Protecting entanglement by detuning: in Markovian environments vs in non-Markovian environments

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The models of two qubits separately trapped in two independent Markovian or non-Markovian environments have been investigated. The distinction of the two-qubit entanglement dynamics in different environments has also been discussed in detail. The results show that, in non-Markovian environments, the possible usage time of entanglement can be extended due to its memory effect. On the other hand, we note that, compared to Markovian environments, the two-qubit entanglement could be protected better in non-Markovian environments by modulating the detuning between qubits and cavities. Finally, an intuitive physical interpretation for these results is given.

Keywords: protecting entanglement, Markovian environment, non-Markovian environment
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1. Introduction

Quantum entanglement, as a main physical resource, plays a central role in the application of quantum communications and quantum information, which has been discussed in detail.[1] Yet, it is very sensitive to the system–environment interaction and the initial state of the system. And in realistic quantum systems, the interactions between the systems and their environments are unavoidable.

As we all know, in Markovian environments, in spite of an exponential decay of the single-qubit coherence, the two-qubit entanglement may become completely disappeared at a finite time.[2−5] This phenomenon which is known as 'entanglement sudden death(ESD)' has been extensively investigated and proved in the experiment.[6,7] While in non-Markovian environments, the two-qubit entanglement dynamics show a very different behaviour.[8,9] That is, although no interaction is present between the qubits, there is a revival of their entanglement, after a finite period of time of its complete disappearance. These revivals, although effectively extend the possible usage time of entanglement, they decrease with time and eventually disappear after a relative long time. This would limit its applications in the quantum information processing, such as quantum teleportation,[10] quantum secret sharing,[11] quantum superdense coding.[12]

It is very important to investigate the physical systems and physical effects that may lead to effective longtime entanglement preservation.[13−17] As we mentioned above, the two-qubit entanglement may completely disappear at a finite time in the Markovian regime, while shows an intriguing phenomenon of revival in the non-Markovian regime. This indicates to us that the entanglement may be more robust in non-Markovian environments. Recently, some schemes, for example, embedding the entangled atoms in photonic band-gap (PBG) materials[18] or using a variant of the quantum Zeno effect on the trapped two qubits in a lossy resonator,[19] have been proposed to protect the entanglement in a non-Markovian regime.[18,19]

However, we also note that the above two schemes have some shortages. For the first scheme, when the atomic transition frequencies are inside the gap, the entanglement is indeed trapped, but it also inhibits the external signals. When we want to implement two-qubit operations for quantum computing, we must move the qubit frequency outside the gap and manipulate it in a very short time then move it back. For the second scheme, the variant quantum Zeno effect for protecting entanglement is not always predictable, since it crucially depends on the environmental spectral density and on the resonance condition. Besides, the two schemes are based on the resonant interactions between qubits and cavities, which have neglected the effect of detuning at the very beginning.

In this study, we focus our attention on the in-

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fluence of non-resonant coupling between qubits and cavities on the two-qubit entanglement dynamics in Markovian or non-Markovian environment. We will show that the entanglement could be protected better in non-Markovian envronments than that in Markovian environments when we consider the effect of detuning between the transition frequency of the two-level system and the centre frequency of the cavity. Compared to the previous schemes, our scheme is easier to implement in experiment and the result is more straightforwardly predictable.

2. The model

We consider a two-qubit system coupling to two uncorrelated vacuum environments at zero temperature. As shown in Fig. 1, the qubit A is always coupled to the environment while the qubit B is isolated in the fourth case. We introduce a parameter $\alpha(\alpha = 0, 1)$ to distinguish them, where $\alpha = 0$ denotes that there is no interaction between the qubit B and its environment.

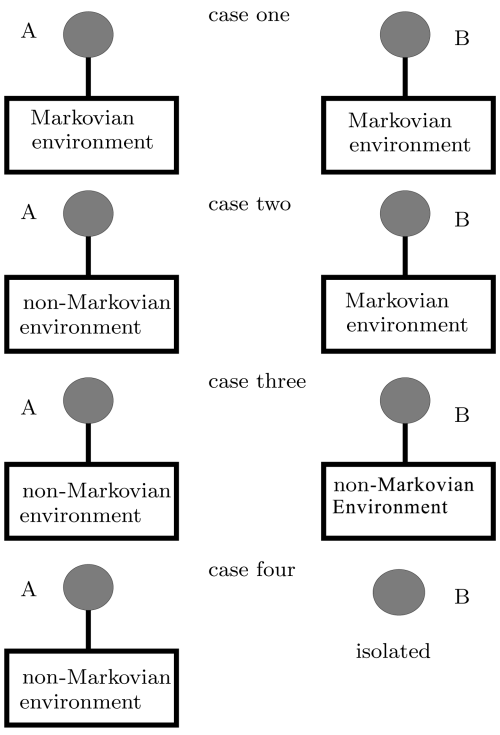


Fig. 1. The models for the four cases.

The total Hamiltonian under the rotating wave approximation is given by ($\hbar = 1$):

$$H = \frac{1}{2}\omega_0(\sigma_z^A + \sigma_z^B) + \sum_{j=1,2}\sum_k \omega_{k_j} b_{k_j}^\dagger b_{k_j} + (\sigma_+^A G_1 + \sigma_-^A G_1^\dagger) + \alpha(\sigma_+^B G_2 + \sigma_-^B G_2^\dagger) \quad (1)$$

with $G_j = \sum_{k_j} g_{k_j} b_{k_j}$, where ω_0 is the transition frequency of the two level system and $\sigma_z, \sigma_+, \sigma_-$ are the system inversion, raising and lowering operators, respectively, while the index k_j labels the kth field mode of the jth reservoir with frequencies ω_k, $b_k^\dagger$ and b_k are the modes' creation and annihilation operators, and g_{k_j} are the coupling constants.

For simplicity, only one excitation of the reservoir will be considered. It follows that any initial state of the form:

$$|\phi(0)\rangle = (c_{01}|1\rangle_1|0\rangle_2 + c_{02}|0\rangle_1|1\rangle_2)|0_{k_1}\rangle_{R_1}|0_{k_2}\rangle_{R_2} \quad (2)$$

evolves after time t into the state

$$\begin{aligned}|\phi(t)\rangle = {} & (c_1(t)|1\rangle_1|0\rangle_2 + c_2(t)|0\rangle_1|1\rangle_2)|0_{k_1}\rangle_{R_1}|0_{k_2}\rangle_{R_2} \\ & + \sum_{k_1} c_{k_1}(t)|0\rangle_1|0\rangle_2|1_{k_1}\rangle_{R_1}|0_{k_2}\rangle_{R_2} \\ & + \sum_{k_2} c_{k_2}(t)|0\rangle_1|0\rangle_2|0_{k_1}\rangle_{R_1}|1_{k_2}\rangle_{R_2}, \end{aligned} \quad (3)$$

where $|1_{k_j}\rangle_{R_j}$ means the state of the jth reservoir with only one excitation in the kth mode. We can solve the Schrödinger equation and get closed equation for $c_1(t)$ and $c_2(t)$

$$\dot{c}_1(t) = -\int_0^t \mathrm{d}t_1 f(t-t_1)c_1(t_1), \quad (4)$$

$$\dot{c}_2(t) = -\alpha^2 \int_0^t \mathrm{d}t_1 f(t-t_1)c_2(t_1), \quad (5)$$

and the correlation function $f(t-t_1)$ is related to the spectral density $J(\omega)$ of the reservoir by

$$f(t-t_1) = \int \mathrm{d}\omega J(\omega)\exp[\mathrm{i}(\omega_0-\omega)(t-t_1)]. \quad (6)$$

Here, the Lorentzian spectral distribution is considered in our calculation,[20]

$$J(\omega) = \frac{1}{2\pi}\cdot\frac{\gamma_0\lambda^2}{(\omega_0-\omega-\Delta)^2+\lambda^2}, \quad (7)$$

where ω_0 is the transition frequency of the two-level system and Δ is the detuning of ω_0 and the centre frequency of the cavity. The parameter λ defines the spectral width of the coupling, which is connected to the reservoir correlation time $\tau_{\mathrm{R}} = \lambda^{-1}$. On the other hand, the parameter γ_0 can be shown to be related to the decay of the excited state of the atom in the Markovian limit.[21] The relaxation time scale τ_{S} over which the state of the system changes is then related to γ_0 by $\tau_{\mathrm{S}} = \gamma_0^{-1}$. In a non-Markovian regime, $\gamma_0 \gg \lambda$, that is, $\tau_{\mathrm{S}} \ll \tau_{\mathrm{R}}$, the reservoir correlation time is greater than the relaxation time and non-Markovian effects become relevant. While in a Markovian regime,

$\gamma_0 << \lambda$, and $\tau_S >> \tau_{\rm R}$, the behaviour of the system is Markovian and irreversible exponential decay occurs.

By using the spectral density $J(\omega)$, we can obtain the exact solutions by residue theorem and Laplace transform

$$c_1(t) = c_{01}\,{\rm e}^{-\frac{(\lambda-{\rm i}\Delta)t}{2}}\left[\cosh\left(\frac{\Omega_1 t}{2}\right)+\frac{\lambda-{\rm i}\Delta}{\Omega_1}\sinh\left(\frac{\Omega_1 t}{2}\right)\right], \tag{8}$$

$$c_2(t) = c_{02}\,{\rm e}^{-\frac{(\lambda-{\rm i}\Delta)t}{2}}\left[\cosh\left(\frac{\Omega_2 t}{2}\right)+\frac{\lambda-{\rm i}\Delta}{\Omega_2}\sinh\left(\frac{\Omega_2 t}{2}\right)\right], \tag{9}$$

where

$$\Omega_1 = \sqrt{(\lambda-{\rm i}\Delta)^2-2\gamma_0\lambda},\quad \Omega_2 = \sqrt{(\lambda-{\rm i}\Delta)^2-2\alpha^2\gamma_0\lambda}.$$

It is obvious that when $\alpha = 0$, the Eq. (10) will reduce to $c_2(t) = c_{02}$, which is in agree with the fourth case.

In the $\{|1\rangle_1|1\rangle_2, |1\rangle_1|0\rangle_2, |0\rangle_1|1\rangle_2, |0\rangle_1|0\rangle_2\}$ basis, the reduced density matrix of the two qubits is given by

$$\boldsymbol{\rho}(t) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & |c_1(t)|^2 & c_1(t)c_2^*(t) & 0 \\ 0 & c_1^*(t)c_2(t) & |c_2(t)|^2 & 0 \\ 0 & 0 & 0 & 1-|c_1(t)|^2-|c_2(t)|^2 \end{pmatrix}. \tag{10}$$

3. Entanglement dynamics

In order to follow the entanglement dynamics of the two qubits, the concurrence[22] is used to measure the entanglement. It is defined as

$$C_\rho(t) = \max\{0, \sqrt{\lambda_1}-\sqrt{\lambda_2}-\sqrt{\lambda_3}-\sqrt{\lambda_4}\}, \tag{11}$$

where $\lambda_i (i = 1, 2, 3, 4)$ are the eigenvalues, in decreasing order, of matrix $\tilde{\boldsymbol{\rho}} = \rho(\boldsymbol{\sigma}_y \otimes \boldsymbol{\sigma}_y)\rho^*(\boldsymbol{\sigma}_y \otimes \boldsymbol{\sigma}_y)$; $\boldsymbol{\sigma}_y$ is the Pauli spin matrix and the asterisk denotes complex conjugation. In particular, the concurrence, for the density matrix of Eq. (10), is given by

$$C(t) = 2|c_1(t)c_2^*(t)|. \tag{12}$$

In Fig. 2, the concurrence as a function of the dimensionless quantity $\gamma_0 t$ for different cases on the resonant condition is plotted. It shows that the Markovian environments lead to an irreversible exponential decay and asymptotical vanish, as shown by case one, while the non-Markovian environments, due to the memory effects, lead to a periodical vanish with a damping of its revival amplitude, as shown by case three. These results are in clear agreement with those in Refs. [8] and [23]. We also note that if one qubit is coupled to a non-Markovian reservoir while another qubit is coupled to a Markovian reservoir, the 'entanglement revival' does not occur but the decay is deferred, just as shown by the solid line in Fig. 2. For the fourth case, as qubit B is isolated and not affected by the environments, so the revival is more remarkable than the third case.

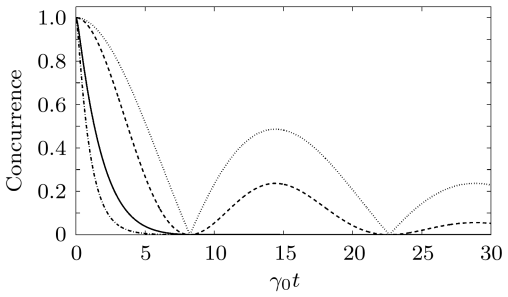


Fig. 2. Concurrence as a function of the dimensionless quantity $\gamma_0 t$ for $\Delta = 0$ in different cases: case one (dotted-dashed line), case two (solid line), case three (dashed line), case four (dotted line) with the parameters $c_{01} = c_{02} = 1/\sqrt{2}$. We have assumed $\lambda = 0.1\gamma_0$ for the non-Markovian regime and $\lambda = 10\gamma_0$ for the Markovian regime.

Next, we turn to investigate the influence of detuning on the entanglement dynamics. For the first case, two qubits are coupled to two independent Markovian environments, as shown in Fig. 3(a), though the rate of decay becomes small, the entanglement still decays exponentially, which is similar to the second case. However, in case three and case four, the entanglement dynamics shows a very interesting phenomenon like as 'entanglement trapping',[18] as shown in Figs. 3(c) and 3(d). The detuning is larger and the

entanglement is protected better. This phenomenon directly depends on the single qubit excited-state population trapping.[24] From Fig. 3, it is easy to note that, in the detuning cases, the entanglement could be protected better in non-Markovian environments than that in Markovian environments.

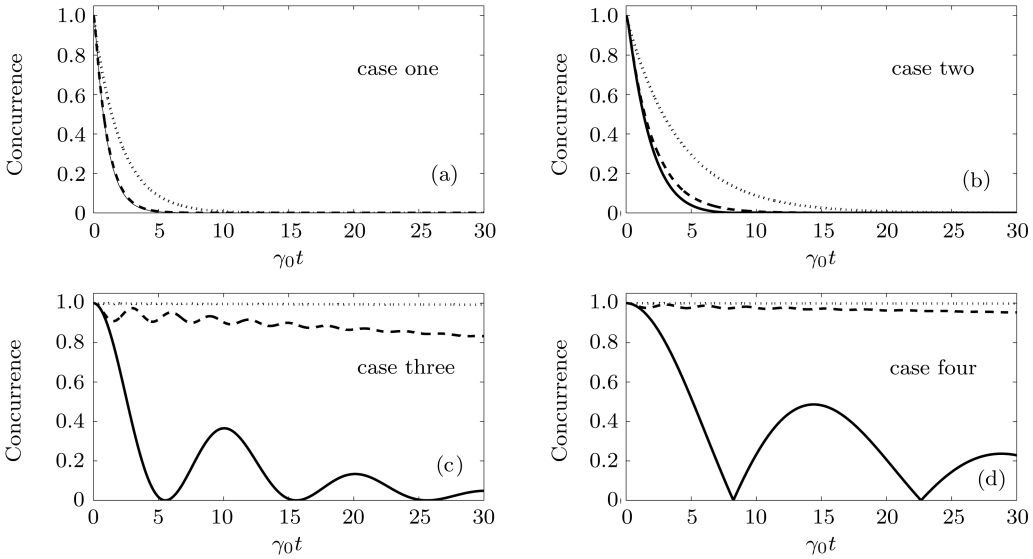


Fig. 3. Concurrence as a function of the dimensionless quantity $\gamma_0 t$ for various detunings in different cases. $\Delta = 0$ (solid curve), $\Delta = 2\gamma_0$ (dashed curve), $\Delta = 10\gamma_0$ (dotted curve) with the parameters $c_{01} = c_{02} = 1/\sqrt{2}$. We have assumed $\lambda = 0.1\gamma_0$ for the non-Markovian regime and $\lambda = 10\gamma_0$ for the Markovian regime.

A more intuitive physical insight on this result could be obtained by the quasimode approach.[25,26] As shown in Fig. 4, according to the quasimode approach, the model of a two-level system coupled to a non-Markovian reservoir could be interpreted as follows: the atom system only couples to a discrete quasimode, while the discrete quasimode interacts with a set of continuum quasimodes, and all these interactions are completely Markovian.

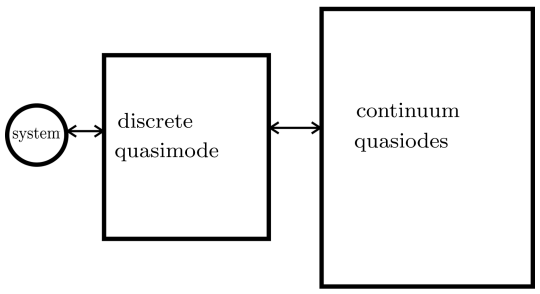


Fig. 4. An diagrammatic representation of the non-Markovian process by quasimode approach.

Since the atom just interacts with the discrete quasimode, but the irreversible dissipative process happens only in the interaction between the discrete mode and the continuum modes. Only when the exciton has been escaped from atom to the discrete mode, then the dissipative process occurs. By modulating the detuning, we can greatly restrain the exciton escaping from atom to the discrete mode, and the population trapping and entanglement could be protected. This is also very liable to understand why the results in Fig. 3(d) is better than those in Fig. 3(c), because an ideal isolated atom never decays.

4. Conclusions

In this paper, we have presented the dynamics of two-qubit entanglement with the qubits in two independent environments. The difference between the Markovian environment and non-Markovian environment has been discussed in detail. What's more, we have presented that the entanglement could be protected better in non-Markovian environments than that in Markovian environments by modulating the detuning between qubits and cavities. This provides a potential method to protect entanglement for a very long time, which would be useful in quantum information processing. In the end, an intuitive physical interpretation has been demonstrated by the quasimode approach.

We argue that the models examined in our study are similar to those typically investigated in quantum information and computation, where qubits are taken to be independent and the couplings between the qubits and reservoirs are strong that should be considered as a non-Markovian process.[27]

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