Generalised synchronisation of spatiotemporal chaos using feedback control method and phase compression*

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Coupled map lattices are taken as examples to study the synchronisation of spatiotemporal chaotic systems. First, a generalised synchronisation of two coupled map lattices is realised through selecting an appropriate feedback function and appropriate range of feedback parameter. Based on this method we use the phase compression method to extend the range of the parameter. So, we integrate the feedback control method with the phase compression method to implement the generalised synchronisation and obtain an exact range of feedback parameter. This technique is simple to implement in practice. Numerical simulations show the effectiveness and the feasibility of the proposed program.

Keywords: coupled map lattice, generalised synchronisation, feedback control, phase compression

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1. Introduction

In 1990, Pecora and Carroll presented the conception of 'chaotic synchronisation'[1] and introduced a method to synchronise two identical chaotic systems with different initial conditions.[2] Since then, the chaos synchronisation has become a topic of great interest. A variety of synchronisation methods have been mentioned.[3,4] For example, Wang *et al.* studied the chaos synchronisation via unidirectional coupling in Ref. [5], and Zhang *et al.* solved the problem of master–slave fault tolerant synchronisation for a class of chaotic system with multiple time-delays in Ref. [6]. But in the recent years, people paid more and more attention to spatiotemporal chaos. There are many models for dealing with spatiotemporal chaos. Coupled map lattice is a kind of efficient model for researching spatiotemporal chaos.[7,8] Coupled map lattice is a discretization of time and space but its status remained under way.[9] Now, the researches of the spatiotemporal chaotic synchronisation based on coupled map lattices has made some progress.[10−15] For example, Liu *et al.* used the nonlinear coupling method to implement the synchronisation of the coupled map lattice in Ref. [14]. Lü *et al.* realised the synchronisation of two unilateral coupled map lattices through the appropriate choice of driving functions in Ref. [15]. But in this area, the using of the feedback control method to implement synchronisation is comparatively lacking. In this paper, we give a simple scheme to synchronise spatiotemporal chaotic systems based on feedback control and phase compression. We also give a range of feedback parameters. In comparison with all methods mentioned above, this technique, in principle, is simple to implement in practice. Numerical simulations show the effectiveness and the feasibility of the proposed program.

2. Feedback control method and numerical simulations

The CML model is mentioned first by Kaneko,[16] it can be described by a dynamics equation

$$x_{n+1}(i) = (1-\varepsilon)f(x_n(i)) + \frac{\varepsilon}{2}(f(x_n(i-1)) + f(x_n(i+1))), \quad (1)$$

where n is the discrete time step, $i(i = 1, 2, \ldots, L)$ is a lattice point, L is the lattice number, $x_n(i)$ is the state variable, and $f(x_n(i))$ is the mapping function that is chosen to be the logistic map and expressed as

$$f(x_n(i)) = 1 - ax_n^2(i), \quad (2)$$

with parameters a and ε representing coupled intensity and nonlinear intensity, respectively. Thus Eq. (1)

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may be represented as

$$x_{n+1}(i) = (1-\varepsilon)(1-ax_n^2(i)) + \frac{\varepsilon}{2}((1-ax_n^2(i-1)) + (1-ax_n^2(i+1))). \quad (3)$$

When the parameter values are $a = 1.8$ and $\varepsilon = 0.1$, system (3) is in the defect chaotic diffusion pattern mentioned in Ref. [17]. In the present paper, we do the research based on this pattern. Figure 1 shows the bifurcation of system (2). Axis a and $x_n(i)$ represent coupled intensity and state variable, respectively. From Fig. 1, we see that the system has a chaotic trend. Now we look at system (3). The initial values of $x_n(i)$ are random numbers between –1 and 1. We implement 200 iteration steps for the 30 lattice sites, after having eliminated 2000 iteration steps. The projections of the spatiotemporal chaotic attractor of the uncontrolled system (3) are shown in Fig. 2. Axes i, n and $x_n(i)$ represent lattice number, iteration number and state variable, respectively.

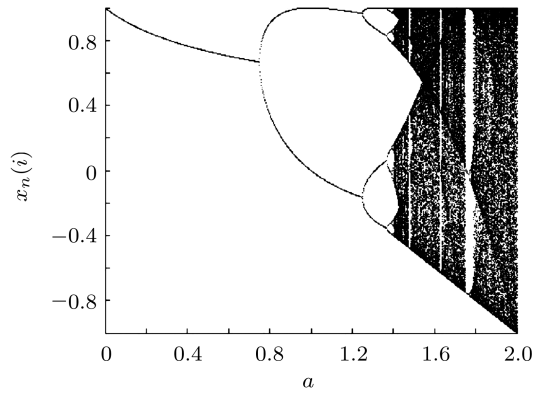


Fig. 1. Bifurcation for logistic map.

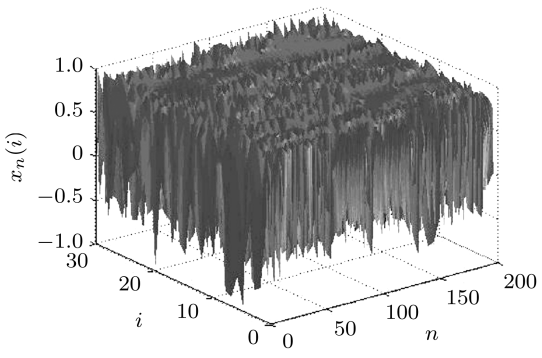


Fig. 2. Spatiotemporal evolution plot.

Construct another independent spatiotemporal chaotic system which has the same structure as system (1), then the equation will be as follows:

$$y_{n+1}(i) = (1-\varepsilon)f(y_n(i)) + \frac{\varepsilon}{2}(f(y_n(i-1)) + f(y_n(i+1))). \quad (4)$$

The error system is represented as

$$e_n(i) = y_n(i) - x_n(i). \quad (5)$$

According to Eqs. (3) and (4), the evolvement of the error equation is as follows:

$$\begin{aligned} & e_{n+1}(i) \\ &= (1-\varepsilon)f'(y_n(i)-x_n(i))e_n(i) \\ &\quad + \frac{\varepsilon}{2}[(f'(y_n(i-1)-x_n(i-1))e_n(i-1)) \\ &\quad + (f'(y_n(i+1)-x_n(i+1))e_n(i+1))]. \end{aligned} \quad (6)$$

The feedback control variable W added to the error system is defined as follows:

$$W = -(1-\varepsilon)f'(y_n(i)-x_n(i))e_n(i)U_n(i) + M, \quad (7)$$

where M is defined as

$$M = re_n(i) - \frac{1}{5}re_{n-1}(i) - a(1-\varepsilon)e_n^2(i) - \frac{1}{5}re_{n-1}(i-1) - \frac{1}{5}re_{n-1}(i+1), \quad (8)$$

where r is the feedback control parameter.

Add the feedback control to Eq. (6), then Eq. (6) will change into

$$\begin{aligned} & e_{n+1}(i) \\ &= (1-\varepsilon)f'(y_n(i)-x_n(i))e_n(i) \\ &\quad + \frac{\varepsilon}{2}[(f'(y_n(i-1)-x_n(i-1))e_n(i-1)) \\ &\quad + (f'(y_n(i+1)-x_n(i+1))e_n(i+1))] \\ &\quad - (1-\varepsilon)f'(y_n(i)-x_n(i))e_n(i)U_n(i) + M, \end{aligned} \quad (9)$$

where in order to prevent the system from diverging after having been added by the feedback control, we restrict the feedback term by defining $U_n(i)$ as

$$U_n(i) = \begin{cases} 1, & |y_n(i)-x_n(i)| < \upsilon, \\ 0, & |y_n(i)-x_n(i)| > \upsilon, \end{cases}$$

where υ has a value between –1 and 1. When the difference between $y_n(i)$ and $x_n(i)$ is in $(-\upsilon, \upsilon)$, $e_{n+1}(i)$ is determined by the following equation next:

$$\begin{aligned} & e_{n+1}(i) \\ &= \frac{\varepsilon}{2}[(f'(y_n(i-1)-x_n(i-1))e_n(i-1)) \\ &\quad + (f'(y_n(i+1)-x_n(i+1))e_n(i+1))] + M, \end{aligned} \quad (10)$$

where

$$f'(y_n(i-1)-x_n(i-1)) = -ae_n(i-1), \quad (11)$$

$$f'(y_n(i+1) - x_n(i+1)) = -ae_n(i+1). \quad (12)$$

Combine Eqs. (8), (11), (12) and (10) yields

$$\begin{aligned} e_{n+1}(i) = {} & re_n(i) - \frac{1}{5}re_{n-1}(i) - a(1-\varepsilon)e_n^2(i) \\ & - \frac{1}{5}re_{n-1}(i-1) - \frac{1}{5}re_{n-1}(i+1) \\ & - \frac{a\varepsilon}{2}e_n^2(i+1) - \frac{a\varepsilon}{2}e_n^2(i-1). \end{aligned} \quad (13)$$

Now, we have a final equation about the error system. Next, we will perform the numerical simulation and find out a range of the feedback parameters. Analysing the error $e_{n+1}(i)$, we know that the error is determined by a range of r. As to the error system, we implement 200 iteration steps for the 30 lattice sites, after having eliminated 2000 iteration steps. We add the control to the error system at the 50th step. Figure 3 shows different spatiotemporal chaotic behaviours for different values of the feedback control parameter, with axes i, n and $e_n(i)$ representing the lattice number, the iteration number and the error system result, respectively.

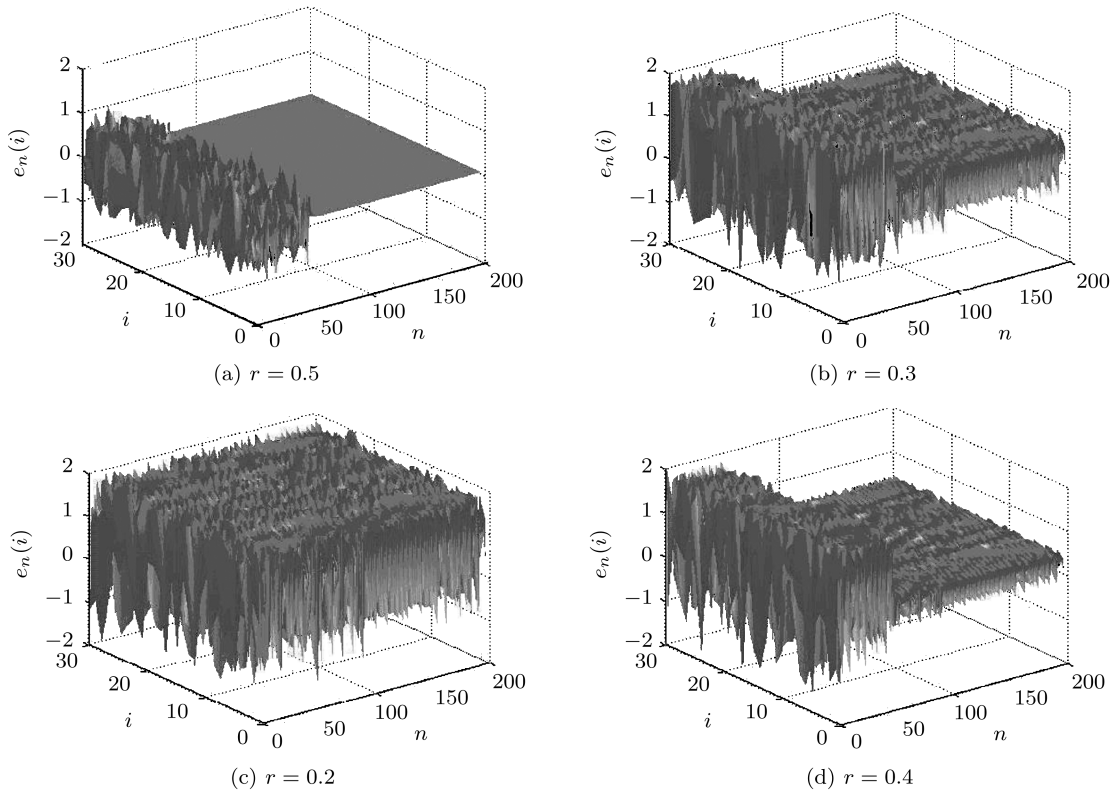


Fig. 3. Spatiotemporal evolution plots for the error system with different feedback control parameters.

From the simulation results obtained through a great number of iteration steps, the conclusions are drawn as follows.

(1) When $r > 0.55$ or $r < 0.15$, the calculation of the error system is overflowing and we cannot obtain the true image.

(2) When $r \in (0.45, 0.55)$, the error system tends to be stabilised after a time, which means that the two spatiotemporal chaotic systems are synchronous just as shown in Fig. 3(a).

(3) When $0.15 \leq r < 0.45$, from the numerical experiments, the space range of the chaotic behaviour becomes smaller but it is still spatiotemporally chaotic as shown in Fig. 3(b). And when $0.15 \leq r < 0.45$, if r is close to 0.45, the error system tend to be stable; on the other hand, if r is close to 0.15, the chaotic behaviour of the error system is obvious as indicated in Figs. 3(c) and 3(d).

In order to prove that the range of parameter r is reasonable, Fig. 4 shows the error system evolution with r value. From the figure, it is obvious that only when $r \in (0.45, 0.55)$, will the error system tend to be stabilised.

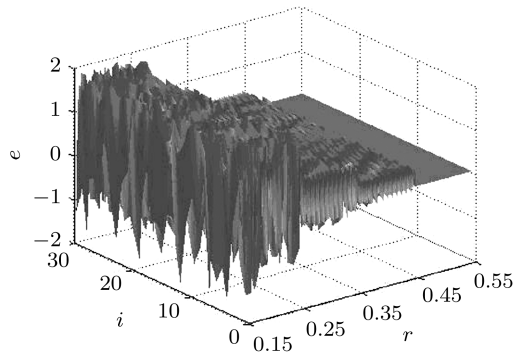


Fig. 4. Spatiotemporal evolution plot for the error system with r value.

3. Phase compression method

Next, we discuss how to reduce the qualification of generalised synchronisation and extend the range of feedback parameter r. According to the numerical experiments, we can implement the synchronisation by using the phase compression method when $r \in (0.15, 0.55)$.

The track of the chaotic system is a geometry graphic which continuously shrinks and expands in the compression process. Perform the phase compression on the error system and restrict its free-shrinking and free-expanding, then the state of the error system will change. Through the numeric experiments, we know the phase compression can change the instability error system to a fixed value.

Using phase compression and limiting the state variable to a certain area, from the above section, we know that the track of error system is in a limited space,A; next choose a subset of A, named B, ($B \subset A$) and limit the error system in B. The phase compression of error system is described as

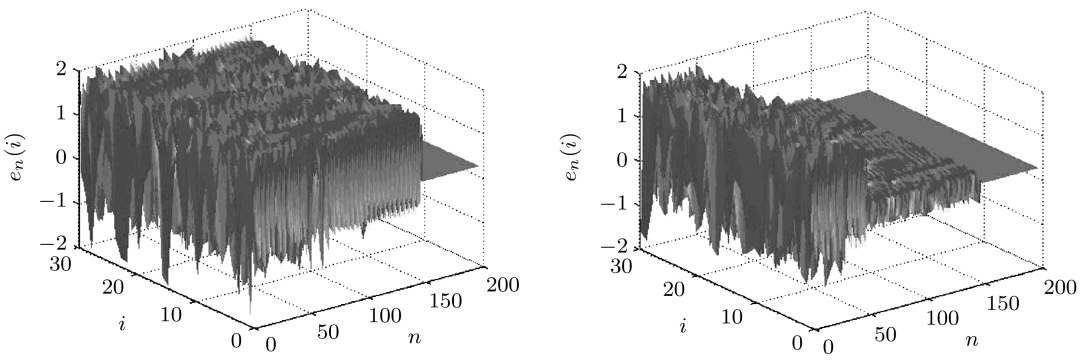
$$e = \begin{cases} e_{\max}, & e > e_{\max} \\ e, & e_{\min} < e < e_{\max} \\ e_{\min}, & e < e_{\min} \end{cases},$$

where $e_{\max}, e_{\min} \subset B$.

In the above section, we know that the value of the error system has a range of (–2, 2), when we limit the value of error system to a range of (–0.5, 0.5) at the 150th step we obtain a new discovery. We know that when the value of r is 0.2 or 0.4, the value of the error system cannot approach to a fixed value. But now using the phase compression method we obtain the chaotic attractor of error system as shown in Fig. 5. From the images, it is obvious that the error systems approach to a fixed value.

In this section, if we give a limited value $(-x, x)$ and an exact range of the limited parameter x, it will be better. But we have not enough evidence now. This respect should be improved.

From a comparison between Fig. 3(c) and Fig. 5(a) it follows that the error system approaches to a fixed value obtained by using phase compression method after 150 iteration steps effectively. From the numerical simulations in the above section, if we use the feedback control method only, when $r \in (0.45, 0.55)$ the error system approaches to a fixed value but when $0.15 \leq r < 0.45$, the error system is still chaotic. But in this section, we propose that when $r \in (0.15, 0.55)$ the error system approaches to a fixed value by entirely using the phase compression method based on the feedback control. Figure 6 proves our scheme.



(a) $r = 0.2$ (b) $r = 0.4$

Fig. 5. Spatiotemporal evolutions for the error system, obtained by phase compression method.

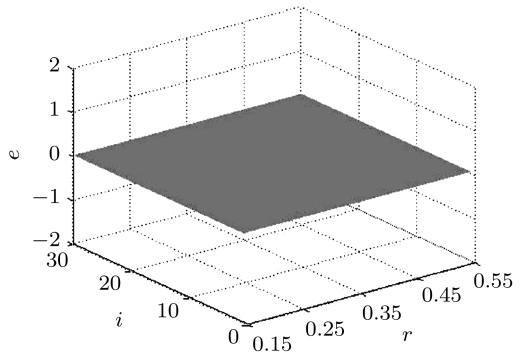


Fig. 6. Spatiotemporal evolutions for the error system under different values of r, obtained by phase compression method.

4. Conclusion

In this paper, we propose a mode to solve synchronisation problems. Generalised synchronisation of spatiotemporal chaos based on coupled map lattices is realised by using feedback control method, choosing an appropriate feedback control term and an appropriate range of the feedback control parameter. On the other hand, using phase compression, generalised synchronisation can be implemented more easily and the range of feedback parameters can be extended. Numerical simulation results show that the proposed method can be successfully used to achieve the synchronisation.

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