Robust generation of qutrit entanglement via adiabatic passage of dark states*

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(Received 9 December 2009; revised manuscript received 24 February 2010)

We propose a scheme for the deterministic generation of qutrit entanglement for two atoms trapped in an optical cavity. Taking advantage of the adiabatic passage, the operation is immune to atomic spontaneous emission as the atomic excited states are never populated; under certain conditions, the probability that the cavity is excited is negligible. We also study the influences of the dissipation due to the atomic spontaneous emission and cavity decay.

Keywords: qutrit entanglement, adiabatic passage, dark states, cavity quantum electrodynamics
PACC: 4250, 0365

1. Introduction

Entanglement, a fundamental feature of quantum mechanics, is a useful resource for many researches in quantum information processing.[1] In particular, certain entangled states have been identified and thus can be directly used for many such researches. Foremost among these are the so-called Bell states (EPR states),[2] Greenberger–Horne–Zeilinger states (GHZ states),[3] and W states.[4] These states are not only relevant to the study of foundations of quantum mechanics, but also the fundamental ingredient of various quantum information protocols.[1,5] As shown by Bell,[2] two particles in Bell states possess correlations that are incompatible with a local realistic theory,[6] no matter what the separation between them is. Greenberger, Horne and Zeilinger showed that three-particle GHZ states possess correlations that violate the local realistic theory much stronger than Bell states.[3] Besides the GHZ states, there is another inequivalent class of triparticle entangled states, i.e., the W states,[4] which is somewhere much more robust than the GHZ states, as it retains biparticle entanglement when any one of the three qubits is traced out.[4] The W states can also be used to test quantum nonlocality.[7] All these states are entangled states that are defined in Hilbert spaces with the dimension d, i.e., entangled qubits. As a general generalisation, entangled states defined in larger Hilber spaces, i.e., entangled qudits ($d \geq 3$), also deserve attention. It has been shown that violations of local realism are stronger for two maximally entangled qudits than for qubits, and the former is more resistant to noise.[8,9] Generation of qudit entanglement using photon pairs in optical systems has been demonstrated and used to verify quantum nonlocal realism.[10−12] Generic Bell inequalities for multipartite qudits have been studied.[13] It has been pointed out that, for quantum key distribution, the usage of qudits offers advantages such as an increased level of tolerance to noise at a given level of security and a higher bit transmission rate compared to the qubit quantum key distribution protocols,[14] and this has recently proved to be true by use of spatial degrees of freedom of photons based on the standard BB84 protocol.[15]

Cavity quantum electrodynamics (CQED), with high excited Rydberg atomic states interacting with superconducting millimeter-wave cavities or low-lying atomic levels interacting with optical cavities (referred to as microwave or optical CQED), offers one of the effective methods for generating entangled states (see Ref. [16] as a review). Based on the CQED technique, a variety of schemes have been proposed for the generation of entangled qubits.[17−27]

Experimental demonstration of some kinds of

*Project supported by the National Natural Science Foundation of China (Grant Nos. 10674025 and 10974028), the Fujian Natural Science Foundation of China (Grant No. 2009J06002), the Doctoral Foundation of the Ministry of Education of China (Grant Nos. 20070386002 and 20093514110009), the Funds from State Key Laboratory Breeding Base of Photocatalysis, Fuzhou University, and the Funds from the Education Department of Fujian Province of China (Grant No. JB08010).

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these states, including the Bell state of two atoms,[28,29] Einstein-Podolsky-Rosen (EPR) state of two cavity modes,[30] and GHZ states (composed of two atoms and one cavity mode),[31] has been reported. Schemes have also been proposed for the generation of different types of entangled qutrits of atoms,[32−37] or of cavity photons.[38,39]

All the above schemes work in the microwave regime.[17−39] In these schemes, the state of the system that evolves under the domination of a given Hamiltonian contains contribution from the excited atomic state, but the atomic spontaneous emission is not a major problem in this regime.[40] For instance, in the optical CQED experiments, coherent-destroying spontaneous emission is always a limiting factor, even though the strong coupling criterion of this regime is satisfied.[41] To avoid this type of detriment, the effective method is to make the state of the system evolve within a dark-state subspace, in which the relevant states contain no contribution from the excited atomic states. The type of the evolution is referred to as *adiabatic passage* of dark states, which has been previously studied in the context of coherent population transfer,[42,43] and coherent atomic-beam deflection,[44] also called the technique of stimulated Raman adiabatic passage (STIRAP) or fractional STIRAP (f-STIRAP).[45] The remarkable property of both techniques is that the atomic excited states only act as virtually-populated intermediate states, thus any possible decay from these states is negligible; moreover, the system parameters are not necessarily to be exactly adjusted, hence it is insensitive to small fluctuations of parameters. Both techniques have been extensively studied for CQED, and used for the preparation of kinds of quantum states, including arbitrary superposition of cavity field Fock states,[46] two-particle (atoms or cavity photons) EPR states,[47,48] W states,[49] etc.

In this paper, we propose a scheme for the deterministic generation of qutrit entanglement for two atoms trapped in an optical cavity. The scheme is based on the adiabatic passage of dark states, which makes the correlative detriment such as atomic spontaneous emission and small parameter errors insignificant.

The following of the paper is organised as follows. In Section 2, we introduce the models for STIRAP and fractional STIRAP (f-STIRAP). In Section 3, we discuss the deterministic generation of qutrit entanglement via STIRAP and f-STIRAP. In Section 4, we give a quantitive analysis on the influences of dissipation due to the atomic spontaneous emission and photon leakage. Section 5 contains the concluding remarks.

2. The model

As shown in Fig. 1(a), two atoms are positioned at different locations inside a high-finesse optical cavity. The distance between the two atoms should be large enough as to ensure individual addressing of each atom and negligible dipole–dipole interaction. The atom has three ground states $|0\rangle$, $|k\rangle$ and $|1\rangle$ and two excited states $|e\rangle$ and $|e'\rangle$ (Fig. 1(b)). The three ground states could be different hyperfine levels or in the same manifold but with different Zeeman sublevels. They can be coupled to the excited state $|e\rangle$ or $|e'\rangle$, through the driving of different light pulses.

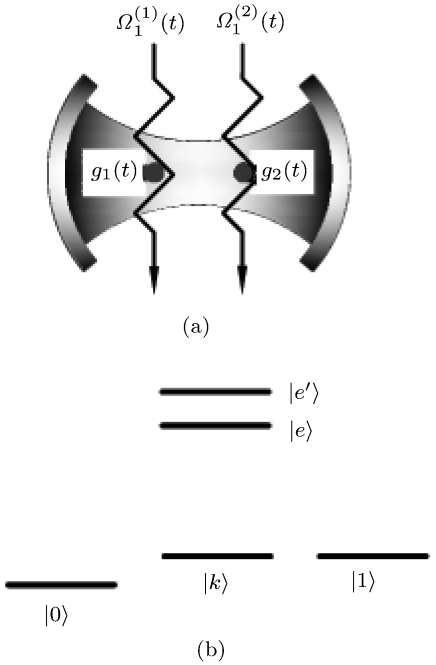


Fig. 1. The setup (a) and the involved atomic levels (b) for the generation of qutrit entanglement.

We first show the coupling between the ground states via the excited state $|e\rangle$, as shown in Fig. 2(a). The transition $|0\rangle_i \leftrightarrow |e\rangle_i$ is coupled to a classical field with the Rabi frequency $\Omega_1^{(i)}$, while the transition $|1\rangle_i \leftrightarrow |e\rangle_i$ is coupled to the quantised cavity field with the Rabi frequency g_i ($i = 1, 2$, used to distinguish between two atoms). The interaction Hamiltonian under the dipole and rotating wave approximation is given by[50−52]

$$H_{\rm I} = \hbar \sum_{i=1,2} (\Omega_1^{(i)} |e\rangle_i \langle 0|_i + g_i a |e\rangle_i \langle 1|_i) + {\rm H.c.}, \quad (1)$$

where a is the annihilation operator for the cavity mode, and H.c. represents the Hermitian conjugate term. Moreover, we assume that all the optical fields are resonant with the corresponding level transitions (Raman resonant condition), thus the Hamiltonian $H_{\rm I}$ is independent of the optical field frequencies. The Hamiltonian has multiple eigenstate subspaces, corresponding to the specific excitation number of the system. The excitation number operator of the system is defined as $N_{\rm e} = \sum_{i=1,2}(|0\rangle_{ii}\langle 0| + |e\rangle_{ii}\langle e|) + a^{+}a$. Since $N_{\rm e}$ commutes with the interaction Hamiltonian $H_{\rm I}$, the excitation number of the system is conserved during the evolution. We are interested in the subspace that possesses one excitation number, i.e., the Hamiltonian $H_{\rm I}$ couples only states within the manifold $\{|0\rangle_1|1\rangle_2|0\rangle_c, |1\rangle_1|0\rangle_2|0\rangle_c, |1\rangle_1|1\rangle_2|1\rangle_c, |e\rangle_1|1\rangle_2|0\rangle_c, |1\rangle_1|e\rangle_2|0\rangle_c\}$, where the first and second kets denote the states of the atoms, and the third ket is an n-photon ($n = 0, 1$) state of the cavity mode. The energy eigenvalues and associated eigenstates of the Hamiltonian are $\varepsilon_0 = 0$, $\varepsilon_k = \pm\hbar\{(|g_1|^2 + |g_2|^2 + \left|\Omega_1^{(1)}\right|^2 + \left|\Omega_1^{(2)}\right|^2) \pm [(|g_1|^2 - |g_2|^2 + \left|\Omega_1^{(1)}\right|^2 - \left|\Omega_1^{(2)}\right|^2)^2 + 4|g_1|^2|g_2|^2]^{1/2}\}^{1/2}/\sqrt{2}$, $|\varepsilon_k\rangle$, $(k = 1, 2, 3, 4)$. Of particular interest here to us is the eigenstate corresponding to null eigenvalue ε_0, which is given by

$$|\varepsilon_0\rangle = N(g_1\Omega_1^{(2)}|0\rangle_1|1\rangle_2|0\rangle_c + g_2\Omega_1^{(1)}|1\rangle_1|0\rangle_2|0\rangle_c - \Omega_1^{(1)}\Omega_1^{(2)}|1\rangle_1|1\rangle_2|1\rangle_c), \quad (2)$$

where N is normalised factor given by

$$N = (|g_1|^2|\Omega_1^{(2)}|^2 + |g_2|^2|\Omega_1^{(1)}|^2 + |\Omega_1^{(1)}|^2|\Omega_1^{(2)}|^2)^{-1/2}. \quad (3)$$

The state $|\varepsilon_0\rangle$ can be called a dark state, as it does not contain any contribution from the atomic excited states.[46,53]

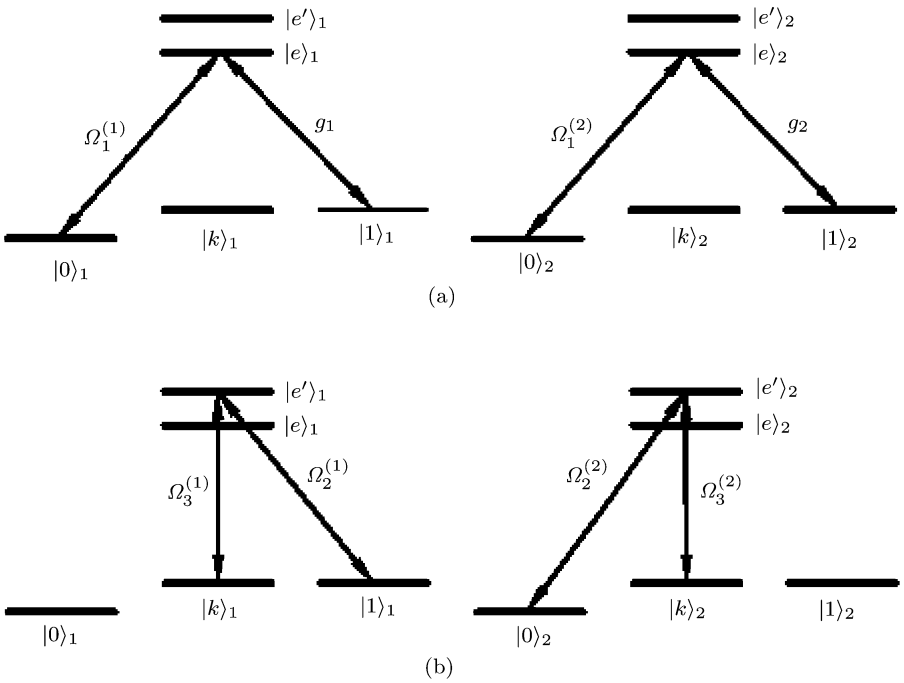


Fig. 2. The coupling of the ground states via the excited state $|e\rangle$ (a), and $|e'\rangle$ (b).

One purpose of our scheme is to utilise the adiabatic passage of the dark state $|\varepsilon_0\rangle$, and thus to produce the approximate f-STIRAP,[44,45] i.e, $|\varepsilon_{\rm f-s}\rangle \sim g_1\Omega_1^{(2)}|0\rangle_1|1\rangle_2|0\rangle_c + g_2\Omega_1^{(1)}|1\rangle_1|0\rangle_2|0\rangle_c$. To do so, one first has to make the adiabatic criterion satisfied to make sure that, the state of the system remains in the dark state $|\varepsilon_0\rangle$ during the whole evolution. This means that any nonadiabatic coupling of $|\varepsilon_0\rangle$ to $|\varepsilon_k\rangle$ should be avoided (adiabatic theorem, mathematically, can be expressed as $|\langle\varepsilon_k \mid \dot{\varepsilon}_0\rangle| << \varepsilon_k$). This can be verified if the condition $\Omega_{1\,\max}^{(i)}T$, $g_{i\,\max}T >> 1$ ('max' denotes peak value, T is pulse duration) is satisfied.[53] Second, the probability of cavity mode being populated (the term $|1\rangle_1|1\rangle_2|1\rangle_c$ being present) should be small enough as to guarantee f-STIRAP as perfect as possible, this can be achieved by setting $g_1 \sim g_2 >> \Omega_1^{(1)}$, $\Omega_1^{(2)}$.

Then we turn to the coupling of the ground states via the excited state $|e'\rangle$, as shown in Fig. 2(b). The transitions $|l\rangle_i \leftrightarrow |e'\rangle_i$ and $|k\rangle_i \leftrightarrow |e'\rangle_i$ ($l = 1$ for atom 1, $l = 0$ for atom 2) are coupled to two classical fields with the Rabi frequencies $\Omega_2^{(i)}$ and $\Omega_3^{(i)}$, respectively. The Hamiltonian describing the Raman resonant coupling in the rotating wave approximation can be expressed as

$$H_{\rm I}' = \hbar(\Omega_2^{(i)}|e'\rangle_{ii}\langle l| + \Omega_3^{(i)}|e'\rangle_{ii}\langle k|) + {\rm H.c.} \qquad (4)$$

The Hamiltonian $H_{\rm I}'$ has a dark state with the null-energy eigenvalue given by

$$|\varepsilon_0'\rangle = \frac{(\Omega_3^{(i)}|l\rangle_i - \Omega_2^{(i)}|k\rangle_i)}{(|\Omega_2^{(i)}|^2 + |\Omega_3^{(i)}|^2)^{1/2}}. \qquad (5)$$

The adiabatic passage of the dark state $|\varepsilon_0'\rangle$ can produce the STIRAP,[42,43,45] i.e., to realise coherence population transfer from $|l\rangle_i$ to $|k\rangle_i$. The adiabatic theorem, which is required for the implementation of the adiabatic transfer, is that the coupling of the dark state $|\varepsilon_0'\rangle$ to other two eigenstates $|\varepsilon_k'\rangle$ ($k = 1, 2$, with the eigenvalues $\varepsilon_k' = \pm\hbar(|\Omega_2^{(i)}|^2 + |\Omega_3^{(i)}|^2)^{1/2}$) cannot take place. This can be done if the condition $\Omega_2^{(i)}\ T'$, $\Omega_3^{(i)}\ T' >> 1$ (T' is pulse duration) is fixed.[42,43]

3. Generation of qutrit entanglement

In order to generate the qutrit entanglement, we have to implement the two cases by stages. This requires that the groups of parameters at each stage should be adjustable (i.e., terminable and restorable) and, both of them could be applied alternately. Furthermore, as to make two stages uninfluenced by each other, the energy spacing of two states $|e\rangle_i$ and $|e'\rangle_i$ should be large enough.

Thus it requires that all the optical pulses be completely adjustable, the implementation of which can follow (and 'recompose') the example of a recent CQED experiment:[54,55] a linearly polarised light beam penetrating one cavity mirror drives the cavity at the frequency of the transition $|1\rangle_i \rightarrow |e\rangle_i$ with time-variable pumping strength g_i, which is atomic-position-dependent; two uniform but independently controllable optical lattices drive two atoms transversely to the cavity axis to provide required addressing classical fields at the corresponding Rabi frequencies $\Omega_j^{(i)}$. This remains a great challenge as we have to trap and control two atoms simultaneously in a cavity, but it can be expected to achieve soon with the development trapped-atom-technique in CQED.[56,57] We will show below how the combined f-STIRAP and STIRAP can be used to prepare a qutrit entanglement of the two atoms.

(i) F-STIRAP for the first time is used to prepare the entangled state

$$|\Im\rangle_1 = (\cos\zeta|0\rangle_1|1\rangle_2 + \sin\zeta|1\rangle_1|0\rangle_2)|0\rangle_c, \quad \zeta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right), \qquad (6)$$

which arises from the following behaviour of the dark state $|\varepsilon_0\rangle : |0\rangle_1|1\rangle_2|0\rangle_c \rightarrow |\Im\rangle_1$ for $\frac{g_2\Omega_1^{(1)}}{g_1\Omega_1^{(2)}} \sim 0 \rightarrow \frac{g_2\Omega_1^{(1)}}{g_1\Omega_1^{(2)}} \sim \tan\zeta$, and $g_1, g_2 >> \Omega_1^{(1)}, \Omega_1^{(2)}$; (ii) STIRAP on each atom is used to realise coherence population transfer, i.e., the adiabatic behaviour of the dark state $|\varepsilon_0'\rangle : |1\rangle_1\ (|0\rangle_2) \rightarrow |k\rangle_1\ (|k\rangle_2)$ for $\frac{\Omega_3^{(i)}}{\Omega_2^{(i)}} \sim 0 \rightarrow \frac{\Omega_2^{(i)}}{\Omega_3^{(i)}} \sim 0$, as to transform the state $|\Im\rangle_1$ to

$$|\Im\rangle_2 = (\cos\zeta|0\rangle_1|1\rangle_2 + \sin\zeta|k\rangle_1|k\rangle_2)|0\rangle_c; \qquad (7)$$

(iii) f-STIRAP for the second time is used to prepare the required state

$$|\Im\rangle_3 = [\cos\zeta(\cos\vartheta|0\rangle_1|1\rangle_2 + \sin\vartheta|1\rangle_1|0\rangle_2) + \sin\zeta|k\rangle_1|k\rangle_2]|0\rangle_c, \quad \vartheta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right). \qquad (8)$$

This can be achieved by the behaviour of the dark state $|\varepsilon_0\rangle : |0\rangle_1\ |1\rangle_2\ |0\rangle_c \rightarrow \cos\vartheta\ |0\rangle_1\ |1\rangle_2 + \sin\vartheta|1\rangle_1|0\rangle_2)|0\rangle_c$ for $\frac{g_2\Omega_1^{(1)}}{g_1\Omega_1^{(2)}} \sim 0 \rightarrow \frac{g_2\Omega_1^{(1)}}{g_1\Omega_1^{(2)}} \sim \tan\vartheta$, and $g_1,\ g_2 >>\ \Omega_1^{(1)},\ \Omega_1^{(2)}$. Therefore, two initially unrelated atoms (in the state $|0\rangle_1|1\rangle_2$) are now prepared in a qutrit entanglement with arbitrary coefficients. The amplitudes of three entangled components can be controlled by the ratio of $g_2\Omega_1^{(1)}$ to $g_1\Omega_1^{(2)}$, and the sign of each element can be decided by Rabi frequencies $\Omega_j^{(i)}$. For $|\Im\rangle_3$, all the entangled bases are long-lived atomic ground states, thus it can be stored for a long time and used for some kinds of purposes. As atomic states can be effectively read out via the detection of atomic fluorescence,[58] any measurement can be performed on these two atoms to demonstrate quantum nonlocality. This has practical significance as it has been shown that violations of local realism are stronger for qutrit entanglement than for qubit entanglement.[8,9] Furthermore, the type of the prepared state is important for quantum information processing,[14] its usage allows one to increase the flux of information. For instance, the correlative parameters can be set in such a way that a qutrit–qutrit maximal entanglement can be prepared and used for

a quantum channel in a procedure for teleportation of a superposition of three orthogonal atomic states.[59] Or it can be used for constructing a special type of qutrit–qutrit non-maximal entanglement, analogous to a paradigm, which we have discussed recently,[60] for probabilistic teleportation of an arbitrary superposition of three atomic states. In addition, optical cavities can be coupled by optical fibres,[61−65] thus such a kind of entanglement preparation might be useful for the distributed quantum communication and computation in multidimensional quantum alphabet. For numerical simulations, we focus on the preparation of the qutrit–qutrit maximal entanglement $|\Im\rangle_3^{\max} = (1/\sqrt{3})(|0\rangle_1|1\rangle_2 + |0\rangle_1|1\rangle_2 + |k\rangle_1|k\rangle_2)|0\rangle_c$.

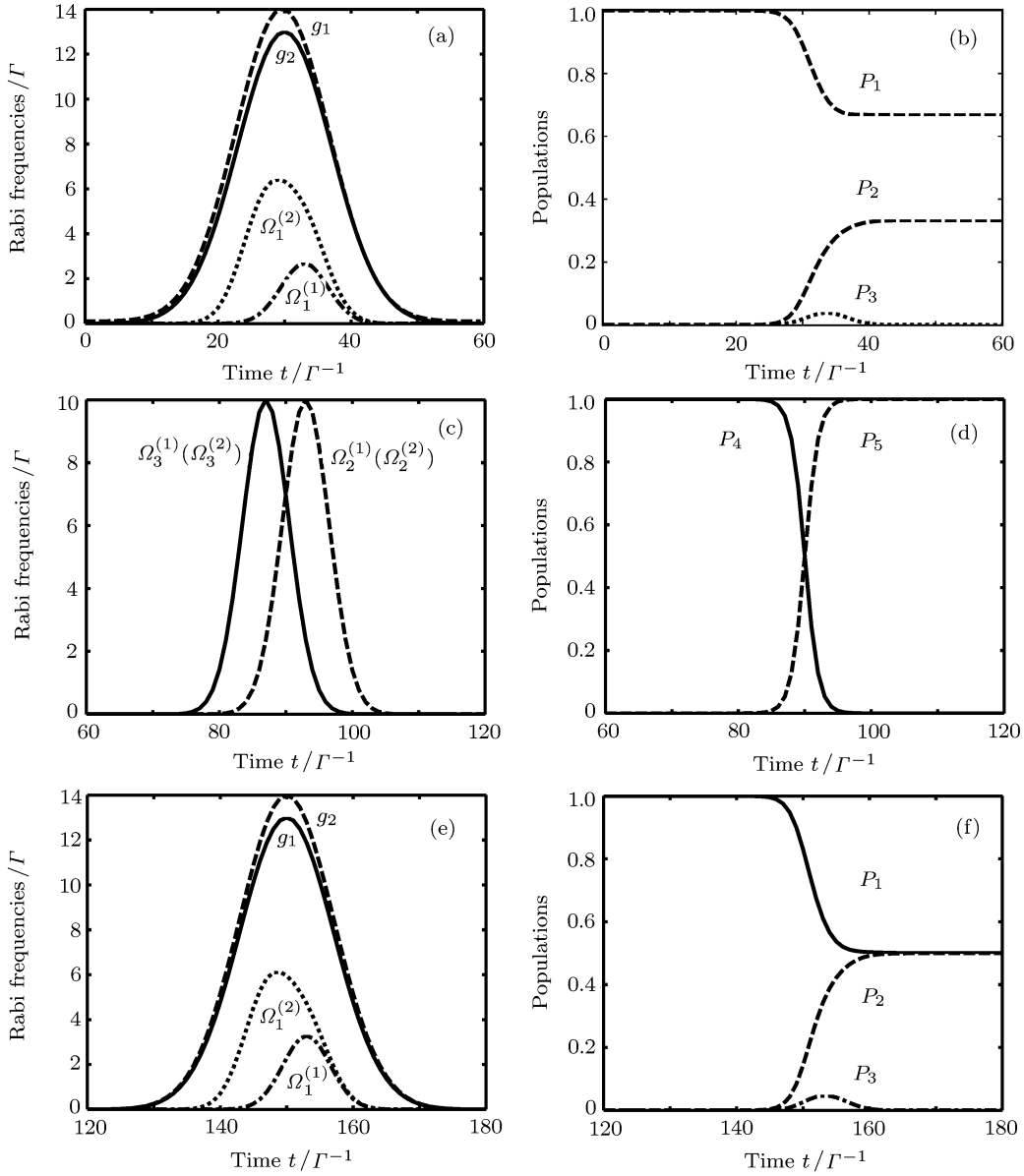


Fig. 3. (a) The Rabi frequencies g_1, g_2, $\Omega_1^{(1)}$ and $\Omega_1^{(2)}$ for the case $g_1 = g_{10}\mathrm{e}^{-(t-\tau_1)^2/t_c^2}$, $g_2 = g_{20}\mathrm{e}^{-(t-\tau_1)^2/t_c^2}$, $\Omega_1^{(1)} = \Omega_{10}^{(1)}\sin\zeta\mathrm{e}^{-(t-\tau_2)^2/t_p^2}$, and $\Omega_1^{(2)} = \Omega_{10}^{(2)}\mathrm{e}^{-(t-\tau_3)^2/t_p^2} + \Omega_{10}^{(2)}\cos\zeta\mathrm{e}^{-(t-\tau_2)^2/t_p^2}$, with parameters $g_{10} = 13\Gamma$, $g_{20} = 14\Gamma$, $\Omega_{10}^{(1)} = \frac{65}{14}\Gamma$, $\Omega_{10}^{(2)} = 5\Gamma$, $\zeta = \arctan(1/\sqrt{2})$, $t_c = 10\Gamma^{-1}$, $t_p = 5\Gamma^{-1}$, $\tau_1 = 30\Gamma^{-1}$, $\tau_2 = 33\Gamma^{-1}$, and $\tau_3 = 27\Gamma^{-1}$, and Γ being the spontaneous emission rate for the excited atomic state. (b) The time evolution of the populations in states $P_1 \equiv |0\rangle_1|1\rangle_2|0\rangle_c$, $P_2 \equiv |1\rangle_1|0\rangle_2|0\rangle_c$, and $P_3 \equiv |1\rangle_1|1\rangle_2|1\rangle_c$ with the same parameters given in (a); (c) The Rabi frequencies $\Omega_2^{(i)}$ and $\Omega_3^{(i)}$ for the case $\Omega_2^{(i)} = \Omega_{20}^{(i)}\sin\xi\mathrm{e}^{-(t-\tau_4)^2/t_{p'}^2}$, and $\Omega_3^{(i)} = \Omega_{30}^{(i)}\mathrm{e}^{-(t-\tau_5)^2/t_{p'}^2} + \Omega_{30}^{(i)}\cos\xi\mathrm{e}^{-(t-\tau_4)^2/t_{p'}^2}$, with parameters $\Omega_{20}^{(i)} = \Omega_{30}^{(i)} = 10\Gamma$, $\xi = \frac{\pi}{2}$, $t_{p'} = 5\Gamma^{-1}$, $\tau_4 = 93\Gamma^{-1}$, and $\tau_5 = 87\Gamma^{-1}$. (d) The time evolution of the populations for atom 1(2) in states $P_4 \equiv |1\rangle_1(|0\rangle_2)$ and $P_5 \equiv |k\rangle_1(|k\rangle_2)$ with the same parameters given in (c); (e) The Rabi frequencies g_1, g_2, $\Omega_1^{(1)}$, and $\Omega_1^{(2)}$ for the same case as in step (i), but, for the different parameters $\xi = \frac{\pi}{4}$ (replaces ζ), $\tau_1 = 150\Gamma^{-1}$, $\tau_2 = 153\Gamma^{-1}$, and $\tau_3 = 147\Gamma^{-1}$; (f) The time evolution of the populations in states $P_1 \equiv |0\rangle_1|1\rangle_2|0\rangle_c$, $P_2 \equiv |1\rangle_1|0\rangle_2|0\rangle_c$, and $P_3 \equiv |1\rangle_1|1\rangle_2|1\rangle_c$ with the same parameters in given (e).

Figure 3 shows the variation of the time-dependent Rabi frequencies and populations for step (i) (a) and (b), step (ii) (c) and (d), and step (iii) (e) and (f). All the Rabi frequencies and the correlative parameters in the figures are chosen to satisfy the condition of the adiabaticity. Thus the influence of the atomic spontaneous emission of the excited atomic states is effectively suppressed. To further suppress the cavity photon excitation, the ideal condition is $g_{i0} >> \Omega_{10}^{(i)}$, which, in our practice, is satisfied approximately with $g_{i0} \sim \frac{14}{5}\Omega_{10}^{(i)}$.

4. The influences of dissipation

In all the above arguments, we assumed that the system is ideally isolated from the environment, and have not considered any dissipation. Here we will discuss the influences of the dissipation due to the atomic spontaneous emission and photon leakage from the cavity on the entangled qutrits preparation.

Subject to the quantum jump approach,[66] we formulate the evolution of the system under the domination of the conditional Hamiltonian, which is based on the condition of the observation of no decay of photon emission (i.e., no quantum jump) induced by the atomic spontaneous emission and cavity photon loss. The conditional Hamiltonian of the system is defined as $H_{\rm con} = H_{\rm I} + H_{\rm I}' - {\rm i}\hbar H_{\rm V}$, with the Hermitian operator $H_{\rm V} = \Sigma_{i=1,2}(\frac{\Gamma}{2}|e\rangle_i\langle e| + \frac{\Gamma'}{2}|{\rm e}'\rangle_{ii}\langle {\rm e}'|) + ka^+a$, where Γ, Γ', and κ are the decay rates of the atomic excited state, $|e\rangle_i$, $|{\rm e}'\rangle_i$ and of the cavity photons. Given an initial state $|\psi(t_0)\rangle$, the state of the system at $t_0 + T$ under the condition of no quantum jump equals $|\psi(t_0 + T)\rangle = U_{\rm con}|\psi(t_0)\rangle/P_{\rm suc}(t_0 + T)$, with $P_{\rm suc}(t_0 + T) = \langle\psi(t_0)|U_{\rm con}^+U_{\rm con}|\psi(t_0)\rangle$ being the probability of no photon emission at time $t_0 + T$. We can expand $|\psi(t_0 + T)\rangle$ with all the eigenstates. Using the Schrödinger equation and eliminating the fast varying coefficients of the nonzero eigenstates, we educe the coefficients of two eigenstates ($|\varepsilon_0\rangle$ and $|\varepsilon_0'\rangle$) with null eigenvalue (say, dark states). For steps (i) and (iii), the result is

$$\alpha_0(T) \simeq \exp\left\{-\kappa\int_0^T \frac{(\Omega_1^{(1)}\Omega_1^{(2)})^2}{g^2[(\Omega_1^{(1)})^2 + (\Omega_1^{(2)})^2]}{\rm d}t - \Gamma\int_0^T \frac{\dot{\Omega}_1^{(1)}\Omega_1^{(2)} - \Omega_1^{(1)}\dot{\Omega}_1^{(2)}}{[(\Omega_1^{(1)})^2 + (\Omega_1^{(2)})^2]^3}{\rm d}t\right\}$$

(we assume $g_1 = g_2 = g$ for convenience); while for step (ii), it is

$$\alpha_0'(T) = \exp\left\{-\Gamma'\int_0^T \frac{\dot{\Omega}_2^{(i)}\Omega_3^{(i)} - \Omega_2^{(i)}\dot{\Omega}_3^{(i)}}{[(\Omega_2^{(i)})^2 + (\Omega_3^{(i)})^2]^3}{\rm d}t\right\},$$

with T being the time required to achieve each step of adiabatic evolution. The first and second parts in $\alpha_0(T)$ respectively result from the effects of cavity photon loss in the dark state $|\varepsilon_0\rangle$ and the nonadiabatic coupling; however, $\alpha_0'(T)$ is just attributed to the effect of the nonadiabatic coupling. Thus the success probability of the combined f-STIRAP and STIRAP is $P = P_{\rm suc}^2P_{\rm suc}'$, with $P_{\rm suc} = |\alpha_0(T)|^2$ and $P_{\rm suc}' = |\alpha_0'(T)|^2$. We can make an optimisation by following the method in Ref. [67], based on the fact that the main contribution to the reduction in success probability is caused by the imperfection in f-STIRAP. By introducing $\Omega_1^{(i)} = gf_i(t/T)$ and applying the Schwarz inequality to $\alpha_0(T)$, we obtain an upper bound $P = {\rm e}^{-5\sqrt{\frac{k\Gamma}{g^2}}}$.

5. Conclusion

In summary, we have proposed a scheme for the deterministic generation of qutrit entanglement for two atoms trapped in an optical cavity via the adiabatic passage of dark states. During the operation, the atoms remain in the ground states and the cavity mode stays in the vacuum state under certain conditions. Thus both the atomic spontaneous emission and the cavity decay are suppressed. Our study on the dissipation due to the atomic spontaneous emission and cavity photon leakage shows that there exists an upper bound for the success probability in the generated qutrit entanglement.

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