Hierarchy property of traffic networks*

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The flourishing complex network theory has aroused increasing interest in studying the properties of real-world networks. Based on the traffic network of Chang–Zhu–Tan urban agglomeration in central China, some basic network topological characteristics were computed with data collected from local traffic maps, which showed that the traffic networks were small-world networks with strong resilience against failure; more importantly, the investigations of assortativity coefficient and average nearest-neighbour degree implied the disassortativity of the traffic networks. Since traffic network hierarchy as an important basic property has been neither studied intensively nor proved quantitatively, the authors are inspired to analyse traffic network hierarchy with disassortativity and to finely characterize hierarchy in the traffic networks by using the n-degree–n-clustering coefficient relationship. Through numerical results and analyses an exciting conclusion is drawn that the traffic networks exhibit a significant hierarchy, that is, the traffic networks are proved to be hierarchically organized. The result provides important information and theoretical groundwork for optimal transport planning.

Keywords: traffic network, hierarchy property, n-clustering coefficient, disassortativity
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1. Introduction

Many theoretical and applied researches on complex networks, among which there are two typical networks, namely, small-world networks and scale-free networks, have been carried out since the pioneering work by Watts and Strogatz,[1] and Barabási and Albert.[2] Many real-world networks such as World Wide Web (WWW) network,[3] scientific collaboration networks[4] reveal a power law distribution. But this is not always true in reality, there also exist networks such as the actor network whose degree distribution follows a power law but has a sharp cut off in its tail.[5]

Up to data, there has been a wave of keen interest in different fields to investigate various properties of transport networks by complex network theory. Most of the studies concentrate on public transport, such as urban bus network and regional rail line transport network. Relying on transportation efficiency, Angeloudis and Fisk[6] investigated the complex features of world subway network. Wu *et al.*[7] analysed the scale-freeness and small-world property for public transportation network of Beijing by constructing an origin–destination network (O–D network). Lu and Shi[8] studied the complex network characteristics of several public transport networks by classifying them into public transportation route networks, public transportation transfer networks and bus station networks. Li and Cai[9] showed that the topological structure of the air traffic network of China had two key characteristics of small world networks. Depending on the adopted definition of a network topology, Sienkiewicz and Holyst[10] examined a topology of 22 public transport networks in Poland, showing that the degree distribution could obey a power laws or could be described by exponential function $p(k) \sim \mathrm{e}^{-\alpha k}$. Ferber, Holovatch *et al.*[11] analysed the statistical properties of public transport networks, and disclosed that larger networks appeared to possess a scale-free structure as demonstrated e.g. by the Zipf law distribution.

In contrast to the above statement, the basic traffic networks such as road network and basic properties

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such as hierarchy in traffic networks, have not received much attention.

Hierarchy is a very common phenomenon in nature. West *et al.*[12] pointed out that hierarchy was a fundamental characteristic of resource distribution networks in biology as can be seen in plants, for instance, the branching of trees, and in animals and humans such as respiratory and cardiovascular systems. The biological hierarchical network structures have been proven to be optimal with respect to maximizing metabolic capacity, that is to say, maximizing the surface area where resources are exchanged, while minimizing internal transport distances. Many investigations reveal the hierarchical organization of some real networks besides traffic network in nature and society.[13] In actor network, the clustering coefficient of a node with k links follows scaling law $C(k) \sim k^{-1}$;[5] and this has been observed and briefly noted for the WWW network.[14] Internet network of autonomous system (AS) level (belongs to the domain level Internet) satisfies $C(k) \sim k^{-0.75}$,[15] and other examples such as metabolic network are observed to self-organize into a hierarchical state.[16]

However, there are very few research papers dealing with traffic network hierarchy. It is necessary and significant to study the hierarchy structure of traffic networks quantitatively.

With the rapid development of regional economy in China, Chang–Zhu–Tan urban agglomeration (CZTUA) in central China has set its goal to integrating the transport networks of the three cities of Changsha, Zhuzhou, and Xiangtan. This offers opportunity and challenge for transport planning. The published research results show that, up to date, much attention has not paid to the phenomenon of hierarchy in transport which is fundamental and important for further research of transport planning dealing with such as connecting of roads at different levels of hierarchy, and the routing of the road at a special hierarchical level.

To meet these challenges, many efforts have been made to investigate the basic significant feature of hierarchy, for the urban agglomeration traffic network. In the present study, basic topological properties of clustering coefficient, assortativity coefficient, and average nearest-neighbours degree are calculated and analysed, and then the coexistence of hierarchy structure and disassortativity of traffic network is investigated. The rest of the present paper is organized as follows. In Section 2 provided is a problem statement, followed by the introduction of basic topological characteristics. In Section 3, the hierarchical property is investigated in the case of Chang–Zhu–Tan urban agglomeration traffic network (CZTUATN), and the relationship of n-clustering coefficient versus the degree of order n, combined with network disassortativity, is originally used to analyse and reveal the hierarchy organisation of urban traffic networks and the urban agglomeration traffic network. Finally, Some conclusions drawn from the present study are presented in Section 4.

2. Topological characteristics of complex network

2.1. Network construction

Because a road network has its own intrinsic spatial structure that must be represented and retained in generalization process, a road network is represented by using a graph where numbered roads are denoted as nodes and road intersections as links of the graph.[17] Note that a numbered road is not a road segment but the entire numbered road as a basic modelling unit. Any graph derived in such a way is connected. A graph $G = (V, E)$ obtained from a road network is defined as a pair of a finite set of nodes $V = \{i, j, \ldots, k\}$ and a finite set of edges $E = \{(i, j) : i, j \in V, i \neq j\}$ with $|V| = N$ that is the number of nodes of the graph and $|E| = M$ is the number of edges (note that the terms that we use, i.e., edge and link, vertex and node, and network and graph, are interchangeable). For computational purposes we represent a connected, undirected and unweighted (i.e. all edges with a unit distance) graph by an adjacency matrix $\boldsymbol{A} = (a_{ij}) \in \{0, 1\}^{N \times N}$, where a_{ij} is one if $(i, j) \in E$ and zero otherwise.

2.2. Notation

For the expressive convenience of this paper, we first present the notation system as follows:

$N^{(n)}(i)$ = set of n-neighbours of node $i \in V$, namely, $N^{(n)}(i) = \{j \in V : (i, i_1), \ldots, (i_{n-1}, j) \in E\}$; $N^{(1)}(i)$ is the set of nearest neighbours, and it is generally referred to as $N(i)$.

$k^{(n)}$ = node degree of order n; $k^{(1)}$ is the so-called node degree. n-degree of node i is denoted by $k_i^{(n)} = \left|N^{(n)}(i)\right|$, and the special case $k_i^{(1)} = k_i = |N(i)| = \sum_{j \in N(i)} a_{ij}$.

$\mathrm{LN}^{(n)}$ = the number of links between neighbours in set $N^{(n)}(i)$; $\mathrm{LN}^{(1)}$ is denoted by LN.

$P(k)$ = the probability of an arbitrary node in the network with degree k.

$C_i^{(n)}$ = n-clustering coefficient of node $i \in V$; $C_i^{(1)}$ is generally referred to as clustering coefficient C_i.

$C^{(n)}(k^{(n)})$ = n-clustering coefficient of node with degree $k^{(n)}$; $C^{(1)}(k^{(1)})$ is the clustering coefficient of degree k node, and it is generally referred to as $C(k)$.

C = clustering coefficient of traffic network.

C_{rand} = clustering coefficient of random grid.

$\hbar$ = the ratio of C to C_{rand}, i.e., C/C_{rand}.

$k_{nn}(k)$ = degree-dependent average nearest-neighbours degree.

PR = assortativity coefficient of traffic network.

Note that all variables are necessarily nonnegative scalar except for PR.

2.3. Network topological characteristics

2.3.1. Clustering coefficient

(1) n-clustering coefficient[1]

The clustering coefficient, defined as

$$C_i = 2\mathrm{LN}/k_i(k_i - 1), \tag{1}$$

where LN denotes the number of direct links connecting the k_i nearest neighbours of node i, is equal to one for a node at the centre of a fully inter-linked cluster, and is zero for a road that is part of a loosely connected group. Clustering coefficient measures the degree to which those roads in traffic network are interconnected each other. The time complexity to compute clustering coefficient is $O(M^2N)$.

To calculate n-clustering coefficient $C_i^{(n)}(n > 1)$ of node i, only LN and k_i in the foregoing formula need to be replaced with $\mathrm{LN}^{(n)}$ and $k_i^{(n)}$ respectively. $C_i^{(n)}$ describes the degree to which those roads within n-neighbourhood in traffic network are connected with each other, and $C_i^{(n)} \in [0, 1]$, $\forall n \geq 1$.

(2) Clustering coefficient C of network

$$C = \sum_k k(k-1)P(k)C(k). \tag{2}$$

2.3.2. Network assortativity coefficient

In an assortative (a disassortative) network, a node tends to connect other nodes with a similar (dissimilar) degree. The assortativity coefficient was introduced as[18]

PR

$$= \frac{M^{-1}\sum_i k_{i^1}k_{i^2} - \left(M^{-1}\sum_i \frac{1}{2}(k_{i^1} + k_{i^2})\right)^2}{M^{-1}\sum_i \frac{1}{2}(k_{i^1}^2 + k_{i^2}^2) - \left(M^{-1}\sum_i \frac{1}{2}(k_{i^1} + k_{i^2})\right)^2} \tag{3}$$

calculated from the degree of endpoint of all edges, to measure the correlation degree between linked nodes, where k_{i^1} and k_{i^2} are degrees of nodes i^1, i^2 of edge $i = (i^1, i^2) \in E$. The measurer PR is positive (negative) for assortative (disassortative) networks, and zero for a random graph. Assortativity coefficient is independent of the weight of each edge in a network.

3. Hierarchy in traffic networks

The following studies are mainly devoted to the hierarchy property based on the case of traffic networks of CZTUA in central China. Using data collected from traffic maps of the three central cities (Changsha, Zhuzhou and Xiangtan), and the CZTUA composed of them, published by Hunan Map Press in 2007, five undirected unweighted graphs were created for the city traffic networks, i.e. CSTN, ZZTN, and XTTN, and the Chang–Zhu–Tan urban agglomeration traffic networks, i.e. CZTUATN1 and CZTUATN2, in which CZTUATN1 was a greatly simplified network, and CZTUATN2 was a relatively complete network composed of 132 roads of all of the main intercity highway, urban arterial roads and most of main streets. According to the following sections, the network scale is enough to investigate and uncover network characteristics.

3.1. Some basic topological characteristics

Some basic network topological characteristics were computed through numerical experiments and some of them were selected (Table 1). Analyses in terms of these results are unfolded below.

(1) The five traffic networks show small average shortest path lengths. The characteristic path lengths of three central city traffic networks are less than 2.5, and those of CZTUATN1 and CZTUATN2 are about 3.0.

By comparison with random grid, each of the five networks exhibits a high degree of clustering coefficient, especially for CZTUATN2. The large clustering coefficient of CZTUATN2 discloses a possible

phenomenon that there exist various sub-graphs with nodes highly interconnected locally. As a matter of fact, CZTUATN is constituted by traffic networks, developed independently in the past, of three central cities of Changsha, Zhuzhou, and Xiangtan, and naturally contains sub-graphs.

According to the small average shortest path length and the high clustering coefficient, the traffic networks of Changsha, Zhuzhou, and Xiangtan exhibit small-world behaviours. The small world effect describes the property that any two nodes in the systems can be connected by relatively short paths. These traffic networks are both globally and locally efficient to some degree, which may be assistantly explained by the good resilience of networks against failure, which is demonstrated by measurer κ computed (refer to Ref. [11]). Most of the resilience values are larger than 2, and especially, the resilience value of the larger network of urban agglomeration, CZTUATN2, is twice as large as the threshold.

Table 1. Some basic topological characteristics of five traffic networks.

	N	$\bar{L}$	$\bar{L}_{\rm rand}$	$\hbar$	κ	PR
CSTN	42	2.0952	1.9010	25	2.5305	–0.2598
ZZTN	36	2.3190	1.9507	33	2.3771	–0.2902
XTTN	43	2.3810	2.2315	15	1.9667	–0.2621
CZTUATN1	41	2.6122	2.1986	25	1.8196	–0.1745
CZTUATN2	132	3.0723	2.3611	125	4.1484	–0.1298

(In Table 1, $\bar{L}$ and $\bar{L}_{\rm rand}$ denote the average shortest path length of traffic network and random grid respectively).

(2) The example of CSTN is taken to investigate the 2-clustering coefficient and compare it with clustering coefficient. It is uncovered that the 2-clustering coefficient of road with zero clustering coefficient is always nonzero. Moreover, a more continuous plot can be obtained if the 2-clustering coefficients are put in decreasing order. This demonstrates that 2-clustering coefficient can distinguish the clustering degree among roads better than clustering coefficient.

(3) That the assortativity coefficients of the five networks are negative demonstrates their disassortativity, namely, larger degree roads tend to connect with smaller ones; the disassortativity of the three central city traffic networks is stronger than that of the urban agglomeration traffic network.

It is worth pointing out the difference of assortativity coefficient between the traffic networks and some other networks. Compared with many other social networks that have the positive assortativity coefficients or assortativity such as human relationship network,[18] the traffic networks are disassortative as exhibited above, which property is consistent with those of most of biological networks and technological networks.[18] But there is still distinctness between traffic networks, biological networks and technological networks. The former has the dominant many-to-many network type, while biological networks, and technological networks, for instance electric power network and water supply network, have one-to-many (or many-to-one) type.

From the aforementioned analyses of Eqs. (2) and (3), the authors are inspired with some ideas: (i) the connection relation between roads may imply that these networks are hierarchically constructed; (ii) the n-clustering coefficient is possible to be utilized to demonstrate the existence of hierarchy in traffic network. Section 3.2 will go deep into investigating the hierarchy feature of traffic networks to substantiate the ideas

3.2. Network hierarchy

3.2.1. Disassortativity and hierarchy of traffic networks

Now we come to probe into the disassortativity phenomenon from another aspect of view.

To investigate the degree correlation of the traffic network according to the study of Pastor-Satorras *et al.*,[19] one can look into the average nearest-neighbours degree of node i, which is defined as

$$k_{nn,i} = \frac{1}{k_i} \sum_{j \in N(i)} k_j. \tag{4}$$

The degree-dependent average nearest-neighbours degree, $k_{nn}(k)$, is the mean of $k_{nn,i}$ restricted to the class of degree k roads. If $k_{nn}(k)$ is an increasing (decreasing) function of k, a network is said to be an

assortative (disassortative) network, in which nodes with high degree tend to connect to other nodes with high (low) degree.

Figure 1 shows obviously that $k_{nn}(k)$ is a decreasing function of k in the three urban traffic networks, CSTN, ZZTN and XTTN. When it comes to CZTUATN1 and CZTUATN2, the tendency is roughly decreasing, though not remarkably. These graphic representations are consistent with the foregoing calculation of assortativity coefficient.

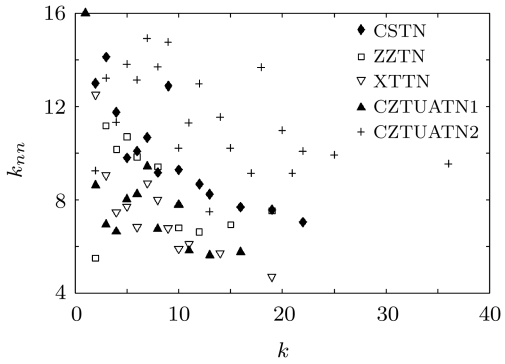


Fig. 1. Average nearest-neighbour degree of five traffic networks.

The connection relation between roads in real-life traffic network is observed further through the relation between degree k and its appropriate number of roads for e.g. CSTN and CZTUATN2.

In CSTN, roads with connectivity $k \in I_1 =: \{k \in \mathbb{Z}_+ | k > 15\}$ are the second and third ring city roads, Furong Road. There are 2 roads with degree $k \in I_2 =: \{k \in \mathbb{Z}_+ | 10 < k \leq 15\}$, namely, Xiangjiang River Road and Shaoshan Road. Degree $k \in I_3 =: \{10\}$ roads, such as Huangxing Road, Laodong Road, etc., sum up to 5 roads. Roads with connectivity $k \in I_4 =: \{k \in \mathbb{Z}_+ | 5 < k < 10\}$ are Renmin Road, Shuguang Road, and so forth, the total number of which is 15. Roads with connectivity $k \in I_5 =: \{k \in \mathbb{Z}_+ | 4 \leq k \leq 5\}$ are Bayi Great Road, Xinyao Road, and so on, summing up to 10 roads. The total number of the lowest degree roads with $k \in I_6 =: \{k \in \mathbb{Z}_+ | k \leq 3\}$ is 7. Together with the traffic information about this region, it is clear to observe again that roads with lower connectivity make up of the vast majority of all roads. Furthermore, roads in intervals of higher connectivity are inclined to link with roads in intervals of lower connectivity.

Similarly, the relationship between different hierarchical levels and disassortativity in CZTUATN2 has been analysed, but the details are omitted here.

Combining these investigations with network disassortativity leads to a conclusion that for a certain road connectivity interval I_i, in which the roads have the tendency to connect to roads in connectivity interval I_j ($\forall j \neq i$). Moreover, these two intervals are, generally, of different importance and technological grades. It is reasonable to argue that there exists a close tie between traffic network disassortativity and the emergence of hierarchy and that it is not difficult to observe the hierarchy of network.

In the following, great efforts have been made to disclose the hierarchy structure of urban agglomeration traffic networks by using the relation between n-clustering coefficient and n-degree.

3.2.2. n-clustering coefficient and hierarchy of traffic networks

Ravasz and Barabási[20] have argued that the scaling law quantifies the coexistence of a hierarchy of nodes with different degrees of clustering and pointed out that in hierarchical networks, the degree of clustering characterizing the different groups follows a strict scaling law and can be used to identify the presence of a hierarchical organisation in real networks.

Together with the discussion about the 2-clustering coefficient in items (2) and (3) of Section 3.1, the relationship between n-clustering coefficient especially 2-clustering coefficient and road connectivity is used to investigate the traffic network hierarchy.

With clustering coefficient data obtained from computation for traffic network of CZTUA, the relationship of n-clustering coefficient versus degree of order n is studied based on the discrete point plot and fitted curve of n-clustering coefficient against degree of order n for the five networks which are shown in Figs. 2–6 (note that, for expressive convenience, $k^{(2)}$ and $C^{(2)}(k^{(2)})$ are represented by k_2 and $C(k_2)$ respectively in these figures).

The two figures of clustering coefficient against degree for traffic network of Xiangtan (XTTN) and one of the traffic networks of CZTUA, i.e., CZTUATN1, are shown in Figs. 4(a) and 5(a), respectively. Figure 4(a) does not reveal the conspicuous scaling law for Xiangtan, so it is hard to observe hierarchy characteristic. One can also find the absence of hierarchy in CZTUA through the investigation of CZTUATN1 from Fig. 5(a) that shows the plot of disorderly scattered points.

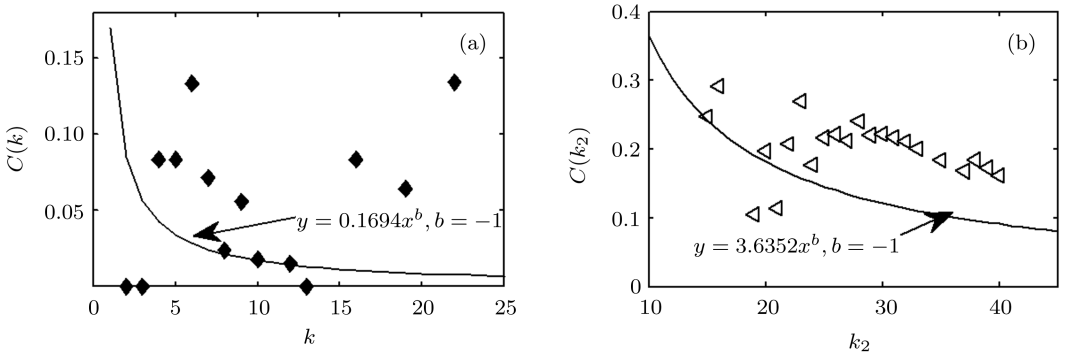


Fig. 2. (a) Clustering coefficient against degree and (b) 2-clustering coefficient against degree of order 2 for the case of CSTN.

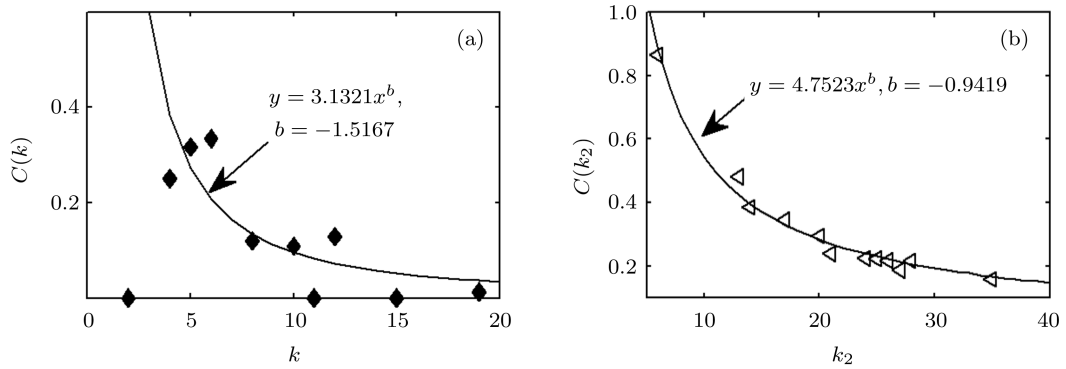


Fig. 3. (a) Clustering coefficient against degree and (b) discrete plot and fitted curve of 2-clustering coefficient against degree of order 2 for the case of ZZTN.

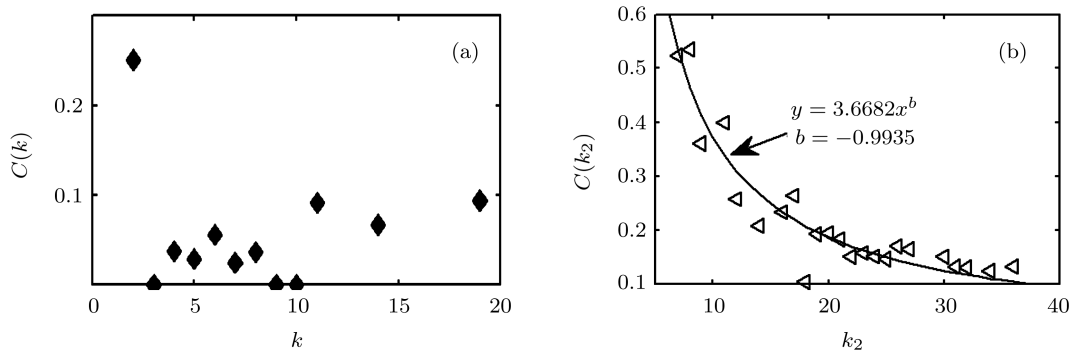


Fig. 4. (a) Discrete plot of clustering coefficient against degree and (b) discrete plot and fitted curve of 2-clustering coefficient against degree of order 2 for the case of XTTN.

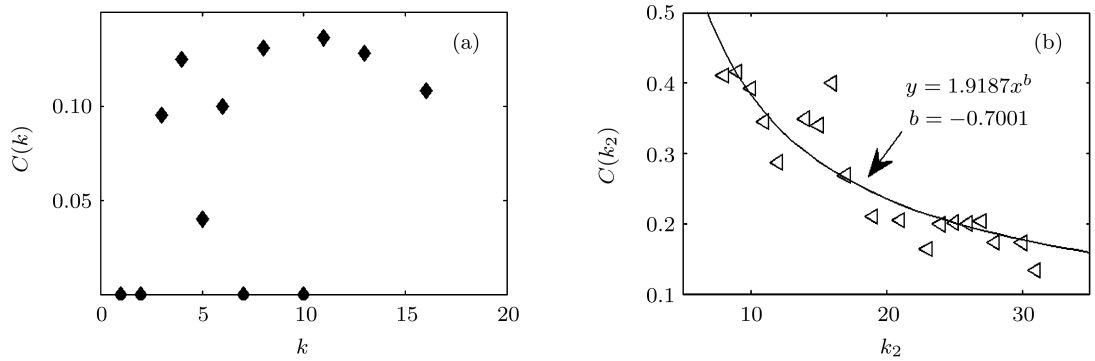


Fig. 5. (a) Discrete plot of clustering coefficient against degree and (b) discrete plot and fitted curve of 2-clustering coefficient against degree of order 2 for the case of CZTUATN1.

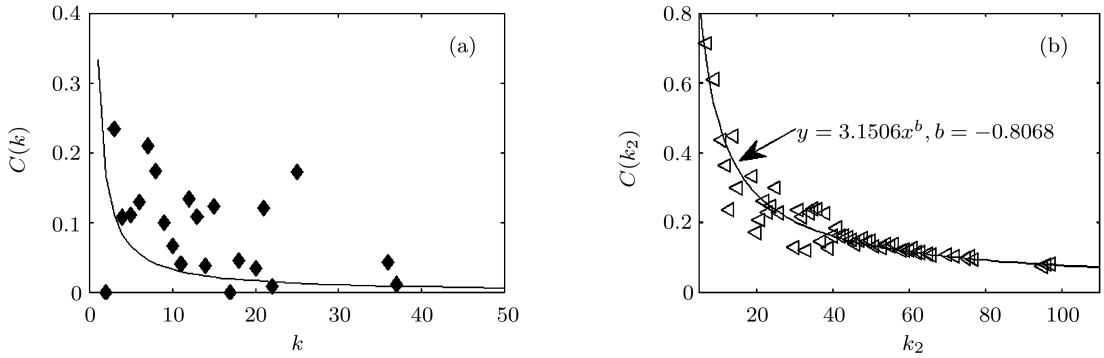


Fig. 6. (a) Discrete plot of clustering coefficient against degree and (b) discrete plot and fitted curve of 2-clustering coefficient against degree of order 2 for the case of CZTUATN2.

The question is whether there exist the present hierarchical architectures for traffic networks of Xiangtan and CZTUA, or whether there is any better approach to disclosing the hierarchy in the two networks?

In the following, we endeavour to handle these problems through first adapting the relationship between 2-clustering coefficient and degree of order 2, i.e. $C^{(2)}(k^{(2)}) - k^{(2)-b}$, and then expanding the Chang–Zhu–Tan urban agglomeration traffic network CZTUATN1, to a larger one, CZTUATN2.

The discrete point plots and fitted curves of the 2-clustering coefficient versus degree of order 2 for XTTN and CZTUATN1 are graphed and exhibited in Figs. 4(b) and 5(b). The equations of fitted curves in the figures are shown as follows:

$$y = 3.6682x^{-0.9935}, \quad y = 1.9187x^{-0.7001},$$

where x and y denote $k^{(2)}$ and $C^{(2)}(k^{(2)})$ respectively. It is easy to see that the 2-clustering coefficient of a node with degree of the second order $k^{(2)}$ approximately follows scaling law $C^{(2)}(k^{(2)}) \sim k^{(2)-b}$, $b \approx 1$. The relation illuminates that nodes with lower degree of order 2 have higher 2-clustering coefficient and constitute a clustering of high degree, while hub nodes with much larger second order degree display relatively small coefficient and play the role of connecting different small clusters. These studies display before us the hierarchical organization for the traffic networks of central city Xiangtan and the urban agglomeration. Hereby, it is well-founded to conclude that the hierarchical characteristic can be depicted more finely by the relationship between 2-clustering coefficient and degree of the second order.

Analogical analyses of the remaining three traffic networks, CSTN, ZZTN, and the network expanded from CZTUATN1 and CZTUATN2, presented as follows.

The figures, Figs. 2(a), 3(a) and 6(a), display the relationships between clustering coefficient and degree of CSTN, ZZTN, and CZTUATN2, in which the three curve equations are given by

$$y = 0.1694x^{-1}, \quad y = 3.1321x^{-1.5167}, \quad y = x^{-1},$$

where x and y denote k and $C(k)$ respectively. These equations display the scaling law of clustering coefficient against road degree for the traffic network. So the three networks of Changsha, Zhuzhou and CZTUA, have hierarchical feature, whose conclusion is easier to draw when network disassortativity is involved in.

Furthermore, 2-clustering coefficient is used to finely describe the hierarchical behaviours of the three traffic networks, i.e. CSTN, ZZTN, and CZTUATN2. Their relationships between 2-clustering coefficient and degree of the second order are shown in Figs. 2(b), 3(b) and 6(b), their corresponding equations are shown as

$$y = 3.6352x^{-1}, \; y = 4.7523x^{-0.9419},$$
$$y = 3.1506x^{-0.8068}.$$

The last two equations above are equations of fitted curves, which exhibit apparently that the 2-clustering coefficient of traffic system can be described in terms of power-law decay as $C^{(2)}(k^{(2)}) \sim k^{(2)-b}$, $b \approx 1$. The result reveals the hierarchy structures of the three networks, CSTN, ZZTN, and CZTUATN2.

Picking out the traffic networks of Chang–Zhu–Tan urban agglomeration, CZTUATN1 and CZTUATN2, for further comparison, one can reach the conclusion that a relatively weak hierarchical organization in CZTUATN1 can be explained by the fact that we have just a partial view of the complete system. In

fact, the analyses obtained by using the relationship between 2-clustering coefficient and degree of the second order show that a generic closed traffic system can exhibit its hierarchy.

There are several outputs achieved according to the intensive and profound analyses of hierarchy feature for all of the five traffic networks. First, some of these traffic networks exhibit notable hierarchy while the others display ambiguous hierarchical state when only the relationship of clustering coefficient versus degree relation is used to analyse the problem. Furthermore, it is clearer to observe the network hierarchy by exploring the relationship of 2-clustering coefficient versus degree of order 2 relation. Thirdly, the reason why the network exhibiting obvious hierarchy when the 2-clustering coefficient versus degree of order 2 relation is involved to deal with the problem does not show notable hierarchy in the initial investigation with lower order clustering coefficient, to a great extent, is the comparatively small network scale. And finally, it is argued that all the traffic networks display hierarchical organisation.

4. Conclusions

The significant basic feature of hierarchy in traffic networks is studied under the background of Chang–Zhu–Tan urban agglomeration in central China. Since the traffic network hierarchy property, which is fundamental phenomenon in traffic system, has not been studied intensively, to meet this challenge, the relationship of n-clustering coefficient versus the degree of order n is proposed to finely characterize hierarchy organisation of the urban agglomeration traffic networks, and it is combined with network disassortativity to gain an insight into the hierarchy. The presented analyses together with numerical and graphical results display scaling law of n-clustering coefficient against road connectivity. It is sufficient to prove that the urban agglomeration traffic networks and the central city traffic networks exhibit hierarchy structures.

In real world and scale-free networks, there are some other important aspects such as rich nodes phenomenon that has been investigated by e.g. Zhao and Gao,[21] in which the relationship of clustering coefficient and assortativity coefficient with the rich level is extensively studied for WWW and protein–protein interaction network. The dynamical behaviour of network traffic flow[22] is also a significant aspect especially at a microscopic level. These investigations and results are beneficial to be integrated with macroscopic consideration for further studying the real transport network hierarchy.

Further research of transport planning dealing with, such as, road connections at different levels of hierarchy, the routing of the road of a special hierarchy, and the optimal transport network design for Chang–Zhu–Tan 'two-oriented society' urban agglomeration can be soundly supported by the theoretical proof of the existence of traffic network hierarchy and corresponding further studies of transport planning based on hierarchical property.

References

[1] Watts D J and Strogatz S H 1998 *Nature* **393** 440
[2] Barabási A L and Albert R 1999 *Science* **286** 509
[3] Albert R, Jeong H and Barabási A L 1999 *Nature* **401** 130
[4] Newman M E J 2001 *Phys. Rev.* E **64** 016132
[5] Albert R and Barabási A L 2000 *Phys. Rev. Lett.* **85** 5234
[6] Angeloudis P and Fisk D 2006 *Physica* A **367** 553
[7] Wu J J, Gao Z Y, Sun H J and Huang H J 2004 *Mod. Phys. Lett.* B **18** 1043
[8] Lu H P and Shi Y 2007 *Tsinghua Sci. and Tech.* **12** 4
[9] Li W and Cai X 2004 *Phys. Rev.* E **69** 046106
[10] Sienkiewicz J and Holyst J A 2005 *Phys. Rev.* E **72** 046127
[11] Ferber C V, Holovatch T, Holovatch Y and Palchykov V 2007 *Physica* A **380** 585
[12] West G B, Brown J H and Enquist B J 1997 *Science* **276** 122
[13] Newman M E J 2003 *SIAM Rev.* **45** 167
[14] Eckmann J P and Moses E 2002 *Proc. Nat. Acad. Sci.* USA **99** 5825
[15] Vazquez A, Pastor-Satorras R and Vespignani A 2002 *Phys. Rev.* E **65** 066130
[16] Ravasz E, Somera A L, Mongru D A and Barabasi A L 2002 *Science* **297** 1551
[17] Rosvall M, Trusina A, Minnhagen P and Sneppenk 2005 *Phys. Rev. Lett.* **94** 028701
[18] Newman M E J 2003 *Phys. Rev.* E **67** 026126
[19] Pastor-Satorras R, Vazquez A and Vespignani A 2001 *Phys. Rev. Lett.* **87** 258701
[20] Ravasz E and Barabási A L 2003 *Phys. Rev.* E **67** 026112
[21] Zhao H and Gao Z Y 2007 *Physica* A **381** 473
[22] Xu M and Gao Z Y 2007 *Chin. Phys.* **16** 1608