2\rangle\left|\alpha e^{-i\theta}\right\rangle, \quad U|V_2\rangle|\alpha\rangle = |V_2\rangle|\alpha\rangle,\\
&U|H_3\rangle|\alpha\rangle = |H_3\rangle|\alpha\rangle, \quad U|V_3\rangle|\alpha\rangle = |V_3\rangle\left|\alpha e^{i\theta}\right\rangle,\\
&U|H_4\rangle|\alpha\rangle = |H_4\rangle\left|\alpha e^{-i\theta}\right\rangle, \quad U|V_4\rangle|\alpha\rangle = |V_4\rangle|\alpha\rangle . \quad (6)
\end{aligned}$$

Then after the cross-Kerr interaction and the second set of PBSs, we have

$$\begin{aligned}
|\psi\rangle_{\rm out} = \frac{1}{4}[&(|HHHH\rangle + |VHHV\rangle + |VHVH\rangle + |VVHH\rangle)_{1234}\left|\alpha e^{-i\theta}\right\rangle\\
&+(|HHVV\rangle + |HVHV\rangle + |HVVH\rangle + |VVVV\rangle)_{1234}\left|\alpha e^{i\theta}\right\rangle\\
&+(|HHHV\rangle + |HHVH\rangle + |HVHH\rangle + |VHVV\rangle + |VVHV\rangle + |VVVH\rangle)_{1234}|\alpha\rangle\\
&+|HVVV\rangle_{1234}\left|\alpha e^{i2\theta}\right\rangle + |VHHH\rangle_{1234}\left|\alpha e^{-i2\theta}\right\rangle]. \quad (7)
\end{aligned}$$

The following step is to perform a homodyne measurement on the quadrature of the auxiliary coherent state. Suppose that the homodyne measurement is performed on the quadrature $\hat{X} = (\hat{a} + \hat{a}^+)/2$. So the measurement cannot distinguish the probe beam $\left|\alpha e^{i\theta}\right\rangle$ and $\left|\alpha e^{-i\theta}\right\rangle$ but can distinguish them from $|\alpha\rangle$ and $\left|\alpha e^{\pm 2\theta}\right\rangle$.[29] Via a feed-forward process, we could make simple local rotations by using phase shifters to obtain the final state

$$\begin{aligned}
|\psi'_{\rm o}\rangle = \frac{1}{4}(&|HHHH\rangle + |HHVV\rangle + |HVHV\rangle + |HVVH\rangle\\
&+|VHHV\rangle + |VHVH\rangle + |VVHH\rangle + |VVVV\rangle)_{1234}. \quad (8)
\end{aligned}$$

From Eq. (7), if the measurement is performed to distinguish the probe beam $\left|\alpha e^{\pm 2\theta}\right\rangle$ from $|\alpha\rangle$ and $\left|\alpha e^{\pm i\theta}\right\rangle$, the state will be transformed into

$$|\psi''_{\rm o}\rangle = \frac{1}{4}(|HVVV\rangle + |VHHH\rangle)_{1234} . \quad (9)$$

A local operation is required to be applied to mode 1, and the state $|\psi''_{\rm o}\rangle$ will be transformed into the four-particle GHZ state.

In order to obtain the genuine four-photon entangled stat $\left|\chi^{00}\right\rangle$, the single-photon in mode 1 and mode 4 pass through the device shown in Fig. 3. A single-photon controlled-Z (CZ) gate will be used in the device, which can be constructed via the same cross-Kerr interaction mechanism.[30,31]

The single-photon CZ gate will have the following transformations:

$$\begin{aligned}
&|V,H\rangle \to |V,H\rangle,\\
&|V,V\rangle \to |V,V\rangle,\\
&|H,H\rangle \to |H,H\rangle,\\
&|H,V\rangle \to -|H,V\rangle . \quad (10)
\end{aligned}$$

Only when mode 1 is in a single-photon state with horizontal polarisation and mode 4 is in a single-photon state with vertical polarisation, will the strength of the Kerr nonlinearity be $\theta = \pi$ for the implementation of a CZ gate[30,31] and the two-photon state acquire a π global phase shift. After the device in Fig. 3, the state $|\psi'_{\rm o}\rangle$ will be transformed into

$$\begin{aligned}
|\psi_{\rm o}\rangle = \frac{1}{2\sqrt{2}}(&|HHHH\rangle - |HHVV\rangle\\
&-|HVHV\rangle + |HVVH\rangle + |VHHV\rangle\\
&+|VHVH\rangle + |VVHH\rangle + |VVVV\rangle)_{1234}, \quad (11)
\end{aligned}$$

which is just the genuine four-particle entangled state[21] to be generated.

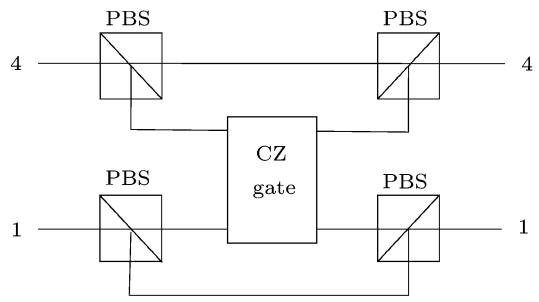


Fig. 3. A single-photon CZ gate for mode 1 and mode 4.

Now we will give a brief analysis on the performance of our scheme. The main imperfection arises from the homodyne detection. Due to the fact that the phase-shifted coherent states are not completely orthogonal to $|\alpha\rangle$, there is an error in the discrimination. The total error probability is $P_{\rm err} = {\rm erfc}[|\alpha|\sin\theta/\sqrt{2}]$ which can be made very small if $\alpha\theta >> 1$.[32] In fact for $\alpha\theta \sim \pi$, the corresponding discrimination error $P_{\rm err} \sim 10^{-3}$. Choosing the mean photon number per pulse to be on the order of 10^{12} (corresponding to $\alpha \sim 10^6$) in the realistic pumps, a weak nonlinearity $\theta \sim 3.14 \times 10^{-6}$ should be sufficient to satisfy $\alpha\theta \sim \pi$. So the proposed scheme here can work well without the requirement for strong nonlinearity.

In summary, we have proposed a scheme for generating a genuine four-particle entangled state. Our scheme involves weak cross-Kerr interaction, polarising beam splitters, a single-photon CZ gate and homodyne detection, and is therefore compatible with current experiment technology. Due to the high efficiency of the homodyne measurement, the success probability is higher than that with using linear optical elements for the engineering of entanglement. We anticipate that the prepared genuine four-particle entanglement with rich entanglement structure has more possible applications in quantum information processing in the future. In addition, our scheme can be easily generalized to the case where more than four particles are involved.

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