Entanglement transfer via the Raman atom–cavity-field interaction

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For the Raman interaction between an atom and a two-mode cavity field prepared in the state $|01\rangle$ or $|10\rangle$, the atom and the field can be disentangled periodically. Such a property of Raman atom–field interaction allows the full entanglement transfer among many atoms and bimodal cavities. In the calculations, each atom is assumed to interact with its own cavity at a different time and so non-identical atoms can be treated conveniently. Entanglement sudden death is discussed too. Though atom–field interaction greatly changes the values of the concurrence for two atoms, configuration of the concurrence is almost not affected. When there is entanglement sudden death, atoms and cavities can still be entangled with one another. However, full entanglement transfer cannot be achieved for such systems with Raman atom–field interaction.

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1. Introduction

Quantum entanglement displays nonlocal correlations between different quantum systems. In quantum information processing such as quantum communication[1−3] and quantum computation,[4−6] entanglement plays the key role. Often, quantum information needs to be transferred between different systems. Therefore, there are many investigations on entanglement transfer from light to qubits[7−18] and from atoms to fields.[11,19] The linear entropy[20,21] and the von Neumann entropy[22−24] are usually used to measure the entanglement in pure states. For two qubits in mixed states, entanglement can be measured by the negativity of the partial transposition of the reduced density matrix[25,26] or the concurrence,[27−29] which is defined as $C = \max\left[0, 2\max\left[\lambda_i\right] - \sum_i^4 \lambda_i\right]$, where λ_i are the square roots of the eigenvalues of the matrix $\boldsymbol{R} = \boldsymbol{\rho S \rho^* S}$ with $\boldsymbol{\rho}$ being the density matrix of the system and $\boldsymbol{S} = \boldsymbol{\sigma}_{1y} \otimes \boldsymbol{\sigma}_{2y}$ ($\boldsymbol{\sigma}_{1y}$ and $\boldsymbol{\sigma}_{2y}$ are Pauli matrices). In the studies of entanglement transfer, atoms are assumed to interact with cavity fields simultaneously. Actually, such a requirement is unnecessary. Within a quantum network, entanglement may evolve among atoms and cavity fields freely. So, we discuss the case that each atom interacts with its own cavity field at a different time.

Recently, the phenomenon of entanglement sudden death[30−34] has become another hot topic. If two atoms, say atoms 1 and 2, are initially entangled, atom–field interactions in lossless cavities can cause the atom–atom entanglement to fall abruptly to zero and remain zero for a period of time before entanglement recovers.[30,31] However, in these cases, the atom–atom entanglement is calculated after tracing over the field variables. So, zero of atom–atom entanglement does not mean that states of atoms 1 and 2 have nothing to do with each other. Actually, atom 1, atom 2 and the field are correlated with one another. To reflect the entanglement properties of the whole system, we further study the atom–field entanglement and the entanglement of each atom with the rest part of the system.

Light fields in maximally entangled number states can maximally entangle atoms initially prepared in ground states.[16] For other initial states of atoms, there is no such result. Intensity-dependent coupling may improve the entanglement transfer.[11] But, such a model of atom–field interaction needs to be tested experimentally. In this paper, we discuss entanglement transfer and entanglement sudden death in systems of atoms and cavities by using the Raman atom–field interaction,[35−37] which turns out to be able to allow full entanglement transfer among atoms and cavities. In the next section, we describe the model we use. The third section is the entanglement transfer among atoms and cavity fields. The phenomenon of

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entanglement sudden death[30−33] is discussed in the fourth section. The final section is a summary.

2. The model

We consider the system with many two-level atoms and high-Q cavities prepared in the zero- and one-photon state $|01\rangle$ or $|10\rangle$. Initially, the atoms and the cavity fields do not interact. Then, each atom is injected into one cavity and interacts with the field independently. Adjusting the velocity of the atom, the time that the atom stays in the cavity or the corresponding atom–field interaction time can be controlled. In the interaction picture, the on-resonant interaction Hamiltonian for the Raman coupled model[35−37] reads

$$H_{\rm int}^{(j)} = \lambda_j (a_j b_j^+ |e_j\rangle \langle g_j| + |g_j\rangle \langle e_j| a_j^+ b_j), \tag{1}$$

where a_j^+, b_j^+ and a_j, b_j are the creation and annihilation operators of the two-mode fields in the j-th cavity, respectively λ_j is the atom–field coupling constant. We have assumed that the j-th atom enters the j-th cavity. The evolution operator is found to be

$$\boldsymbol{U}_j(t) = \exp\left(-\mathrm{i}\,\boldsymbol{H}_{\rm int}^{(j)} t\right) = \begin{bmatrix} \cos(A_j t), & -\mathrm{i}\,\dfrac{\sin(A_j t)}{A_j}\lambda_j a_j b_j^+ \\ -\mathrm{i}\,\lambda_j a_j^+ b_j \dfrac{\sin(A_j t)}{A_j}, & \cos(B_j t) \end{bmatrix}, \tag{2}$$

where $A_j = \lambda_j \sqrt{a_j a_j^+ b_j^+ b_j}$ and $B_j = \lambda_j \sqrt{a_j^+ a_j b_j b_j^+}$. By some calculations, we have the following results

$$\boldsymbol{U}_j(t_j)\,|g_j\rangle\,|01\rangle_j = |g_j\rangle\,|01\rangle_j\,,\ \boldsymbol{U}_j(t_j)\,|e_j\rangle\,|10\rangle_j = |e_j\rangle\,|10\rangle_j\,, \tag{3}$$

$$\boldsymbol{U}_j(t_j)\,|g_j\rangle\,|10\rangle_j = |g_j\rangle\,|10\rangle_j \cos(\lambda_j t_j) - \mathrm{i}\,|e_j\rangle\,|01\rangle_j \sin(\lambda_j t_j), \tag{4}$$

$$\boldsymbol{U}_j(t_j)\,|e_j\rangle\,|01\rangle_j = |e_j\rangle\,|01\rangle_j \cos(\lambda_j t_j) - \mathrm{i}\,|g_j\rangle\,|10\rangle_j \sin(\lambda_j t_j), \tag{5}$$

where t_j is the time of the j-th atom interacting with the j-th cavity field. For the benefit of following discussions, we define the scaled time $T_j = \lambda_j t_j$. For the scaled time, the period is 2π. It is easy to see that at the times $T_j = \pi/2$, π, $3\pi/2$, and 2π, the atom and the field are disentangled. Such properties allow full entanglement transfer among atoms and cavities, which is displayed in the following section.

3. Entanglement transfer among atoms and cavities

Here, we first consider an example that there are two atoms (named atoms 1 and 2) and one cavity (named cavity 1). The two atoms are initially in the entangled state

$$|\psi_{12}\rangle = \cos\alpha\,|g_1 g_2\rangle + \sin\alpha\,|e_1 e_2\rangle\,. \tag{6}$$

The concurrence of this state is $C_{12} = 2\,|\cos\alpha \sin\alpha|$. Assuming that atom 1 enters cavity 1 prepared in the state $|01\rangle$, the wave function of the whole system evolves into

$$\begin{aligned} |\psi(t_1)\rangle &= U_1(t_1)\,|\psi_{12}\rangle\,|01\rangle_1 \\ &= |g_1 g_2\rangle\,|01\rangle_1 \cos\alpha + |e_1 e_2\rangle\,|01\rangle_1 \sin\alpha \cos T_1 \\ &\quad - \mathrm{i}\,|g_1 e_2\rangle\,|10\rangle_1 \sin\alpha \sin T_1, \end{aligned} \tag{7}$$

where $T_1 = \lambda_1 t_1$ is the scaled time. When $T_1 = \pi/2$, the state (7) becomes

$$|\psi(t_1)\rangle = |g_1\rangle\,(\cos\alpha\,|g_2\rangle\,|01\rangle_1 - \mathrm{i}\sin\alpha\,|10\rangle_1\,|e_2\rangle), \tag{8}$$

which means that atom 1 is disentangled from the subsystem formed by atom 2 and cavity 1, which has the concurrence $2\,|\cos\alpha \sin\alpha|$. The entanglement of atoms 1 and 2 is completely transferred to atom 2 and cavity 1. For the state (7), the reduced density matrix for the two atoms is

$$\rho_{12}(t_1) = \begin{bmatrix} \sin^2\alpha\cos^2 T_1, & 0, & 0, & \dfrac{1}{2}\sin(2\alpha)\cos T_1 \\ 0, & 0, & 0, & 0 \\ 0, & 0, & \sin^2\alpha \sin^2 T_1, & 0 \\ \dfrac{1}{2}\sin(2\alpha)\cos T_1, & 0, & 0, & \cos^2\alpha \end{bmatrix}, \tag{9}$$

where the matrix basis is chosen as $|e_1e_2\rangle$, $|e_1g_2\rangle$, $|g_1e_2\rangle$ and $|g_1g_2\rangle$. The concurrence is $C_{12}(T_1) = 2|\cos\alpha\sin\alpha\cos T_1|$. Within one period, this concurrence is zero only when $T_1 = \pi/2$, $3\pi/2$. So, there is no phenomenon of entanglement sudden death.[30,31]

If there is another cavity (say cavity 2) and atom 2 is injected into this cavity, the wave function of the two atoms and the two cavities will become

$$\begin{aligned}|\psi(t_1,t_2)\rangle &= \boldsymbol{U}(t_2)\,|\psi(t_1)\rangle\,|10\rangle_2\\ &= |g_1g_2\rangle\,|01\rangle_1\,|10\rangle_2\cos\alpha\cos T_2\\ &\quad -\mathrm{i}\,|g_1e_2\rangle\,|01\rangle_1\,|01\rangle_2\cos\alpha\sin T_2\\ &\quad +|e_1e_2\rangle\,|01\rangle_1\,|10\rangle_2\sin\alpha\cos T_1\\ &\quad -\mathrm{i}\,|g_1e_2\rangle\,|10\rangle_1\,|10\rangle_2\sin\alpha\sin T_1, \quad (10)\end{aligned}$$

where cavity 2 is supposed to be in the state $|10\rangle$ initially. At T_1, $T_2 = \pi/2$, $3\pi/2$, the two atoms and the two cavities are disentangled. The two cavities are now in the state

$$|01\rangle_1\,|01\rangle_2\cos\alpha \pm |10\rangle_1\,|10\rangle_2\sin\alpha. \quad (11)$$

The state of the two atoms is $|g_1e_2\rangle$, or one atom is in the ground state and the other in the excited state. The concurrence of state (11) is also $2|\cos\alpha\sin\alpha|$. That is to say, entanglement of the two atoms is fully transferred to the two cavities. Here, $\lambda_1 = \lambda_2$ and $T_1 = T_2$ are not needed. At T_1, $T_2 = \pi$, 2π, the two atoms and the two cavities are also disentangled. The state of the two atoms is

$$\cos\alpha\,|g_1g_2\rangle \pm \sin\alpha\,|e_1e_2\rangle\,, \quad (12)$$

which has the same degree of entanglement as that of the cavity state (11). In other words, if treating the cavity state (11) and the atomic state $|g_1e_2\rangle$ as the initial states, entanglement of the field can be fully transferred to the atoms. When the state of the two atoms is $|e_1e_2\rangle$ or $|g_1g_2\rangle$, full entanglement transfer can be achieved too. From the atomic state (6) to the cavity state (11) and then to the atomic state (12), full entanglement transfer happens among the atoms and the cavities. The one-mode atom–field interaction cannot induce the complete entanglement transfer when the two atoms are not both in the ground states.[16]

Next, we consider the case that atoms are initially in mixed states. To be specific, we suppose that there are three atoms and three cavities. The evolution operator is $\boldsymbol{U}(t_1,t_2,t_3) = \boldsymbol{U}(t_1)\boldsymbol{U}(t_2)\boldsymbol{U}(t_3)$. We assume that initially each cavity is in the state $|10\rangle$ and the three atoms are in a general state represented by the density matrix

$$\boldsymbol{\rho}_a(0) = \sum_{\mu,\nu=1}^{8}\rho_{\mu\,\nu}\,|\phi_{\mu\,a}\rangle\,\langle\phi_{\nu\,a}|\,, \quad (13)$$

where $|\phi_{1a}\rangle = |e_1e_2e_3\rangle$, $|\phi_{2a}\rangle = |e_1e_2g_3\rangle$, $|\phi_{3a}\rangle = |e_1g_2e_3\rangle$, $|\phi_{4a}\rangle = |g_1e_2e_3\rangle$, $|\phi_{5a}\rangle = |e_1g_2g_3\rangle$, $|\phi_{6a}\rangle = |g_1e_2g_3\rangle$, $|\phi_{7a}\rangle = |g_1g_2e_3\rangle$, $|\phi_{8a}\rangle = |g_1g_2g_3\rangle$ form a basis for an eight-dimensional space. The matrix elements $\rho_{\mu\,\nu}$ are all constants. For a pure state, the density operator for the three atoms has the form $\rho_a(0) = |\psi_a(0)\rangle\,\langle\psi_a(0)|$, which is a special case of the mixed state (13). Two typical pure states are the Greenberger–Horne–Zeilinger (GHZ) state $(1/\sqrt{2})(|e_1e_2e_3\rangle + |g_1g_2g_3\rangle)$ and the W state $(1/\sqrt{3})(|e_1e_2g_3\rangle + |e_1g_2e_3\rangle + |g_1e_2e_3\rangle)$. The initial state of the whole system is described by the density operator

$$\boldsymbol{\rho}(0) = \boldsymbol{\rho}_a(0)\rho_f(0),\ \rho_f(0) = \prod_{j=1}^{3}|10\rangle_j\,\langle 10|_j\,. \quad (14)$$

Choosing $t_1 = t_2 = 0$ means that atoms 1 and 2 are not injected into the corresponding cavities and only atom 3 interacts with cavity 3. For $T_3 = \pi/2$, the density matrix of the whole system becomes

$$\begin{aligned}\boldsymbol{\rho}(0,0,\pi/2) &= \boldsymbol{U}(0,0,\pi/2)\boldsymbol{\rho}(0)\boldsymbol{U}^{+}(0,0,\pi/2)\\ &= \boldsymbol{\rho}_{a\,f}(0,0,\pi/2)\\ &\quad\times\left(\prod_{j=1}^{2}|10\rangle_j\,\langle 10|_j\right)|e_3\rangle\,\langle e_3|\,, \quad (15)\end{aligned}$$

where

$$\boldsymbol{\rho}_{a\,f}(0,0,\pi/2) = \sum_{\mu,\,\nu=1}^{8}\rho_{\mu\nu}\,|\phi_{\mu\,a\,f},\pi/2\rangle\,\langle\phi_{\nu\,a\,f},\pi/2|\,. \quad (16)$$

The functions $|\phi_{\mu\,a\,f},\pi/2\rangle$ are derived from $|\phi_{\mu\,a}\rangle$ by making the following transformations: $|e_3\rangle \to |10\rangle_3$ and $|g_3\rangle \to -\mathrm{i}\,|01\rangle_3$. In matrix form, the density operator $\boldsymbol{\rho}_{a\,f}(0,0,\pi/2)$ in Eq. (16) and the initial atomic density operator $\rho_a(0)$ in Eq. (13) have the same matrix elements $\rho_{\mu\,\nu}$ and thus have the same entanglement, as the entanglement is determined by the density matrix for a multipartite system.[20,38] So, entanglement of the three atoms is fully transferred to atoms 1, 2, and cavity 3.

After atom 3 exits cavity 3, we inject atom 2 into cavity 2. If choosing the interaction time $T_2 = 3\pi/2$, the state of the whole system will further become

$$\boldsymbol{\rho}(0,3\pi/2,\pi/2) = \boldsymbol{U}(0,3\pi/2,0)\boldsymbol{\rho}(0,0,\pi/2)\boldsymbol{U}^{+}(0,3\pi/2,0) = \boldsymbol{\rho}_{af}(0,3\pi/2,\pi/2)\Big(\prod_{j=2}^{3}|e_j\rangle\langle e_j|\Big)(|10\rangle_1\langle 10|_1). \quad (17)$$

where

$$\boldsymbol{\rho}_{af}(0,3\pi/2,\pi/2) = \sum_{\mu,\nu=1}^{8}\rho_{\mu\nu}\,|\phi_{\mu af},3\pi/2,\pi/2\rangle\langle\phi_{\nu af},3\pi/2,\pi/2|\,. \quad (18)$$

The functions $|\phi_{\mu af},3\pi/2,\pi/2\rangle$ are obtained from $|\phi_{\mu af},\pi/2\rangle$ by carrying out the transformations: $|e_2\rangle\rightarrow|10\rangle_2$ and $|g_2\rangle\rightarrow i\,|01\rangle_2$. Now, the entanglement is fully transferred to atom 1 and cavities 2 and 3.

Further, injecting atom 1 into cavity 1 after atom 2 is out of cavity 2 and choosing the interaction time to be $T_1 = 3\pi/2$, the state of the whole system is then

$$\boldsymbol{\rho}(3\pi/2,3\pi/2,\pi/2) = \boldsymbol{U}(3\pi/2,0,0)\boldsymbol{\rho}(0,0,\pi/2)\boldsymbol{U}^{+}(3\pi/2,0,0) = \boldsymbol{\rho}_{af}(3\pi/2,3\pi/2,\pi/2)\Big(\prod_{j=1}^{3}|e_j\rangle\langle e_j|\Big), \quad (19)$$

where

$$\boldsymbol{\rho}_{af}(3\pi/2,3\pi/2,\pi/2) = \sum_{\mu,\nu=1}^{8}\rho_{\mu\nu}\,|\phi_{\mu af},3\pi/2,3\pi/2,\pi/2\rangle\langle\phi_{\nu af},3\pi/2,3\pi/2,\pi/2|\,. \quad (20)$$

The functions $|\phi_{\mu af},3\pi/2,3\pi/2,\pi/2\rangle$ are obtained from $|\phi_{\mu af},3\pi/2,\pi/2\rangle$ by the substitutions: $|e_1\rangle\rightarrow|10\rangle_1$ and $|g_1\rangle\rightarrow \mathrm{i}\,|01\rangle_1$. Entanglement of the initial atoms is completely transferred to the cavities.

If injecting the three atoms simultaneously into the corresponding cavities, the states (16), (18) and (20) will correspond to choosing the interaction times (a) $T_1 = 0,\ 2\pi$, $T_2 = 0,\ 2\pi$, $T_3 = \pi/2$; (b) $T_1 = 0,\ 2\pi$, $T_2 = 3\pi/2$, $T_3 = \pi/2$; and (c) $T_1 = 3\pi/2$, $T_2 = 3\pi/2$, $T_3 = \pi/2$ respectively. If choosing $T_1 = 2k_1\pi$, $T_2 = 2k_2\pi$, $T_3 = 2k_3\pi$ (k_1, k_2, k_3 are all integers), the system will return to its initial state. So, through the Raman atom–cavity-field interaction, entanglement can be transferred among the atoms and cavities freely and completely. For more atoms and cavities, there are similar results.

4. Quantum states when entanglement sudden death happens

At first, we still focus our attention on the case that there are two atoms (named atoms 1 and 2) and one cavity (named cavity 1). The initial state of the cavity is assumed to be $|n+1,n\rangle_1$ (n is an integer) and the two atoms are initially in the maximally entangled state

$$|\psi_{12}\rangle = (|e_1g_2\rangle + |g_1e_2\rangle)/\sqrt{2}. \quad (21)$$

Injecting atom 1 into cavity 1, the wave function at any time is found to be

$$\begin{aligned}|\psi(T_1)\rangle = {} & x_1(T_1)\,|e_1g_2\rangle\,|n+1,n\rangle_1 \\ & +x_2(T_1)\,|g_1e_2\rangle\,|n+1,n\rangle_1 \\ & +x_3(T_1)\,|g_1g_2\rangle\,|n+2,n-1\rangle_1 \\ & +x_4(T_1)\,|e_1e_2\rangle\,|n,n+1\rangle_1\,, \end{aligned} \quad (22)$$

where

$$\begin{aligned} x_1(T_1) &= \frac{1}{\sqrt{2}}\cos(\sqrt{n(n+2)}T_1), \\ x_2(T_1) &= \frac{1}{\sqrt{2}}\cos((n+1)T_1), \\ x_3(T_1) &= -\frac{\mathrm{i}}{\sqrt{2}}\sin(\sqrt{n(n+2)}T_1), \\ x_4(T_1) &= -\frac{\mathrm{i}}{\sqrt{2}}\sin((n+1)T_1). \end{aligned} \quad (23)$$

Tracing over the field variables, the reduced density matrix for the two atoms is derived

$$\boldsymbol{\rho}_{\mathrm{A1A2}}(T_1) = \begin{bmatrix} |x_4(T_1)|^2 & 0 & 0 & 0 \\ 0 & |x_1(T_1)|^2 & x_1(T_1)x_2^*(T_1) & 0 \\ 0 & x_2(T_1)x_1^*(T_1) & |x_2(T_1)|^2 & 0 \\ 0 & 0 & 0 & |x_3(T_1)|^2 \end{bmatrix}. \quad (24)$$

The concurrence for state (24) is found to be[30]

$$C_{\rm A1A2} = 2\max\{0, |x_1(T_1)x_2(T_1)| - |x_3(T_1)x_4(T_1)|\} = \max\{0, \left|\cos((n+1)T_1)\cos(\sqrt{n(n+2)}T_1)\right| - \left|\sin((n+1)T_1)\sin(\sqrt{n(n+2)}T_1)\right|\}. \quad (25)$$

When $n = 0$, the concurrence reduces to $C_{\rm A1A2} = |\cos T_1|$, which is similar to the case in the above section. For $n = 1$, the concurrence versus the scaled time T_1 is plotted in Fig. 1. Clearly, entanglement sudden death appears.

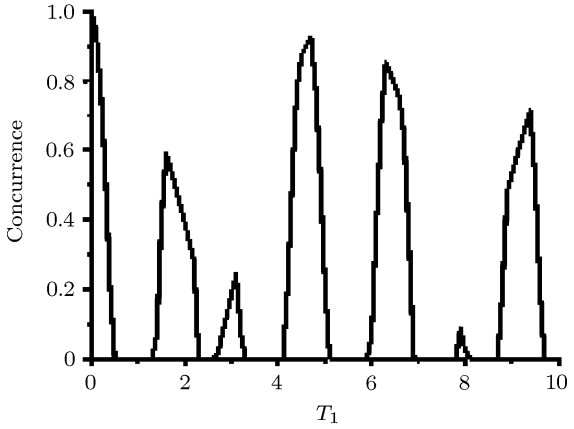


Fig. 1. Concurrence for atoms 1 and 2 versus the scaled time T_1.

However, that the concurrence is zero does not mean that states of atom 1 and atom 2 have nothing to do with each other, as the system described by the density matrix $\boldsymbol{\rho}_{\rm A1A2}(T_1)$ is not completely equivalent to that described by the wave function (22). For example, when $n = 1$ and $T_1 = \pi/4$, the matrix element $x_2(T_1) = 0$, which results in $C_{\rm A1A2} = 0$. Now, the wave function (22) has the form

$$|\psi(T_1)\rangle = \frac{1}{\sqrt{2}}\cos(\sqrt{3}\pi/4)\,|e_1g_2\rangle\,|n+1,n\rangle_1 - \frac{\rm i}{\sqrt{2}}\sin(\sqrt{3}\pi/4)\,|g_1g_2\rangle\,|n+2,n-1\rangle_1 - \frac{\rm i}{\sqrt{2}}\,|e_1e_2\rangle\,|n,n+1\rangle_1\,, \quad (26)$$

which means that atoms 1, 2, and the cavity field are entangled with one another. To reflect such properties of the original wave function (22), we should further study the entanglement between each part and the rest of the system.

Next, we use the von Neumann entropy[22−24] to measure the entanglement between the two atoms and the cavity field, which can be derived from the reduced density matrix (24)

$$S_{\rm A1A2}(T_1) = -{\rm Tr}[\boldsymbol{\rho}_{\rm A1A2}(T_1){\rm lb}\boldsymbol{\rho}_{\rm A1A2}(T_1)] = -\sum_{\mu=1}^{4}[\beta_\mu(T_1){\rm lb}\beta_\mu(T_1)], \quad (27)$$

where $\beta_\mu(T_1)$, $\mu = 1, 2, 3, 4$, are the roots of $\boldsymbol{\rho}_{\rm A1A2}(T_1)$

$$\beta_1(T_1) = |x_4(T_1)|^2\,, \quad \beta_2(T_1) = 0,$$
$$\beta_3(T_1) = |x_1(T_1)|^2 + |x_2(T_1)|^2\,,$$
$$\beta_4(T_1) = |x_3(T_1)|^2\,. \quad (28)$$

As $\beta_2(T_1) = 0$, the corresponding term in the sum in Eq. (27) is taken as zero. For a pure state containing two parts A and B, it is known that the von Neumann entropy for part A is equal to that for part B.[22−24] The von Neumann entropy for part A or part B measures the entanglement between these two parts. For the present system described by the wave function (22), if part A is chosen as the two atoms, part B will be cavity 1. Hence, the von Neumann entropy (27) for the two atoms is equal to the von Neumann entropy $S_{\rm C1}(T_1)$ for cavity 1.

The reduced density matrix for atom 2 is further calculated as

$$\boldsymbol{\rho}_{\rm A2}(T_1) = {\rm Tr}_{\rm A1}[\boldsymbol{\rho}_{\rm A1A2}(T_1)] = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}. \quad (29)$$

Entanglement of atom 2 with the subsystem formed by atom 1 and the cavity is measured by the von Neumann entropy $S_{\rm A2}(T_1) = -{\rm Tr}[\boldsymbol{\rho}_{\rm A2}(T_1){\rm lb}\boldsymbol{\rho}_{\rm A2}(T_1)] = 1$. That is to say, atom 2 is always maximally entangled with the subsystem formed by atom 1 and cavity 1. The reduced density matrix for atom 1 has the form

$$\boldsymbol{\rho}_{\rm A1}(T_1) = {\rm Tr}_{\rm A2}[\boldsymbol{\rho}_{\rm A1A2}(T_1)] = \begin{bmatrix}|x_2(T_1)|^2 + |x_4(T_1)|^2 & 0\\ 0 & |x_1(T_1)|^2 + |x_3(T_1)|^2\end{bmatrix}. \quad (30)$$

Entanglement of atom 1 with the subsystem formed by atom 2 and the cavity is measured by the von Neumann entropy

$$\begin{aligned} S_{\rm A1}(T_1) &= -{\rm Tr}[\boldsymbol{\rho}_{\rm A1}(T_1){\rm lb}\boldsymbol{\rho}_{\rm A1}(T_1)] \\ &= -[|x_2(T_1)|^2+|x_4(T_1)|^2]{\rm lb}[|x_2(T_1)|^2 \\ &\quad +|x_4(T_1)|^2]-[|x_1(T_1)|^2 \\ &\quad +|x_3(T_1)|^2]{\rm lb}[|x_1(T_1)|^2 \\ &\quad +|x_3(T_1)|^2]. \end{aligned} \tag{31}$$

The von Neumann entropies (27) and (31) are shown in Fig. 2. Besides the initial time, the entropies are nonzero all the time. Thus, each part of the system is always entangled with the rest of the system. In other words, atom 1, atom 2 and the cavity are inseparable forever. Now, the initial entanglement of atoms 1 and 2 cannot be completely transferred to atom 2 and cavity 1 even if the concurrence (25) is zero.

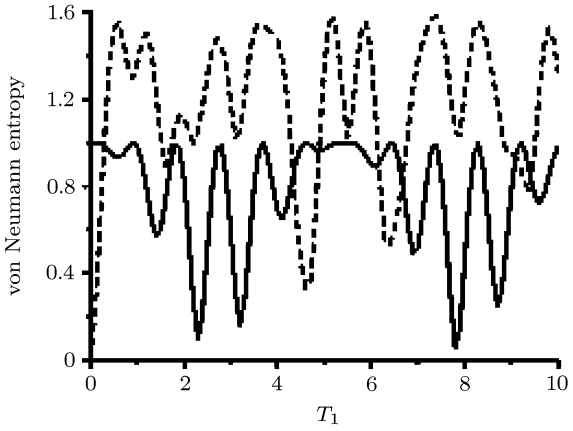


Fig. 2. von Neumann entropies versus the scaled time T_1. The solid and dashted curves correspond to $S_{\rm A1}(T_1)$ and $S_{\rm A1A2}(T_1)=S_{\rm C1}(T_1)$ respectively.

Now, we turn to the case that there is another cavity, say cavity 2 prepared in the state $|m+1,m\rangle_2$ (m is an integer), and atom 2 is injected into this cavity. The wave function (22) further evolves into

$$\begin{aligned} |\psi(T_1,T_2)\rangle &= x_1(T_1)\cos[(m+1)T_2]\,|e_1g_2\rangle\,|n+1,n\rangle_1\,|m+1,m\rangle_2 \\ &\quad -{\rm i}\,x_4(T_1)\sin[\sqrt{m(m+2)}T_2]\,|e_1g_2\rangle\,|n+1,n\rangle_1\,|m+2,m-1\rangle_2 \\ &\quad -{\rm i}\,x_3(T_1)\sin[(m+1)T_2]\,|g_1e_2\rangle\,|n+2,n-1\rangle_1\,|m,m+1\rangle_2 \\ &\quad +x_2(T_1)\cos[\sqrt{m(m+2)}T_2]\,|g_1e_2\rangle\,|n+1,n\rangle_1\,|m+1,m\rangle_2 \\ &\quad +x_3(T_1)\cos[(m+1)T_2]\,|g_1g_2\rangle\,|n+2,n-1\rangle_1\,|m+1,m\rangle_2 \\ &\quad -{\rm i}\,x_2(T_1)\sin[\sqrt{m(m+2)}T_2]\,|g_1g_2\rangle\,|n+1,n\rangle_1\,|m+2,m-1\rangle_2 \\ &\quad -{\rm i}\,x_1(T_1)\sin[(m+1)T_2]\,|e_1e_2\rangle\,|n+1,n\rangle_1\,|m,m+1\rangle_2 \\ &\quad +x_4(T_1)\cos[\sqrt{m(m+2)}T_2]\,|e_1e_2\rangle\,|n,n+1\rangle_1\,|m+1,m\rangle_2\,. \end{aligned} \tag{32}$$

The reduced density matrix for the two atoms is derived as

$$\boldsymbol{\rho}_{\rm A1A2}(T_1,T_2)=\begin{bmatrix} z_1 & 0 & 0 & 0 \\ 0 & z_2 & z_{23} & 0 \\ 0 & z_{23}^* & z_3 & 0 \\ 0 & 0 & 0 & z_4 \end{bmatrix}, \tag{33}$$

where the matrix elements are

$$\begin{aligned} z_1 &= |x_4|^2\cos^2(\sqrt{m(m+2)}T_2)+|x_1|^2\sin^2((m+1)T_2), \\ z_2 &= |x_4|^2\sin^2(\sqrt{m(m+2)}T_2)+|x_1|^2\cos^2((m+1)T_2), \\ z_3 &= |x_2|^2\cos^2(\sqrt{m(m+2)}T_2)+|x_3|^2\sin^2((m+1)T_2), \\ z_4 &= |x_2|^2\sin^2(\sqrt{m(m+2)}T_2)+|x_3|^2\cos^2((m+1)T_2), \\ z_{23} &= x_1x_2^*\cos((m+1)T_2)\cos(\sqrt{m(m+2)}T_2). \end{aligned} \tag{34}$$

The square roots of the eigenvalues of the matrix $\boldsymbol{R}=\boldsymbol{\rho}_{\rm A1A2}(T_1,T_2)S\boldsymbol{\rho}_{\rm A1A2}(T_1,T_2)^*S$ are

$$\lambda_1 = |z_{23}| + \sqrt{z_2 z_3},\ \lambda_2 = ||z_{23}| - \sqrt{z_2 z_3}|\,,$$
$$\lambda_3 = \lambda_4 = \sqrt{z_1 z_4}. \quad (35)$$

The concurrence is significantly affected by the interaction of atom 2 with the cavity 2. When $(m+1)T_2 = (2k_2+1)\pi/2$, the matrix element z_{23} and so the concurrence are zero for arbitrary T_1. The density matrix (33) now describes a disentangled mixed system for the two atoms. For $T_2 = 0.5\pi$ and 2π, the concurrence is plotted in Fig. 3. Entanglement sudden death still happens. For different interaction times T_2, extreme values of the concurrence are different. But the configuration keeps basically the same. Change of m does not change the configuration either.

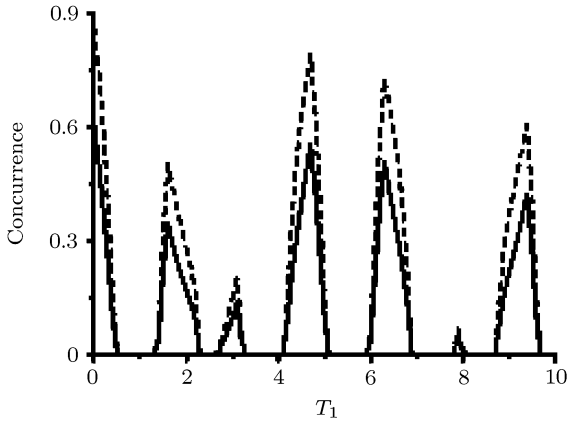


Fig. 3. Concurrence for atoms 1 and 2 versus the scaled time T_1 when $T_2 = 0.5\pi$ (solid line), 2π (dashed line).

The wave function still reveals the permanent correlation among the atoms and cavities. For example, for the parameters $m = n = 1$, $T_1 = 2T_2 = \pi/2$, the concurrence is zero and the wave function (32) takes the form

$$\begin{aligned}|\psi(T_1,T_2)\rangle \;=\; & -\frac{1}{\sqrt{2}}\sin[\sqrt{3}\pi/4]\,|g_1e_2\rangle\,|n+2,n-1\rangle_1\,|m,m+1\rangle_2 \\ & -\frac{1}{\sqrt{2}}\cos[\sqrt{3}\pi/4]\,|g_1e_2\rangle\,|n+1,n\rangle_1\,|m+1,m\rangle_2 + \frac{\mathrm{i}}{\sqrt{2}}\,|g_1g_2\rangle\,|n+1,n\rangle_1\,|m+2,m-1\rangle_2 \\ & -\frac{\mathrm{i}}{\sqrt{2}}\cos[\sqrt{3}\pi/4]\,|e_1e_2\rangle\,|n+1,n\rangle_1\,|m,m+1\rangle_2\,, \end{aligned} \quad (36)$$

which shows that the atoms and the cavities are entangled with one another. The von Neumann entropy for atom 1 is the same as that given by Eq. (31). Changing n and T_1 in Eq. (31) into m and T_2 respectively, we obtain the von Neumann entropy for atom 2. From the above discussions related to Fig. 2, we know that each atom is always entangled with the rest part of the system. Entanglement of atoms with the cavities is quantified by the entropy

$$\begin{aligned}S_{\mathrm{A1A2}}(T_1,T_2) &= -\mathrm{Tr}[\boldsymbol{\rho}_{\mathrm{A1A2}}(T_1,T_2)\mathrm{lb}\boldsymbol{\rho}_{\mathrm{A1A2}}(T_1,T_2)] \\ &= -\sum_{\mu=1}^{4}[\alpha_\mu(T_1,T_2)\mathrm{lb}\alpha_\mu(T_1,T_2)], \end{aligned} \quad (37)$$

where $\alpha_\mu(T_1)$, $(\mu = 1, 2, 3, 4)$ are the roots of $\boldsymbol{\rho}_{\mathrm{A1A2}}(T_1,T_2)$ given by Eq. (33)

$$\begin{aligned}&\alpha_1(T_1,T_2) = z_1(T_1,T_2),\\ &\alpha_2(T_1,T_2) = \frac{1}{2}(z_2+z_3) - \frac{1}{2}\sqrt{(z_2-z_3)^2+4z_{23}^2},\\ &\alpha_3(T_1,T_2) = \frac{1}{2}(z_2+z_3) - \frac{1}{2}\sqrt{(z_2-z_3)^2+4z_{23}^2},\\ &\alpha_4(T_1) = z_4(T_1,T_2). \end{aligned} \quad (38)$$

For $T_1 = \pi/2$, the entropy versus the interaction time T_2 is plotted in Fig. 4. Numerical calculations show that this entropy is nonzero except that at the initial time. That is to say, atoms and cavities are always entangled.

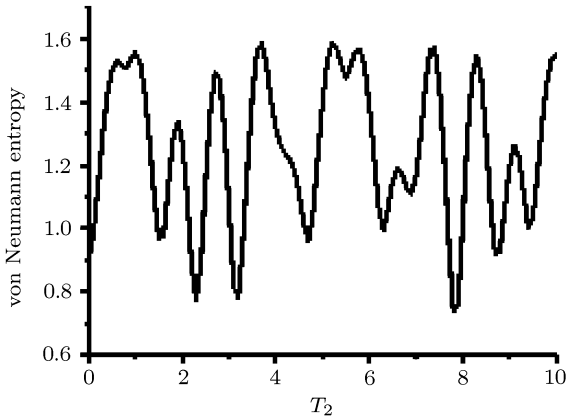


Fig. 4. von Neumann entropy $S_{\mathrm{A1A2}}(T_1,T_2)$ versus the scaled time T_2 for $T_1 = \pi/2$.

5. Summary

Transfer and sudden death of entanglement in systems of atoms and bimodal cavities are investigated. When the cavity is initially prepared in the state $|01\rangle$

or $|10\rangle$, the atom and the field can be disentangled periodically for Raman atom–cavity-field interaction. Such a property of atom–field interaction allows full entanglement to transfer among atoms and bimodal cavities, no matter whether the atoms are initially in excited or ground states. Each atom is assumed to interact with its own cavity at a different time, and atoms can be non-identical. When the cavity field is not prepared in the state $|01\rangle$ or $|10\rangle$, entanglement sudden death may occur. Atom–field interaction greatly changes the values of the concurrence for two atoms, but configuration of the concurrence is almost not affected. Entanglement of each atom with the rest part of the system and the atom–field entanglement are calculated too. These calculations show that atoms and cavities are correlated with one another even when there is entanglement sudden death, which is also true for many other systems such as that discussed in Refs. [30] and [31].

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