Theoretical and experimental study of Chen chaotic system with notch filter feedback control*

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(Received 31 January 2009; revised manuscript received 19 April 2010)

Since the past two decades, the time delay feedback control method has attracted more and more attention in chaos control studies because of its simplicity and efficiency compared with other chaos control schemes. Recently, it has been proposed to suppress low-dimensional chaos with the notch filter feedback control method, which can be implemented in a laser system. In this work, we have analytically determined the controllable conditions for notch filter feedback controlling of Chen chaotic system in terms of the Hopf bifurcation theory. The conditions for notch filter feedback controlled Chen chaoitc system having a stable limit cycle solution are given. Meanwhile, we also analysed the Hopf bifurcation direction, which is very important for parameter settings in notch filter feedback control applications. Finally, we apply the notch filter feedback control methods to the electronic circuit experiments and numerical simulations based on the theoretical analysis. The controlling results of notch filter feedback control method well prove the feasibility and reliability of the theoretical analysis.

Keywords: notch filter feedback control, Chen chaotic system, Hopf bifurcation
PACC: 0545, 0547

1. Introduction

During the past few years, many efforts have been devoted to the researches of chaos and spatiotemporal chaos control.[1−26] However, there are still some problems of applications and theories for controlling chaos that need to be further investigated. Recently, Ahlborn and Parlitz[1] proposed the notch filter feedback control (NFFC) scheme based on the time delay feedback control (i.e. the TDFC) method.[2] This method had been successfully applied in controlling chaos in a laser model. The NFFC signals are generated from the difference between the incoming signal (namely, original chaotic series) and output signal of the band-pass filter. If the controlling signal is transformed into the frequency domain, the frequency–amplitude characteristics in the central frequency domain and its neighbourhood show the notch-like distribution. The main idea of the NFFC method is to delete the components in the centre frequency domain and its neighbourhood and to control chaos into periodic or steady state with the other frequency components. Actually, the controlling signals of TDFC also possess the notch-like distributions in the frequency-amplitude characteristic domains. However, there are more than one notches in TDFC, namely, there are many notches in certain fundamental frequency and fractional frequency domains.[2,3,27−30] Although this notch filter controlling method can be applied in experiments in both low- and high-frequency systems, currently, there seems no analysis of the theoretically controllable parameter conditions having been reported.

In this work, we analyse the controllable conditions for the NFFC method theoretically. Actually, in some chaotic systems with notch filter method, whether they are capable to be controlled into a periodic state, can be analysed by the Hopf bifurcation theories. In the following sections, we adopt the NFFC method reported in Ref. [1] to control the Chen chaotic system. The controllable conditions for parameters are analytically determined, which provide a reliable basis for the applications of NFFC method. And we also apply this control scheme in circuit experiments and numerical simulations. All these control results confirm the theoretical analysis.

2. Controlling of Chen system

We consider the Chen chaotic system[31]

$$\dot{u}_1 = a(u_2 - u_1),$$

*Project supported by the National Natural Science Foundation of China (Grant Nos. 70571053, 10405018 and 10747147).
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$$\begin{aligned}\dot{u}_2 &= (c-a)u_1 + cu_2 - u_1u_3,\\ \dot{u}_3 &= u_1u_2 - bu_3,\end{aligned} \tag{1}$$

where $\dot{u}_i = \mathrm{d}u_i/\mathrm{d}t$ $(i = 1, 2, 3)$; a, b, and c are constants. Under the conditions of $a = 35$, $b = 3$, $c = 28$, Eqs. (1) have a chaotic solution. In order to apply the Chen model to the electronic circuit, taking into account the amplitude limit of realistic electronic components, we first perform the linear transformation as follows:

$$u_1 = \varepsilon_1 v_1, \quad u_2 = \varepsilon_2 v_2, \quad u_3 = \varepsilon_3 v_3. \tag{2}$$

Taking the parameters ε_i $(i = 1,\ 2,\ 3)$ properly, the terminal voltages of electronic components can be effectively constrained, which ensures all devices to work normally. In this paper, we let $\varepsilon_1 = \varepsilon_2 = 5$, $\varepsilon_3 = 10$. The electronic circuit corresponding to Eqs. (1) is shown in Fig. 1(a). The dynamical equations of this circuit with NFFC signal w_k are

$$\begin{aligned}\dot{v}_1 &= \frac{1}{RC}(av_2 - av_1),\\ \dot{v}_2 &= \frac{1}{RC}((c-a)v_1 + cv_2 - \varepsilon_3 v_1 v_3),\\ \dot{v}_3 &= \frac{1}{RC}\left(\frac{\varepsilon_1\varepsilon_2}{\varepsilon_3}v_1v_2 - bv_3 - w_k\right),\end{aligned} \tag{3}$$

in which $\dot{v}_i = \mathrm{d}v_i/\mathrm{d}t$ $(i = 1, 2, 3)$, $a = R/R_1$, $b = R/R_7$, $(a - c) = R/R_3$, $\varepsilon_3 = 0.1R/R_4$, $\varepsilon_1\varepsilon_2/\varepsilon_3 = 0.1R/R_6 = 2.5$. And we adopt the parameter sets $R_1 = R_2 = 2.86$ kΩ, $R_3 = 14.29$ kΩ, $R_4 = 1$ kΩ, $R_5 = 3.57$ kΩ, $R_6 = 4$ kΩ, $R_7 = 33.33$ kΩ, $R = 100$ kΩ, $C = 10$ nF.

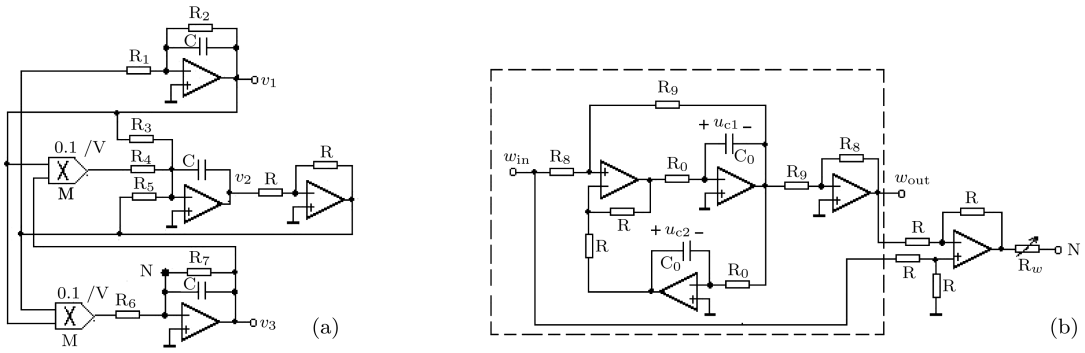


Fig. 1. Electronic circuits of Chen chaotic system and band-pass filter: (a) Electronic circuit for Chen chaotic system. The point 'N' denotes the signal entrance from the band-pass filter. (b) Electronic circuit of the band-pass filter, in which w_{in} and w_{out} are the input and output end-points, respectively. The point 'N' denotes the final output NFFC signals.

The general form of NFFC signal w_k can be described as

$$w_k = k(w_{\mathrm{in}} - w_{\mathrm{out}}), \tag{4}$$

where w_{in} is the time series of a certain state variable (i.e., v_1, v_2 or v_3) in Eqs. (3), which can be used as the incoming signals of the band-pass filter (see the dashed rectangular box in Fig. 1(b)); w_{out} is the output signals of the band-pass filter; k is the feedback gain factor. The parameter w_{out} varies according to the following differential equations

$$\begin{aligned}\dot{u}_{\mathrm{c1}} &= \frac{1}{R_0C_0}\left(\frac{2Q-1}{Q}w_{\mathrm{in}} - \frac{1}{Q}u_{\mathrm{c1}} - u_{\mathrm{c2}}\right),\\ \dot{u}_{\mathrm{c2}} &= \frac{1}{R_0C_0}u_{\mathrm{c1}},\\ w_{\mathrm{out}} &= \frac{1}{2Q-1}u_{\mathrm{c1}},\end{aligned} \tag{5}$$

where u_{c1} and u_{c2} are the capacitor voltages of the band-pass filter (see the dashed rectangular box in Fig. 1(b)), respectively. Q is the quality factor of the band-pass filter. In our case, we take $Q = (1 + R_9/R_8)/2$, $R_8 = 10$ kΩ, $R_9 = 36$ kΩ, $R_0 = 9.3$ kΩ and $C_0 = C = 10$ nF. The resistances R in both circuits (i.e., Figs. 1(a) and 1(b)) have the same value.

Now, taking v_3 as the incoming signals of the band-pass filter (i.e., let $w_{\mathrm{in}} = v_3$), we substitute Eqs. (4) and (5) into Eqs. (3). With the following variable substitution, $x_1 = v_1$, $x_2 = v_2$, $x_3 = v_3$, $x_4 = u_{\mathrm{c1}}/(2Q - 1)$, $x_5 = u_{\mathrm{c2}}/(2Q - 1)$, $t' = t/RC$, $\dot{x}_i = \mathrm{d}x_i/\mathrm{d}t'$ $(i = 1, 2, 3, 4, 5)$, we can obtain the notch-feedback controlled Chen system

$$\begin{aligned}\dot{x}_1 &= ax_2 - ax_1,\\ \dot{x}_2 &= (c-a)x_1 + cx_2 - \varepsilon_3 x_1x_3,\\ \dot{x}_3 &= \frac{\varepsilon_1\varepsilon_2}{\varepsilon_3}x_1x_2 - bx_3 - k(x_3 - x_4),\\ \dot{x}_4 &= \frac{R}{R_0}\left(\frac{1}{Q}x_3 - \frac{1}{Q}x_4 - x_5\right),\end{aligned}$$

$$\dot{x}_5 = \frac{R}{R_0} x_4. \tag{6}$$

In the following sections, we analyse the conditions for Eqs. (6) which have a stable periodic solution in terms of Hopf bifurcation theory, indicating that the Chen chaotic system (1) can be successfully controlled into a periodic state by using the NFFC method. And the corresponding Hopf bifurcation direction of Eqs. (6) can give the parameter modulation direction in NFFC applications. According to the NFFC method, the central frequency of band-pass filter should be equal (or approximated) to the period of target periodic orbit. So, with the method reported in Ref. [32], we first compute the elementary period of Eqs. (1) to be $T_1 = 0.5867$. Then, the central frequency of the band-pass filter can be taken as $f_r = 1/T_1$. Meanwhile, the parameters R_0 and C_0 can also be determined according to $f_r = (2\pi\sqrt{R_0 C_0})^{-1}$.

3. Theoretical analysis

3.1. The conditions for Hopf bifurcation

In this section, we analyse the Hopf bifurcation occurring conditions of the controlled system (6), which have three fixed points

$$\begin{aligned}
&\boldsymbol{O} = (0,\ 0,\ 0,\ 0,\ 0)^{\mathrm{T}},\\
&\boldsymbol{S}^{+} = (0.2S_{\mathrm{c}},\ 0.2S_{\mathrm{c}},\ 2.1,\ 0.0,\ 0.913)^{\mathrm{T}},\\
&\boldsymbol{S}^{-} = (-0.2S_{\mathrm{c}},\ -0.2S_{\mathrm{c}},\ 2.1,\ 0.0,\ 0.913)^{\mathrm{T}},
\end{aligned}$$

where $S_{\mathrm{c}} = \sqrt{63 + 21k}$ and T means the transposition for the state variable. Linearising Eqs. (6) around the fixed point $\boldsymbol{S}^{+}$, the Jacobian matrix is

$$\boldsymbol{J} = \begin{pmatrix}
-35 & 35 & 0 & 0 & 0\\
-28 & 28 & -2S_{\mathrm{c}} & 0 & 0\\
0.5S_{\mathrm{c}} & 0.5S_{\mathrm{c}} & -3-k & k & 0\\
0 & 0 & 0.4348 & -0.4348 & -1\\
0 & 0 & 0 & 1 & 0
\end{pmatrix}. \tag{7}$$

Then the characteristic equation of Eq. (7) is

$$\lambda^5 + a_1\lambda^4 + a_2\lambda^3 + a_3\lambda^2 + a_4\lambda + a_5 = 0, \tag{8}$$

where $a_1 = k + 10.44,\ a_2 = 28k + 89.5,\ a_3 = 4456.0 + 1480k,\ a_4 = 667.2k + 2002.0,\ a_5 = 4410.0 + 1470.0k$. Take $\lambda = \mathrm{i}\omega$ to be the simple pure imaginary root and substitute it into Eq. (8). After separating the real and imaginary parts, we can obtain

$$\omega = 35.696, \tag{9a}$$

$$k_{\mathrm{c}} = 43.179. \tag{9b}$$

Here k_{c} is the critical gain factor of the notch feedback control. Apart from the above two simple pure imaginary roots $\lambda_{1,2} = \pm 35.696\mathrm{i}$, all the other solutions to Eq. (8) have negative real parts

$$\lambda_{3,4} = -0.2179 \pm 0.9769\mathrm{i},\ \lambda_5 = -53.178. \tag{10}$$

Therefore, according to Eq. (8), we can also derive the transversality condition

$$\begin{aligned}
&\lambda'\\
&= \frac{\mathrm{d}\lambda}{\mathrm{d}k}\\
&= -\frac{\dfrac{\mathrm{d}a_1}{\mathrm{d}k}\lambda^4 + \dfrac{\mathrm{d}a_2}{\mathrm{d}k}\lambda^3 + \dfrac{\mathrm{d}a_3}{\mathrm{d}k}\lambda^2 + \dfrac{\mathrm{d}a_4}{\mathrm{d}k}\lambda + \dfrac{\mathrm{d}a_5}{\mathrm{d}k}}{5\lambda^4 + 4a_1\lambda^3 + 3a_2\lambda^2 + 2a_3\lambda + a_4},
\end{aligned} \tag{11}$$

and we have

$$\mathrm{Re}(\lambda_1'(k_{\mathrm{c}})) = \mathrm{Re}\left[\left(\frac{\mathrm{d}\lambda_1}{\mathrm{d}k}\right)_{\lambda=\mathrm{i}\omega,\ k=k_{\mathrm{c}}}\right] = -0.155 \neq 0.$$

Given the above results, the following conditions can be satisfied

(a) $\lambda_1(k_{\mathrm{c}}) = \overline{\lambda}_2(k_{\mathrm{c}}) = \mathrm{i}\omega$,

(b) $\mathrm{Im}(\lambda_1(k_{\mathrm{c}})) = \omega \neq 0$,

(c) $\mathrm{Re}(\lambda_1'(k_{\mathrm{c}})) \neq 0$,

(d) $\mathrm{Re}(\lambda_j(k_{\mathrm{c}})) < 0 \quad (j = 3,\ 4,\ 5)$.

According to the Hopf bifurcation theory,[33] when $k = k_{\mathrm{c}}$, a Hopf bifurcation occurs in the neighbourhood of the fixed point $\boldsymbol{S}^{+}$ in Eqs. (6). Similarly, we can also find the Hopf bifurcation near the other fixed point $\boldsymbol{S}^{-}$. It means that Eqs. (1) may be controlled by the NFFC scheme (i.e., Eqs. (4) and (5)) from chaotic state into a one-period state. In the following section, we shall analyse whether the limit cycle solution generated from the notch feedback controlled system (6) is stable and the bifurcating direction in the parameter space, using the normal form method.

3.2. Stability and direction of Hopf bifurcation

Let $k = k_{\mathrm{c}}$, we can solve the eigenvectors corresponding to the eigenvalues $\lambda_{1,2} = \pm\mathrm{i}\omega = \pm 35.696\mathrm{i}$, $\lambda_{3,4} = -0.2179 \pm 0.9769\mathrm{i},\ \lambda_5 = -53.178$, and con-

struct the transforming matrix

$$\boldsymbol{P} = \begin{pmatrix} 1 & 0 & 113.3 & -1.393 & -5729.0 \\ 0.9997 & -1.020 & 104.3 & -1.385 & 2976 \\ 0.5844 & 0.1146 & 1 & -0.0014 & 6454 \\ -0.0013 & 0.007 & -77.41 & 1 & -53.18 \\ 0 & 0 & -192.4 & -0.2175 & 1 \end{pmatrix}. \tag{12}$$

For the fixed point $\boldsymbol{S}^+$, we apply the following transformation

$$\boldsymbol{x} = \boldsymbol{x}_0(k_\mathrm{c}) + \boldsymbol{P} \cdot \boldsymbol{y} \tag{13}$$

with $\boldsymbol{x} \in \mathbb{R}^5$, $\boldsymbol{y} \in \mathbb{R}^5$ and $\boldsymbol{x}_0(k_\mathrm{c}) = \boldsymbol{S}^+(k_\mathrm{c})$. Substituting Eq. (13) into Eqs. (6), we have

$$\dot{\boldsymbol{y}} = \boldsymbol{A} \cdot \boldsymbol{y} + \boldsymbol{F}(\boldsymbol{y}), \tag{14}$$

where

$$\boldsymbol{A} = \begin{pmatrix} 0 & -35.69 & 0 & 0 & 0 \\ 35.69 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.1345 & -0.004553 & 0.002 \\ 0 & 0 & 236.9 & -0.5704 & 0 \\ 0 & 0 & -0.0001 & 0 & -53.18 \end{pmatrix},$$

and

$$\boldsymbol{F}(\boldsymbol{y}) = (F^1 \quad F^2 \quad F^3 \quad F^4 \quad F^5)^\mathrm{T},$$

in which

$$\begin{aligned}
F^1 &= -0.03 + 0.9395y_1^2 - 1.431y_1y_2 + 244.3y_1y_3 - 3.147y_1y_4 - 5718.0y_1y_5 - 162.1y_3y_2 + 15620.0y_3^2 \\
&\quad - 400.3y_3y_4 - 827900.0y_3y_5 + 1.993y_4y_2 + 2.561y_4^2 + 11000.0y_4y_5 + 8197.0y_5y_2 + 1930000.0y_5^2, \\
F^2 &= 7.214y_1^2 - 0.992y_1y_2 + 1033.0y_1y_3 - 12.80y_1y_4 + 22390.0y_1y_5 - 112.5y_3y_2 + 24400.0y_3^2 - 612.0y_3y_4 \\
&\quad + 5.985 \times 10^6 y_3y_5 + 1.382y_4y_2 - 72970.0y_4y_5 + 5688.0y_5y_2 - 364900000.0y_5^2 + 3.834y_4^2, \\
F^3 &= 0.00004131y_1^2 + 0.000008650y_1y_2 + 0.004703y_1y_3 - 5.701 \times 10^{-5} y_1y_4 + 0.2231y_1y_5 + 0.0009798y_3y_2 \\
&\quad + 0.002738y_3^2 + 0.0000262y_3y_4 + 51.96y_3y_5 - 7.357 \times 10^{-7} y_4^2 - 0.6435y_4y_5 - 2634.0y_5^2 \\
&\quad - 0.00001204y_4y_2 - 0.04956y_5y_2, \\
F^4 &= -0.03782y_1^2 - 0.007309y_1y_2 + 0.05290y_1y_4 - 200.3y_1y_5 - 0.8278y_3y_2 - 8.360y_3^2 + 0.1268y_3y_4 \\
&\quad - 46820.0y_3y_5 + 0.01017y_4y_2 - 0.0002958y_4^2 + 2389000.0y_5^2 + 579.6y_4y_5 - 4.358y_1y_3 + 41.86y_5y_2, \\
F^5 &= -2 \times 10^{-6} + 0.0001740y_1^2 - 0.0002478y_1y_2 + 0.04381y_1y_3 - 0.0005635y_1y_4 - 0.9452y_1y_5 \\
&\quad - 0.06993y_3y_4 + 0.0003453y_4y_2 + 0.0004471y_4^2 - 132.1y_3y_5 + 1.767y_4y_5 + 1.420y_5y_2 - 297.0y_5^2 \\
&\quad - 0.02809y_3y_2 + 2.730y_3^2.
\end{aligned}$$

According to the Hopf bifurcation theory,[33] we have

$$\begin{aligned}
g_{11} &= [F^1_{y_1y_1} + F^1_{y_2y_2} + \mathrm{i}(F^2_{y_1y_1} + F^2_{y_2y_2})]/4 \\
&= 0.4698 + 3.608\mathrm{i}, \\
g_{02} &= \frac{1}{4}\begin{bmatrix} F^1_{y_1y_1} - F^1_{y_2y_2} - 2F^2_{y_1y_2} + \\ \mathrm{i}(F^2_{y_1y_1} - F^2_{y_2y_2} + 2F^1_{y_1y_2}) \end{bmatrix} \\
&= 0.9658 + 2.892\mathrm{i}, \\
g_{20} &= \frac{1}{4}\begin{bmatrix} F^1_{y_1y_1} - F^1_{y_2y_2} + 2F^2_{y_1y_2} + \\ \mathrm{i}(F^2_{y_1y_1} - F^2_{y_2y_2} - 2F^1_{y_1y_2}) \end{bmatrix} \\
&= -0.0262 + 4.322\mathrm{i}, \\
G_{21} &= \frac{1}{8}\begin{bmatrix} F^1_{y_1y_1y_1} + F^1_{y_1y_2y_2} + F^2_{y_1y_1y_2} + F^2_{y_2y_2y_2} + \\ +\mathrm{i}(F^2_{y_1y_1y_1} + F^2_{y_1y_2y_2} - F^1_{y_1y_1y_2} - F^1_{y_2y_2y_2}) \end{bmatrix} \\
&= 0.0, \\
h^1_{11} &= (F^3_{y_1y_1} + F^3_{y_2y_2})/4 = 0.00002066, \\
h^2_{11} &= (F^4_{y_1y_1} + F^4_{y_2y_2})/4 = -0.01891, \\
h^3_{11} &= (F^5_{y_1y_1} + F^5_{y_2y_2})/4 = 0.000087, \\
h^1_{20} &= (F^3_{y_1y_1} - F^3_{y_2y_2} - 2\mathrm{i}F^3_{y_1y_2})/4 \\
&= 0.00002066 - 0.000004325\mathrm{i}, \\
h^2_{20} &= (F^4_{y_1y_1} - F^4_{y_2y_2} - 2\mathrm{i}F^4_{y_1y_2})/4 \\
&= -0.01891 + 0.003654\mathrm{i}, \\
h^3_{20} &= (F^5_{y_1y_1} - F^5_{y_2y_2} - 2\mathrm{i}F^5_{y_1y_2})/4 \\
&= 0.000087 + 0.0001239\mathrm{i}.
\end{aligned}$$

Then solve the linear systems

$$\boldsymbol{D}\cdot\boldsymbol{w}_{11}=-\boldsymbol{h}_{11},$$
$$(\boldsymbol{D}-2\mathrm{i}\omega_0)\cdot\boldsymbol{w}_{20}=-\boldsymbol{h}_{20},$$

where

$$\boldsymbol{D}=\begin{pmatrix}0.1345 & -0.004553 & 0.002\\ 236.9 & -0.5704 & 0\\ -0.0001 & 0 & -53.18\end{pmatrix}.$$

We thus have vectors

$$\boldsymbol{w}_{11}=\begin{pmatrix}0.0000977\\ 0.007425\\ 0.000001636\end{pmatrix},$$

$$\boldsymbol{w}_{20}=\begin{pmatrix}0\\ 0.00004811+0.0002655\mathrm{i}\\ 0.0000017\end{pmatrix}.$$

With

$$\begin{aligned}
G^1_{110} &= [F^1_{y_1y_3}+F^2_{y_2y_3}+\mathrm{i}(F^2_{y_1y_3}-F^1_{y_2y_3})]/2\\
&= 65.95+597.5\mathrm{i},\\
G^2_{110} &= [F^1_{y_1y_4}+F^2_{y_2y_4}+\mathrm{i}(F^2_{y_1y_4}-F^1_{y_2y_4})]/2\\
&= -0.8830-7.395\mathrm{i},\\
G^2_{110} &= [F^1_{y_1y_4}+F^2_{y_2y_4}+\mathrm{i}(F^2_{y_1y_4}-F^1_{y_2y_4})]/2\\
&= -15+7095\mathrm{i},\\
G^1_{101} &= [F^1_{y_1y_3}-F^2_{y_2y_3}+\mathrm{i}(F^1_{y_2y_3}+F^2_{y_1y_3})]/2\\
&= 178.4+435.4\mathrm{i},\\
G^2_{101} &= [F^1_{y_1y_4}-F^2_{y_2y_4}+\mathrm{i}(F^1_{y_2y_4}+F^2_{y_1y_4})]/2\\
&= -2.265-5.405\mathrm{i},\\
G^3_{101} &= [F^1_{y_1y_5}-F^2_{y_2y_5}+\mathrm{i}(F^1_{y_2y_5}+F^2_{y_1y_5})]/2\\
&= -5703+15300\mathrm{i},\\
g_{21} &= G_{21}+\sum_{k=1}^{3}2G^k_{110}w^k_{11}+G^k_{101}w^k_{20}\\
&= -0.009252+0.05501\mathrm{i},
\end{aligned}$$

we obtain

$$\begin{aligned}
c_1(0) &= \frac{\mathrm{i}}{2\omega_0}\left(g_{20}g_{11}-2\left|g_{11}\right|^2-\frac{1}{3}\left|g_{02}\right|^2\right)+\frac{g_{21}}{2}\\
&= -0.03175-0.6055\mathrm{i}, \qquad (15)
\end{aligned}$$

and

$$\mu_2 = -\mathrm{Re}(c_1(0))/\alpha'(0)=-0.2046, \qquad (16a)$$
$$\beta_2 = 2\mathrm{Re}(c_1(0))=-0.0635, \qquad (16b)$$
$$T = T_0(1+\tau_2\varepsilon^2+O(\varepsilon^4)), \qquad (16c)$$

where

$$T_0=\frac{2\pi}{\omega_0}=0.176,$$
$$\tau_2=-(\mathrm{Im}(c_1(0))+\mu_2\omega'(0))/\omega_0=0.01785,$$
$$\alpha'(0)=\mathrm{Re}(\lambda'_1(k_\mathrm{c}))=-0.155,$$
$$\omega'(0)=\mathrm{Im}(\lambda'_1(k_\mathrm{c}))=0.155,$$
$$\varepsilon^2=\frac{k-k_\mathrm{c}}{\mu_2}+O(k-k_\mathrm{c})^2.$$

According to the general theorem in Ref. [33], if $\beta_2<0$ and $\mu_2<0$, the bifurcating periodic solution is asymptotically stable and the direction of Hopf bifurcation is along the direction of $k<k_\mathrm{c}$. It means that Eqs. (1) can be successfully controlled by the notch feedback control scheme, i.e. Eqs. (4) and (5) when $k<k_\mathrm{c}$. And T is the period of the stable limit cycle solution in controlled system (6). Actually, according to the Hopf bifurcation theory, the above results are valid only when the gain factor k is in the neighbourhood of k_c. However, when the gain factor k is changed along the Hopf bifurcation direction, even if k is far from k_c, we can also find the conditions for Eqs. (6) to have a stable periodic solution. Through numerical simulations, we find that when $k\in(2.025,\ 39.15)\cup(41.175,\ 43.16)$, Eqs. (6) has a stable one-periodic solution. It means that Chen chaotic system can be stabilised into one periodic orbit with NFFC method, as the gain factor $k\in(2.025,\ 39.15)\cup(41.175,\ 43.16)$.

4. Experimental applications and numerical simulations of NFFC method

The above analysis for Hopf bifurcation occurrence conditions, stability and direction of the notch-feedback controlled system provides the reliable parameter conditions for the application of electronic circuit experiments and the numerical simulations. Actually, besides the neighbourhood of the critical gain factor k_c, we can also find more controllable parameter sets along the Hopf bifurcation direction, which present a more wider controllable parameter region. In this section, we shall use some parameter sets and employ the notch feedback control scheme in electronic devices according to the above results. At the same time, the numerical simulations will also be used to check the circuit experiments.

In the electronic circuit experiments, we take v_3 in Eq. (3) as the incoming signals of the band-pass filter,

which means to connect the end-point v_3 in Fig. 1(a) with $w_{\rm in}$ in Fig. 1(b). The output signals will be fed back into the original Chen chaotic circuit (i.e., connecting the two 'N' points in Figs. 1(a) and 1(b)). We take the central frequency and quality factor as $f_0 = 1/T_1 = 1.7$ kHz and $Q = 2.3$, respectively. And the gain factor k is taken as $k = R/R_w$, in which R_w is a variable resistance used to change the gain factor k. In the circuit experiment, we use the electronic oscilloscope to observe and display the chaos controlling results.

Keeping the Chen circuit system evolving freely (i.e., taking the feedback gain factor $k = 0$), we can observe the Chen chaotic attractor as shown in Fig. 2(a). Meanwhile, Fig. 2(d) also demonstrates the Chen chaotic solution, with numerical simulations, which is the same as the circuit experiment results. Turn on the notch feedback control device, i.e., connect both 'N' end-points in Figs. 1(a) and 1(b). We set the gain factor k in the range of $k \in (2.025,\ 39.15) \cup (41.175,\ 43.16)$. Here, selecting the variable resistance to be $R_w = 2.38$ kΩ (or $R_w = 3.33$ kΩ), which corresponds to $k = 42$ (or $k = 30$), we can find one-period solutions in both cases (see Figs. 2(b) and 2(c)). In order to check the control results of our circuit experiments, we also compute numerically the notch-feedback controlled system (6) when $k = 42$ and 30. The numerical results are demonstrated in Figs. 2(e) and 2(f). From Figs. 2(b) and 2(c), and Figs. 2(e) and 2(f), we can see that the circuit experiment results coincide with the numerical simulations very well.

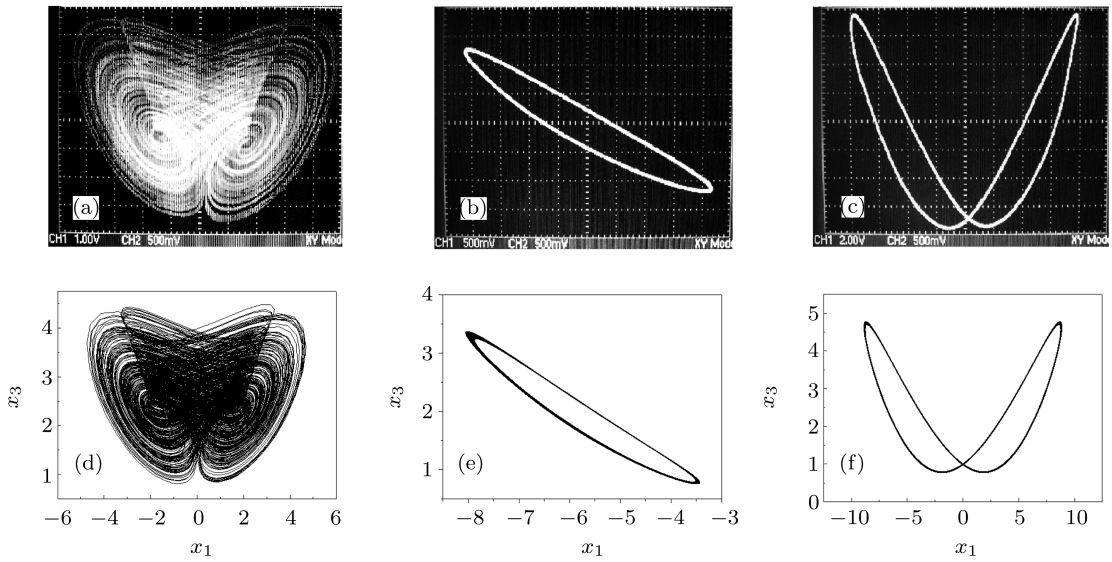


Fig. 2. Electronic circuit experiments and numerical simulation results of NFFC method. (a)–(c) Electronic circuit experiment results: (a) Chen chaotic attractor as $k = 0$ in Eqs. (6) (i.e., turning off the NFFC signal); (b) the periodic solution for the gain factor $k = 42$ (i.e., $R_w = 2.38$ kΩ); (c) the periodic solution for the gain factor $k = 30$ (i.e., $R_w = 3.33$ kΩ). (d)–(f) Numerical simulation results: (d) Chen chaotic attractor for $k = 0$ in Eqs. (6); (e) the periodic solution for the gain factor $k = 42$; (f) the periodic solution for the gain factor $k = 30$.

5. Conclusion

In this work, we analytically determine the controllable conditions of controlling Chen chaotic system by the notch feedback control method according to the Hopf bifurcation theory. Actually, when we gain the conditions for the original chaotic system to be controlled into a periodic state, the conditions for the Chen system to be confined to a stationary state can also be determined. This is because Hopf bifurcation results from the destabilisation of fixed point and results in the emergence of a stable periodic limit cycle. In our case, when $k > 43.179$, the Chen system can be controlled into a fixed point (either $\boldsymbol{S}^+$ or $\boldsymbol{S}^-$, which is determined by the initial conditions).

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