A super-high resolution frequency standard measuring approach based on phase coincidence characteristics between signals*

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A method for super high resolution comparison measurement is proposed in this paper with a comparison between the frequency standards of different nominal frequencies, which is based on phase coincidence detection of the two compared signals. It utilizes the regular phase shift characteristics between the signals. The resolution of the measurement approach can reach 10^{-13}/s at 5 MHz, and the self-calibration resolution can achieve 10^{-14}/s in the comparison between 10 MHz and 100 MHz, or even can reach 10^{-15}/s in the comparison between 10 MHz and 190 MHz. This method implies significant progress in the development of the high precision frequency standard comparison technology.

Keywords: frequency measurement, resolution, phase coincidence, equivalent phase comparison frequency

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1. Introduction

Conventional high-resolution frequency standard measurement methods are phase comparison, frequency difference multiplication, dual mixer time difference method and so on. The highest accuracy of these frequency measurement methods with complex devices can reach $10^{-13}/\tau$.[1] With the development of time-frequency technology, new measuring methods and higher precision measuring instruments become necessary.[2] Much higher measuring resolution can be obtained by combining the characteristics of regular phase difference change with available instruments. This method can be widely used to measure and process any periodic signals.[3]

2. Principle analysis

A serial of researches shows that between two arbitrary frequency signals there exists a key relationship, which can be expressed in terms of greatest common factor frequency, least common multiple period, equivalent phase comparison frequency, and quantized phase variation resolution and so on.[4] And its corresponding equation is as follows:[5]

$$f_{\rm equ} = ABf_{\max c}, \tag{1}$$

where $f_{\max c}$ is the greatest common factor frequency of the compared frequencies f_1 and f_2, and f_{equ} is the equivalent phase comparison frequency of f_1 and f_2. Its corresponding period represents the periodic phase shift between two contiguous phase coincidences of the two frequency signals when the phase difference varies along one direction. We called it quantized phase variation resolution.

$$\Delta T = \frac{f_{\max c}}{f_1 f_2}, \tag{2}$$

where f_1 and f_2 are the frequencies of the two compared signals respectively, ΔT is the quantized phase variation resolution, $f_{\max c}$ is much lower than the frequencies of the two signals, ΔT, the period corresponding to the equivalent phase comparison frequency, is much smaller than the periods corresponding to two compared frequencies.

It is ΔT that can best reflect the measurement resolution. Stochastic phase synchronization will occur between some phases. Phase coincidences will appear periodically between two arbitrary frequency signals because of the above correlation. Phase coincidences usually depend on frequency relationship and corresponding ΔT between two signals. If one utilizes the time interval between any two phase coincidences to form a count gate. And ± 1 count error existing

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in usual measurement instruments can be eliminated. Therefore, measurement resolution can be greatly improved. We apply two compared signals to the input of the phase coincidence detector. Based on a reference gate time which can be set, separately, as 1s, 100 ms or 10 ms, etc, two phase coincidences are selected after the rising edge and the falling edge of reference gate time signal respectively. And this yields the very count gate. Since the rising edge and the falling edge of the gate signal are simultaneous with the two signals, and ±1 count error can be eliminated. The system block diagram constructed by this method is shown in Fig. 1. The waveform of the system is shown in Fig. 2.

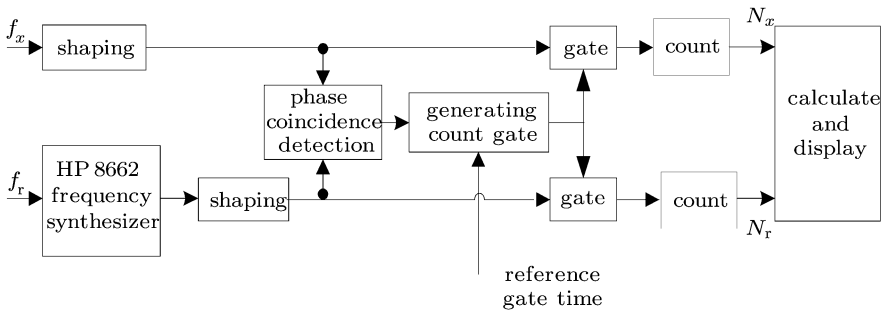


Fig. 1. Block diagram of frequency measurement system.

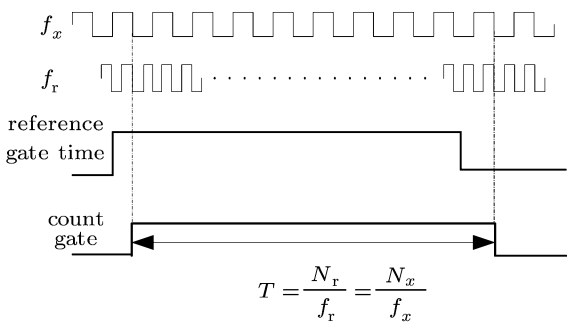


Fig. 2. Waveform of the frequency measurement system.

For some special frequency relationships like approximately multiple relations, comparatively concentrated phase difference closer to the phase coincidence can be induced. The condition combining with the circuit stability is more suitable to realizing higher coincidence detection precision and frequency measurement resolution. The calculation equation is shown as follows:

$$f_x = f_r \frac{N_x}{N_r}, \tag{3}$$

where f_r is the frequency of reference standard signal, f_x is the frequency of measured signal, N_r is the count number of reference signals in the count gate, N_x is the count number of measured signals in the count gate.

3. Test results and error analysis

We use the comparative measurement to make the comparative test of high-stability standard frequency signals on the system. First, in self-calibration tests (with one high-stable 8607 OCXO used as both f_r and f_x) the theoretical analyses are in accordance with test results. Secondly, in order to achieve the higher measurement resolution, we not only control the frequency difference of the reference signal, but also choose a more suitable frequency.

When the frequencies of the two compared signals are almost the same or have strict multiple relations, it is difficult to obtain the phase coincidence and the accuracy can be affected. Therefore a frequency synthesizer is necessary to be used as assistant equipment. The frequency synthesizer can be adjusted according to measured frequency, required accuracy and sampling time. In self-calibration experiment, a 10 MHz standard frequency signal is compared with the signal locked by HP8662A frequency synthesizer. Different self-calibration results are obtained according to the different chosen frequency outputs of synthesizer. The test results are shown in Table 1, and the frequency stabilities are calculated by Allan.[6]

Table 1. Self-calibration results of the system.

self-calibration frequencies/MHz		ΔT/fs	$f_{\rm equ}$/GHz	frequency stabilities	frequency stabilities	frequency stabilities
$f_{\rm r}$	f_x			σ/s	σ/0.1 s	σ/10 s
10.000000	5.000001	19.999996	50000.01	9.13×10^{-14}	7.26×10^{-13}	2.43×10^{-14}
10.000000	5.00001	199.9996	5000.01	1.30×10^{-12}	9.37×10^{-12}	1.61×10^{-13}
10.000000	5.0001	1999.96	500.01	1.31×10^{-11}	8.53×10^{-11}	1.29×10^{-12}
10.000000	5.001	19996	50.01	1.82×10^{-10}	7.92×10^{-10}	1.63×10^{-11}
10.000000	10.000010	99.9999	10000.01	2.77×10^{-12}	7.14×10^{-12}	3.02×10^{-13}
10.000000	20.000010	49.999975	20000.01	7.34×10^{-13}	1.25×10^{-12}	8.21×10^{-14}
10.000000	100.00001	9.999999	100000.01	2.62×10^{-14}	3.96×10^{-13}	9.65×10^{-15}
10.000000	190.00001	5.263	190000.01	9.18×10^{-15}	1.38×10^{-14}	6.37×10^{-15}

If $f_{\rm r}$ = 10 MHz and f_x = 190.00001 MHz, the quantized phase variation resolution is about 0.005 ps and the equivalent phase comparison frequency is 190.00001 THz, which are shown in Table 1. We can obtain the resolution of 0.005 ps in theory by this method. Due to the limited performance of the device and the noises of frequency synthesizer HP8662A and the detection circuits, the theoretical accuracy cannot be reached. But the measuring resolution for frequency stability can reach 10^{-14}/s through further improvement.

Because of the limited operating frequency of the circuit, the highest compared frequency in our system is 190.00001MHz. Self-calibration resolution is affected by the synthetic frequency, which is shown in Eq. (2) as well as Table 1.

In order to reflect the system precision, a supper-high stability 8607 5MHz OCXO made by OSA Company is used as the reference signal. And another super-high stability 8607 OCXO, a Cs atomic frequency standard, an Rb atomic frequency standard, and high-stability 1250 A OCXO are used as the measured signals respectively. The system in Fig. 1 is adopted in this comparison test. One of the two compared signals from 8607 is offset in order to generate the frequency difference of 1 Hz between two compared signals. The measurement resolution of the other comparison tests in which frequency synthesizer HP8662A is used, are ensured by high equivalent phase comparison frequency. The measured signals are compared with 20.00001 MHz output signal of HP8662A which is locked by super-high stability 8607 OCXO. The test results are shown in Table 2.

Table 2. Measurement results of frequency stability.

compared frequency standard/MHz		ΔT/fs	$f_{\rm equ}$/THz	frequency stabilities	frequency stabilities	frequency stabilities
$f_{\rm r}$	f_x			σ/1 s	σ/0.1 s	σ/10 s
8607 OCXO 5.000000	8607 OCXO 5.000001	39.99	25.00005	3.53×10^{-13}	1.26×10^{-12}	1.23×10^{-13}
8607 OCXO with HP8662A 20.00001	5585B Cs atomic frequency standard 10.000000	49.999975	20.00001	9.53×10^{-12}	7.58×10^{-11}	3.15×10^{-12}
8607 OCXO with HP8662A 20.00001	SR625 Rb atomic frequency standard 10.000000	49.999975	20.00001	1.99×10^{-11}	1.12×10^{-10}	5.28×10^{-12}
8607 OCXO with HP8662A 20.00001	1250A OCXO 5.000000	99.99995	10.000005	2.16×10^{-12}	1.34×10^{-11}	1.10×10^{-12}

The frequency stability of the two 8607 OCXOs is 2×10^{-13}/s which is appraised by HP5390A high-resolution frequency stability measurement system. The comparison result between the two oscillators should be about 3×10^{-13}/s. Besides, the following test results in our test are in accordance with the ones based on other high-resolution measuring systems.

High-stability frequency standard is needed in the measurement. Meanwhile, the influence of those major factors concerning accuracy, such as the trigger er-

ror, the frequency relationship between the measured signal and the standard signal, and the stability of frequency standard should be considered.[7] The effect of trigger error can be reduced by reshaping small signals. The emitter coupled logic (ECL) circuit is selected to improve the gradient of the signal.[8] At the same time, the switch stability and its velocity can affect the measurement result,[9,10] the former is one of the major measurement errors. In the coincidence detection, the phase coincidence is the ideal phase coincidence when the phase difference becomes the smallest one. When only one coincidence can be detected in one $T_{\min c}$, the measurement resolution can be extremely high. But in one $T_{\min c}$, the higher the frequency of the emergence of the situation adjacent to ideal phase coincidence is, the lower the accuracy of the measurement becomes. The measurement result will be affected, if there is a very small phase variation resolution such as less than 10^{-17} s between the measured frequency and the frequency standard, where the frequency values of the two signals are almost the same or the integral multiple. In this case, group periods are very big. Therefore, we find that the above-mentioned measurement method should be connected with the simple frequency synthesizer technique, thus the wide range frequency measurement can be realized.[11]

4. Conclusion

The regular variation of phase difference between two frequency signals can be used in the super-high resolution measurement of frequency standard signals on the basis of quantized phase variation resolution. We keep a certain frequency difference between two compared nomination frequencies in this measurement method, and can obtain an even higher resolution combining the application of a frequency standard with a special nomination frequency. The measurement accuracy can be improved by catching phase coincidences between two signals with an appropriate frequency relationship. The self-calibration measurement resolution can reach 10^{-14}/s. High-resolution frequency measurement in a wide range can be realized in the measurement if assistant equipment is adopted. As a super-high resolution and effective measuring or processing approach, this method can help achieve an even higher accuracy if we improve its circuit design further.

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