Radiation energy flux of Dirac field of static spherically symmetric black holes*

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By the statistical entropy of the Dirac field of the static spherically symmetric black hole, the result is obtained that the radiation energy flux of the black hole is proportional to the quartic of the temperature of its event horizon. That is, the thermal radiation of the black hole always satisfies the generalised Stenfan–Boltzmann law. The derived generalised Stenfan–Boltzmann coefficient is no longer a constant. When the cut-off distance and the thin film thickness are both fixed, it is a proportional coefficient related to the space–time metric near the event horizon and the average radial effusion velocity of the radiation particles from the thin film. Finally, the radiation energy fluxes and the radiation powers of the Schwarzschild black hole and the Reissner–Nordström black hole are derived, separately.

Keywords: static spherically symmetric black hole, thin film model, generalised Stenfan–Boltzmann law, radiation energy flux

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1. Introduction

Since the pioneering work done by Bekenstein[1] and Hawking,[2] the thermal dynamics of the black hole has been a most concerned research area. The brick-wall model proposed by 't Hooft gave an explanation for the statistical origin of the black hole entropy.[3] By the brick-wall model, the entropies of the static and the stationary black holes are calculated out separately, and the results are obtained that the entropy of the black hole is proportional to the area of its event horizon. Because the Hawking radiation originates from the vacuum fluctuation near the event horizon, the brick-wall model has been developed into the thin film model.[4−7] According to this thin film model, the entropy of the black hole comes from the contribution of quantum field in an infinitesimal thin film near its event horizon. Many researchers have adopted the thin film model to calculate the black hole entropy, and obtained the same result that black hole entropy is proportional to the area of the event horizon.[8−12] Where the event horizon is, there are black hole entropy and Hawking radiation.[13] However, the Hawking radiation spectrum is precisely thermal, which means that the quantum pure state in the radiation process will be evolved into the thermal mixed states. This result is contrary to the unitary principle of quantum mechanics. In 2000, considering the self-gravitation action of the radiation particles and regarding the Hawking radiation as a quantum tunnelling process, Parikh and Wilczek[14] obtained that the quantum tunnelling rate of the radiation particles near the event horizon is related to the change of the Bekenstein–Hawking entropy, and that the radiation spectrum deviates from the pure thermal spectrum and the unitary principle can be satisfied. Then, some authors have obtained the same result as that of Parikh.[15−23] It is shown that there must be an intrinsic relation between the entropy and the thermal radiation of the black hole. Therefore, it is of significance to further study this intrinsic relation.[24] Recently, we have studied the thermal radiation of the black hole by the entropy density near its event horizon, and found that the thermal radiation of the black hole satisfies the generalised Stenfan–Boltzmann law.[25−34] In order to make these results more universally significant, we study the radiation energy flux and the radiation power of the static spherically symmetric black hole, and reveal the thermal radiation law of the black hole in curved space–time.

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2. Generalised Stenfan–Boltzmann law of Dirac field of static spherically symmetric black hole

The line element of the non-extreme static spherically symmetric black hole is given by[35]

$$\mathrm{d}s^2 = g_{00}\mathrm{d}t^2 + g_{11}\mathrm{d}r^2 - r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2\right), \quad (1)$$

where $g_{00} = -g_{11}^{-1}$, the space–time is spherically symmetric, $g_{00} = 0$ is the equation of the event horizon, so

$$g_{00} = f(r)(r - r_{\mathrm{H}}), \quad (2)$$

where r_{H} is the radius of the event horizon. By the thin film model, the statistical entropy of the Dirac field of the static spherically symmetric black hole is given by[35]

$$S = \frac{7\pi^2\delta A_{\mathrm{H}}}{45\beta_{\mathrm{H}}^3\varepsilon(\varepsilon+\delta)f_{\mathrm{H}}^2}, \quad (3)$$

where $\beta_{\mathrm{H}} = 1/T$, $A_{\mathrm{H}} = 4\pi r_{\mathrm{H}}^2$ is the area of the event horizon, ε is the cut-off distance, δ is the thickness of the thin film, and $f_{\mathrm{H}}^2 = f^2(r)\big|_{r=r_{\mathrm{H}}}$. From Eq. (2), one infers that f_{H} is a function related to the space–time metric near the event horizon. Let $\delta = n\varepsilon$, one can obtain from Eq. (3)

$$S_n = \frac{7\pi^2 A_{\mathrm{H}}}{45\varepsilon f_{\mathrm{H}}^2}\frac{n}{n+1}T^3. \quad (4)$$

When $n \to \infty$, the corresponding entropy can be denoted by S_∞. From Eq. (4), one can obtain $S_n = (n/n+1)S_\infty$. If the cut-off distance is set as the Planck length (the same as the 't Hooft intrinsic thickness), that is, $\varepsilon = l_p$, and when $\delta = 10l_p$, the derived entropy ($S_{10} = \frac{10}{11}S_\infty$) is approximately equal to the whole entropy of the black hole. Therefore, the entropy of the black hole comes mainly from the contribution of quantum field in an infinitesimal thin film near its event horizon. According to the thin film model, the entropy density of the Dirac field in the thin film near the event horizon can be obtained from Eq. (3) as

$$s = \frac{S}{V} = \frac{7\pi^2}{45\varepsilon(\varepsilon+\delta)f_{\mathrm{H}}^2\sqrt{g_{11}}\big|_{r_{\mathrm{H}}+\varepsilon}}T^3, \quad (5)$$

where V is the volume of the thin film. Since the value of r_{H} is much greater than those of ε and δ, the volume of the thin film can be adopted as $V = A_{\mathrm{H}}\Delta l_r = A_{\mathrm{H}}\sqrt{g_{11}}\big|_{r_{\mathrm{H}}+\varepsilon}\delta$.

Based on the local equivalence principle, in the local area of the infinitesimal thin film near the event horizon, the basic thermodynamic equations still hold. The energy density ρ, the entropy density s in the thin film near the event horizon, and the temperature in local area T satisfy the following relations:[25]

$$\rho = bT^4, \quad (6)$$

$$s = \frac{4}{3}bT^3. \quad (7)$$

Combining Eqs. (5) and (7), one can obtain

$$b = \frac{7\pi^2}{60\varepsilon(\varepsilon+\delta)f_{\mathrm{H}}^2\sqrt{g_{11}}\big|_{r_{\mathrm{H}}+\varepsilon}}. \quad (8)$$

Substituting Eq. (8) into Eq. (6), one can have

$$\rho = \frac{7\pi^2}{60\varepsilon(\varepsilon+\delta)f_{\mathrm{H}}^2\sqrt{g_{11}}\big|_{r_{\mathrm{H}}+\varepsilon}}T^4. \quad (9)$$

Considering the physical mechanism of Hawking radiation of the black hole, due to the virtual particle pairs caused by the vacuum fluctuation near the event horizon, the energy of the black hole will decrease when the virtual particles with negative energy go back to the black hole by the tunnelling effects. At the same time, the virtual particles with positive energy will emit out of the gravitational area of the black hole and fly far away to form Hawking radiation. In fact, the motion of the particles with positive energy is very complicated in the thin film ($r_{\mathrm{H}} + \varepsilon \to r_{\mathrm{H}} + \varepsilon + \delta$). The world line of the particles with zero rest mass is a kind of light, while that of the particles of rest mass is a kind of time. In order to make this question simple, we assume that the average radial effusion velocity of the radiation particles with positive energy is

$$v_{\mathrm{e}} = \bar{v}_{\mathrm{r}}^{(+)} = \int_0^\infty v_{\mathrm{r}}F(v_{\mathrm{r}})\mathrm{d}v_{\mathrm{r}}, \quad (10)$$

where v_{r} is the radial velocity of the particles with positive energy, $F(v_{\mathrm{r}})$ is the radial velocity distribution function of the particles with positive energy, and the superscript $(+)$ denotes the integral range $v_{\mathrm{r}} > 0$. The value of v_{e} is related to not only the species of radiation particle, but also space–time metric near the event horizon. The greater the black hole mass is, the stronger the gravitational field near its event horizon is, the smaller the value of v_{e} is. The gravitational field near the event horizon is extremely strong, so the value of v_{e} is very small. The average time which it takes for the particles with positive energy to reach

the radiation spherical area of the black hole is expressed as

$$\bar{t} = \frac{\lambda\sqrt{g_{11}}\big|_{r_{\rm H}+\varepsilon}\,\delta}{2v_{\rm e}}, \tag{11}$$

where λ is a revised constant. Combining Eqs. (9) and (11), one can obtain the radiation energy flux of the Dirac field near the black hole event horizon as

$$M_{\rm H}^{(\rm D)} = \frac{\rho V}{A\bar{t}} = \frac{7\pi^2 v_{\rm e}}{30\lambda\varepsilon\,(\varepsilon+\delta)\,f_{\rm H}^2\,\sqrt{g_{11}}\big|_{r_{\rm H}+\varepsilon}} T^4. \tag{12}$$

Letting

$$\sigma = \frac{7\pi^2 v_{\rm e}}{30\lambda\varepsilon\,(\varepsilon+\delta)\,f_{\rm H}^2\,\sqrt{g_{11}}\big|_{r_{\rm H}+\varepsilon}}, \tag{13}$$

equation (12) can be rewritten as

$$M_{\rm H}^{(\rm D)} = \sigma T^4. \tag{14}$$

It can be seen that the radiation energy flux of the black hole is proportional to the quartic of temperature of its event horizon when ε, δ and $v_{\rm e}$ are all fixed. Equation (14) can be called the generalised Stefan–Boltzmann law of the static spherically symmetric black hole. And Eq. (13) corresponds to the generalised Stefan–Boltzmann coefficient. The derived σ is no longer a constant, but a proportional coefficient related to the space–time metric and the average radial effusion velocity of the radiation particles near the event horizon when the cut-off distance and the thin film thickness are both fixed. Because Hawking radiation is related to the vacuum fluctuation near the event horizon, the space–time metric near the event horizon will affect the vacuum fluctuation, and then affect the value of σ. When the average radial effusion velocity of radiation particles is greater, their ability to escape from the black hole is stronger, and then the value of σ will be larger. This result is consistent with that obtained from Eq. (13). The radiation power of the Dirac field near the event horizon of the static spherically symmetric black hole can be obtained from Eq. (12) as

$$P^{(\rm D)} = \frac{14\pi^3 v_{\rm e} r_{\rm H}^2}{15\lambda\varepsilon\,(\varepsilon+\delta)\,f_{\rm H}^2\,\sqrt{g_{11}}\big|_{r_{\rm H}+\varepsilon}} T^4. \tag{15}$$

3. Radiation energy flux of Dirac field of two kinds of typical static spherically symmetric black holes

3.1. Radiation energy flux of Dirac field of Schwarzschild black hole

For the Schwarzschild black hole, one can easily obtain $f_{\rm H} = 1/r_{\rm H} = 1/2m$, where m is the black hole mass, and $\sqrt{g_{11}}\big|_{r_{\rm H}+\varepsilon} = \sqrt{(r_{\rm H}+\varepsilon)/\varepsilon}$. From Eq. (12), the radiation energy flux of the Dirac field near the event horizon of the Schwarzschild black hole is

$$M_{\rm H}^{(\rm D)} = \frac{14\pi^2 v_{\rm e} m^2}{15\lambda\,(\varepsilon+\delta)\,\sqrt{\varepsilon\,(r_{\rm H}+\varepsilon)}} T^4. \tag{16}$$

Equation (16) can be called the generalised Stefan–Boltzmann law of the Dirac field of the Schwarzschild black hole. The corresponding generalised Stefan–Boltzmann coefficient is

$$\sigma = \frac{14\pi^2 v_{\rm e} m^2}{15\lambda\,(\varepsilon+\delta)\,\sqrt{\varepsilon\,(r_{\rm H}+\varepsilon)}}. \tag{17}$$

It can be seen that σ is no longer a constant, but a proportional coefficient related to the black hole mass and the average radial effusion velocity of the radiation particles near the event horizon when the cut-off distance and the thin film thickness are both fixed. The greater the black hole mass is, and the smaller the cut-off distance and thin film thickness are, the more significant the vacuum fluctuation is. Then, the value of σ is greater. When the average radial effusion velocity of radiation particles is greater, their ability to escape from the black hole is stronger, and then the value of σ will be larger. This result is consistent with that obtained from Eq. (17). One can verify the following formulas by dimensional analysis:

$$\begin{aligned} l_{\rm P} &= \sqrt{\frac{\hbar G}{c^3}}, \quad m_{\rm P} = \sqrt{\frac{\hbar c}{G}}, \\ T_{\rm P} &= \sqrt{\frac{c^5\hbar}{G k_{\rm B}^2}}, \quad \sigma_{\rm P} = \frac{k_{\rm B}^4}{\hbar^3 c^2}, \end{aligned} \tag{18}$$

where $l_{\rm P}$ is the Planck length, $m_{\rm P}$ the Planck mass, $T_{\rm P}$ the Planck temperature, $\sigma_{\rm P}$ the Planck generalised Stefan–Boltzmann coefficient, $\hbar$ the Planck constant, G the gravitational constant, c the light speed in vacuum, $k_{\rm B}$ the Boltzmann constant. In the common unit system, the relation between the temperature of the Schwarzschild black hole and its mass is

$$T = \frac{\hbar c^3}{8\pi k_{\rm B} G m}. \tag{19}$$

Combining Eqs. (18) and (19), the radiation energy flux of the Dirac field near the event horizon of the Schwarzschild black hole can be obtained from Eq. (16) in the common unit system as

$$M_{\rm H}^{(\rm D)} = \frac{7\hbar c^5 v_{\rm e}}{30720\pi^2 G^2 \lambda\,(\varepsilon+\delta)\, m^2 \sqrt{\varepsilon\,(r_{\rm H}+\varepsilon)}}. \tag{20}$$

It can be seen that when the cut-off distance and the thin film thickness are both fixed, the radiation energy flux of the Schwarzschild black hole is proportional to the average radial effusion velocity of the radiation particles in the thin film, and inversely proportional to the square of the black hole mass.

In the common unit system, the radiation power of Dirac field of the Schwarzschild black hole can be derived from Eq. (20) as

$$P^{(\rm D)} = \frac{7\hbar c v_{\rm e}}{1920\pi\lambda\,(\varepsilon+\delta)\,\sqrt{\varepsilon\,(r_{\rm H}+\varepsilon)}}. \tag{21}$$

It can be seen that when the cut-off distance and the thin film thickness are both fixed, the radiation power of the Schwarzschild black hole is proportional to the average radial effusion velocity of the radiation particles in the thin film.

3.2. Radiation energy flux of Dirac field of Reissner–Nordström black hole

For the Reissner–Nordström black hole, the radii of the inner horizon and the outer horizon are expressed as

$$r_- = m - \sqrt{m^2 - Q^2}, \quad r_{\rm H} = m + \sqrt{m^2 - Q^2}, \tag{22}$$

where m is the black hole mass, and Q the black hole charge. One can easily obtain

$$f_{\rm H} = \frac{r_{\rm H} - r_-}{r_{\rm H}^2} = \frac{2\sqrt{m^2-Q^2}}{\left(m+\sqrt{m^2-Q^2}\right)^2}, \tag{23}$$

$$\sqrt{g_{11}}|_{r_{\rm H}+\varepsilon} = \frac{m+\sqrt{m^2-Q^2}+\varepsilon}{\sqrt{\varepsilon\left(2\sqrt{m^2-Q^2}+\varepsilon\right)}}. \tag{24}$$

Substituting Eqs. (23) and (24) into Eq. (12), we can obtain the radiation energy flux of Dirac field near the event horizon of the Reissner–Nordström black hole as

$$M_{\rm H}^{(\rm D)} = \frac{7\pi^2 v_{\rm e}\left(m+\sqrt{m^2-Q^2}\right)^4\sqrt{\varepsilon\left(2\sqrt{m^2-Q^2}+\varepsilon\right)}}{120\lambda\varepsilon\,(\varepsilon+\delta)\,(m^2-Q^2)\left(m+\sqrt{m^2-Q^2}+\varepsilon\right)} T^4. \tag{25}$$

Equation (25) can be called the generalised Stefan–Boltzmann law of Dirac field of the Reissner–Nordström black hole. The corresponding generalised Stefan–Boltzmann coefficient is

$$\sigma = \frac{7\pi^2 v_{\rm e}\left(m+\sqrt{m^2-Q^2}\right)^4\sqrt{\varepsilon\left(2\sqrt{m^2-Q^2}+\varepsilon\right)}}{120\lambda\varepsilon\,(\varepsilon+\delta)\,(m^2-Q^2)\left(m+\sqrt{m^2-Q^2}+\varepsilon\right)}. \tag{26}$$

The derived σ is no longer a constant, but a proportional coefficient related to the black hole mass, the black hole charge, the average radial effusion velocity of the radiation particles near the event horizon when the cut-off distance and the thin film thickness are both fixed. For an extreme Reissner–Nordström black hole, $r_{\rm H} = r_- = m$, the inner and the outer horizons are the same. Then, the one-way membrane region reduces into an infinitesimal thin film. And the generalised Stefan–Boltzmann coefficient for an extreme Reissner–Nordström black hole tends to infinity according to Eq. (26). By using Eq. (18), Eq. (26) can rewritten in the common unit system as

$$\sigma = \frac{7\pi^2 k_{\rm B}^4 v_{\rm e}}{120 c^6 \hbar^3 \lambda} \frac{\left[Gm+\sqrt{(Gm)^2-GQ^2}\right]^4\sqrt{\varepsilon\left[2\sqrt{(Gm)^2-GQ^2}+\varepsilon c^2\right]}}{\varepsilon\,(\varepsilon+\delta)\left[(Gm)^2-GQ^2\right]\left[Gm+\sqrt{(Gm)^2-GQ^2}+\varepsilon c^2\right]}. \tag{27}$$

In the common unit system, the temperature of the event horizon of Reissner–Nordström black hole is[32]

$$T = \frac{\hbar c^3}{2\pi k_{\rm B}} \frac{\sqrt{(Gm)^2 - GQ^2}}{\left[Gm + \sqrt{(Gm)^2 - GQ^2}\right]^2}. \tag{28}$$

By Eqs. (27) and (28), the radiation energy flux of the Dirac field near the event horizon of the Reissner–Nordström black hole can be obtained from Eq. (25) in the common unit system as

$$M_{\rm H}^{(\rm D)} = \frac{7\hbar c^6 v_{\rm e}}{1920\pi^2\lambda} \frac{\left[(Gm)^2 - GQ^2\right]\sqrt{\varepsilon\left[2\sqrt{(Gm)^2 - GQ^2} + \varepsilon c^2\right]}}{\varepsilon(\varepsilon+\delta)\left[Gm + \sqrt{(Gm)^2 - GQ^2}\right]^4\left[Gm + \sqrt{(Gm)^2 - GQ^2} + \varepsilon c^2\right]}. \tag{29}$$

It can be seen that the radiation energy flux of the Dirac field of the Reissner–Nordström black hole is related to not only the black hole mass, but also the black hole charge. This result indicates that the gravitational field and the electromagnetic field around the black hole will affect the thermal radiation of the black hole. When $Gm^2 = Q^2$, the Reissner–Nordström black hole reduces into an extreme Reissner–Nordström black hole. From Eq. (29), the radiation energy flux of the extreme Reissner–Nordström black hole is equal to zero. The reason is that the temperature of the event horizon of the extreme black hole is equal to zero, which is consistent with the known theory.

By using Eq. (29), the radiation power of the Dirac field of the Reissner–Nordström black hole is

$$P^{(\rm D)} = \frac{7\hbar c^2 v_{\rm e}}{480\pi\lambda} \frac{\left[(Gm)^2 - GQ^2\right]\sqrt{\varepsilon\left[2\sqrt{(Gm)^2 - GQ^2} + \varepsilon c^2\right]}}{\varepsilon(\varepsilon+\delta)\left[Gm + \sqrt{(Gm)^2 - GQ^2}\right]^2\left[Gm + \sqrt{(Gm)^2 - GQ^2} + \varepsilon c^2\right]}. \tag{30}$$

It can be seen that when the cut-off distance and the thin film thickness are both fixed, the radiation power of the Dirac field of the Reissner–Nordström black hole is relate not only to the mass and the charge of the black hole, but also to the average radial effusion velocity of the radiation particles. For an extreme Reissner–Nordström black hole, according to Eq. (30), its radiation power is equal to zero.

4. Conclusion

The results indicate that the thermal radiation of the static spherically symmetric black hole always satisfies the generalised Stenfan–Boltzmann law. The derived generalised Stenfan–Boltzmann coefficient is no longer a constant, but a proportional coefficient which is related to the space–time metric near the event horizon and the average radial effusion velocity of the radiation particles in the thin film when the cut-off distance and the thin film thickness are both fixed. The radiation energy fluxes and the radiation powers of the Dirac field near the event horizon of both the Schwarzschild black hole and the Reissner–Nordströsm black hole have been obtained, separately. For an extreme Reissner–Nordström black hole, its radiation energy flux and radiation power are both equal to zero. Although one can obtain more thermal properties of the black hole by the thin film model, the cut-off factor cannot be avoided. This indicates that there exists a restriction on the calculating of the black hole entropy in the semi-classical approximation. The paper puts forward a new method for studying the thermal radiation of the black hole.

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