Generation of GHZ state and cluster state with atomic ensembles via the dipole–blockade mechanism*

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This paper proposes scalable schemes to generate the Greenberger–Horne–Zeilinger (GHZ) state and the cluster state with atomic ensembles via the dipole blockade mechanism on an atom chip, where the qubit is not carried by a single atom but an atomic ensemble. In the protocols, multiqubit entangled states are determinately prepared. Needlessness for single-photon source further decreases the complexity of the experiment. Based on the present laboratory technique, the schemes may be realized. The achieved results reveal a prospect for large-scale quantum communication and quantum computation.

Keywords: dipole–blockade mechanism, n-qubit GHZ state, n-qubit cluster
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1. Introduction

Quantum entanglement is a crucial feature of quantum mechanics. It plays an important role not only in the demonstration of quantum nonlocality,[1,2] but also in quantum teleportation,[3] quantum cryptography,[4] and quantum dense coding.[5] As we know, multiparticle entangled states can achieve more efficient quantum information processing. For example, the Greenberger–Horne–Zeilinger (GHZ) state can be employed as a quantum channel for quantum key distribution[6,7] and quantum secret sharing;[8] the cluster state can implement the entire quantum computation through the local, single-qubit projective measurements and feedforward of their outcomes.[9] So the generation of GHZ state and cluster state has attracted much attention in the theory and the laboratory. Recently, many physical systems for preparation of GHZ state and cluster state have been investigated, such as trapped ions,[10] cavity quantum electrodynamics (QED),[11−15] linear optics.[16] Remarkably, atomic ensemble system is an ideal candidate for quantum information processing since it has the following obvious advantages: laser manipulation of atomic ensembles without separately addressing the individual atoms is easier than the coherent control of single particles; atomic ensembles could have some kind of collectively enhanced coupling to a certain optical mode due to the many-atom interference effects.[17] A lot of schemes for the generation of GHZ state and cluster state have been proposed based on atomic ensembles.[18−20]

Rydberg atoms have strong long-range dipole–dipole interactions, as a result of their large dipole moments. This strong dipole–dipole interaction gives rise to the so-called dipole blockade mechanism,[21,22] which provides an interesting approach for manipulating mesoscopic quantum system, since the large dipole–dipole interaction strongly shifts the atomic energy levels with the separation of several μm, so that the presence of one Rydberg atom is enough to prevent the excitation of all other atoms in the ensemble.[23] Remarkably, the dipole blockade mechanism has been locally observed in cooled atomic systems.[23−27] Based on the background, many protocols for quantum information processing via the dipole–blockade mechanism have been proposed.[20,28−30]

In this paper, we utilise the dipole blockade mechanism to present schemes for generation of the GHZ state and the cluster state with atomic ensembles on an atom chip, which can precisely control and manipulate neutral atoms.[31,32] Compared with previous pro-

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tocol for generating cluster-state[20], our schemes do not require single-photon source, which lowers experimental difficulties since deterministically ideal single-photon sources still remain daunting experimental challenges. In addition, our schemes deterministically prepare multiqubit entangled states replacing probabilistic creation in Ref. [20], and the advantage would be more prominent with the number of the qubits.

2. The fundamental model

The basic building block of the schemes is an ensemble of n identical atoms on an atom chip. Each atom has two internal ground states$|g\rangle$, $|s\rangle$, and an Rydberg state $|e\rangle$. As shown in Fig. 1, we define the state $|0\rangle$ with all the atoms in the ground state $|g\rangle$, the state $|1\rangle$ with only one atom in the ground state $|s\rangle$, and the state $|r\rangle$ with only one atom in the Rydberg state $|e\rangle$. The qubit involved in our schemes is encoded by the states $|0\rangle$ and $|1\rangle$.

As illustrated in Fig. 2(a), the state $|0\rangle$ can be prepared by pumping all atoms to the state $|g\rangle$. The state $|r\rangle$ can be prepared by a π pulse coupling the state $|g\rangle$ to the Ryberg state $|e\rangle$ via Rydberg blockade mechanism, namely, $|r\rangle_i = R_i^1 |0\rangle_i$ (where i denotes the ith atomic ensemble). The state $|1\rangle$ can be prepared by a π pulse coupling the Ryberg state $|e\rangle$ to the state $|s\rangle$, namely, $|1\rangle_i = R_i^2 |r\rangle_i$. As plotted in Fig. 2(b), the state $|\psi\rangle = (1/\sqrt{2})(|0\rangle + |r\rangle)$ and $|\psi'\rangle = (1/\sqrt{2})(|0\rangle - |r\rangle)$ can be prepared by a $\pi/2$ pulse coupling the state $|g\rangle$ to the Ryberg state $|e\rangle$, namely, $|\psi\rangle_i = R_i^3 |0\rangle_i$, $|\psi'\rangle_i = R_i^3 |r\rangle_i$. Here, R_i^1, R_i^2, and R_i^3 mark the corresponding single qubit operations on the ith qubit.

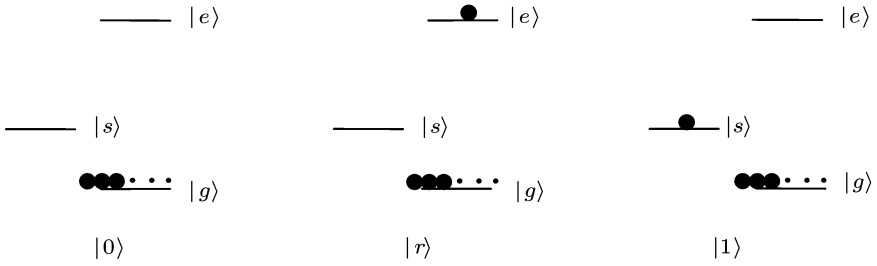


Fig. 1. Scheme for the quantum states encoding.

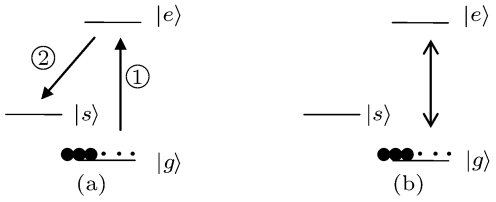


Fig. 2. Scheme for the initial states preparation. Here and the following, single-arrow line denotes a π pulse and double-arrow line represents a $\pi/2$ pulse.

Suppose that the ith qubit is prepared in the state $(1/\sqrt{2})(|0\rangle_i \pm |r\rangle_i)$ via the Rydberg blockade mechanism, the jth qubit is in the states $|0\rangle_j$, and the other qubits $k(k \neq i, j)$ are in the states $|0\rangle_k$ or $|1\rangle_k$, where the ith and the jth qubits are adjacent. As sketched in Fig. 3, a π pulse couples the state $|g\rangle$ to the unshifted Ryberg state $|e\rangle$ in the jth qubit via Rydberg blockade mechanism. If the ith qubit is in the state $|r\rangle_i$, then the jth qubit is still in the state $|0\rangle_j$ due to the dipole blockade mechanism, whereas if the ith qubit is in the state $|0\rangle_i$, then the jth qubit will be in the state $|r\rangle_j$. Thus the ith and jth qubits will become an entangled state, i.e., $|\psi\rangle_{i,j} = R_{i,j}^4 (1/\sqrt{2})(|0\rangle_i \pm |r\rangle_i)\,|0\rangle_j = (1/\sqrt{2})(|0\rangle_i\,|r\rangle_j \pm |r\rangle_i\,|0\rangle_j)$, where $R_{i,j}^4$ denotes the corresponding two-qubit operation on the ith and the jth qubits. No dipole–dipole interaction between the jth and the kth qubits will occur due to all atoms of the kth atomic ensemble being in the ground states.

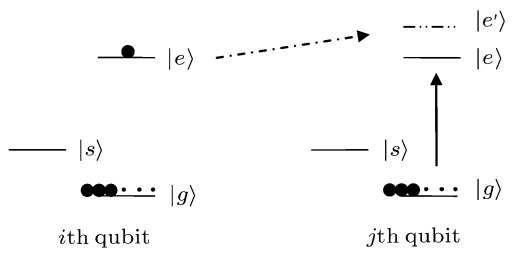


Fig. 3. The dipole–dipole interaction between the ith and jth qubits.

3. The generation of n-qubit GHZ state

Now we describe in detail how to prepare a three-qubit GHZ state, which can be realized by six laser pulses. The three-qubit are initially prepared in the state $|000\rangle$.

As charted in Fig. 4, the operation R_1^3 with a $\pi/2$ pulse is first performed on the 1th qubit, which leads the initial state to

$$|\psi\rangle_{31} = \frac{1}{\sqrt{2}}(|0\rangle_1 |0\rangle_2 |0\rangle_3 + |r\rangle_1 |0\rangle_2 |0\rangle_3). \quad (1)$$

Second, the operation $R_{1,2}^4$ with a π pulse is performed on the 1th and the 2th qubits. The state of the three-qubit will become

$$|\psi\rangle_{32} = \frac{1}{\sqrt{2}}(|0\rangle_1 |r\rangle_2 |0\rangle_3 + |r\rangle_1 |0\rangle_2 |0\rangle_3). \quad (2)$$

Third, the operation R_1^2 with a π pulse is performed on the 1th qubit. This procedure transfers the state Eq. (2) to

$$|\psi\rangle_{33} = \frac{1}{\sqrt{2}}(|0\rangle_1 |r\rangle_2 |0\rangle_3 + |1\rangle_1 |0\rangle_2 |0\rangle_3). \quad (3)$$

Fourth, the operation $R_{2,3}^4$ with a π pulse is performed on the 2th and the 3th qubits. One has

$$|\psi\rangle_{34} = \frac{1}{\sqrt{2}}(|0\rangle_1 |r\rangle_2 |0\rangle_3 + |1\rangle_1 |0\rangle_2 |r\rangle_3). \quad (4)$$

Finally, the operation $R_3^2 R_2^2$ with two π pulses is performed on the 2th and the 3th qubits. The state of the three-qubit will become

$$|\psi\rangle_{35} = \frac{1}{\sqrt{2}}(|0\rangle_1 |1\rangle_2 |0\rangle_3 + |1\rangle_1 |0\rangle_2 |1\rangle_3), \quad (5)$$

which is the three-qubit GHZ state.

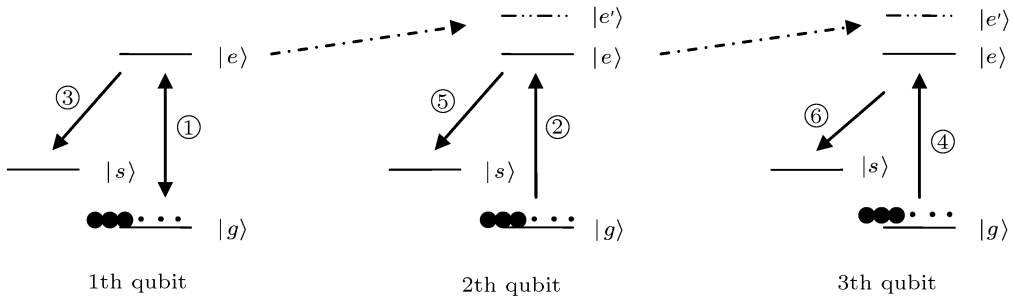


Fig. 4. The generation of a three-qubit GHZ state.

Next we show how to prepare an n-qubit GHZ state. Similarly, suppose that the n-qubit is initially prepared in the state $|00\ldots0\rangle_n$. The operation $R_1^2 R_{1,2}^4 R_1^3$ is applied in sequence on the first two qubits. One will obtain the state

$$|\psi\rangle_{n1} = \frac{1}{\sqrt{2}}(|0\rangle_1 |r\rangle_2 + |1\rangle_1 |0\rangle_2) \bigotimes_{i=3}^{n} |0\rangle_i . \quad (6)$$

Then the manipulation $R_i^2 R_{i,i+1}^4$ $(i = 2, \ldots, n-1)$ is repeated $(n-2)$ times step by step, one has

$$|\psi\rangle_{n2} = \frac{1}{\sqrt{2}}(|0101\ldots010\rangle_n + |1010\ldots10r\rangle_n) \quad (7)$$

with n being odd or

$$|\psi'\rangle_{n2} = \frac{1}{\sqrt{2}}(|0101\ldots010r\rangle_n + |1010\ldots1010\rangle_n) \quad (8)$$

with n being even. After the operation R_n^2 with a π pulse is applied on the nth qubit, the states Eq. (7) or Eq. (8) will become

$$|\psi\rangle_{n3} = \frac{1}{\sqrt{2}}(|0101\ldots010\rangle_n + |1010\ldots101\rangle_n), \quad (9)$$

or

$$|\psi'\rangle_{n3} = \frac{1}{\sqrt{2}}(|0101\ldots0101\rangle_n + |1010\ldots1010\rangle_n), \quad (10)$$

which both are the n-qubit GHZ states.

4. The generation of n-qubit cluster state

We can prepare an n-qubit cluster state by making a small modification on the scheme above. A two-qubit cluster state can be realized by five laser pulses, as shown in Fig. 5. At the beginning, the two

qubits are prepared in the state $|00\rangle$, and the operation $R^4_{1,2}R^3_1$ with two pulses is applied on the 1th and the 2th qubits. Thus, one has

$$|\Psi\rangle_{21} = \frac{1}{\sqrt{2}}(|0\rangle_1 |r\rangle_2 + |r\rangle_1 |0\rangle_2). \quad (11)$$

Subsequently, the operation R^2_1 with a π pulse is applied on the 1th qubit. The state of the two-qubit will become

$$|\Psi\rangle_{22} = \frac{1}{\sqrt{2}}(|0\rangle_1 |r\rangle_2 + |1\rangle_1 |0\rangle_2). \quad (12)$$

Next, the operation R^3_2 with a $\pi/2$ pulse is applied on the 2th qubit. One has

$$\begin{aligned}|\Psi\rangle_{23} &= \frac{1}{2}[|0\rangle_1 (|0\rangle_2 - |r\rangle_2) + |1\rangle_1 (|0\rangle_2 + |r\rangle_2)] \\ &= \frac{1}{2}(|1\rangle_1 + |0\rangle_1 \ell^{(2)}_z)(|r\rangle_2 + |0\rangle_2), \end{aligned} \quad (13)$$

where $\ell^{(2)}_z = |0\rangle_2 \langle 0| - |r\rangle_2 \langle r|$. Finally, the operation R^2_2 with a π pulse is applied on the 2th qubit. Thus we obtain the two-qubit cluster state

$$|\Psi\rangle_{24} = \frac{1}{2}(|1\rangle_1 + |0\rangle_1 \sigma^{(2)}_Z)(|1\rangle_2 + |0\rangle_2), \quad (14)$$

where $\sigma^{(2)}_z = |0\rangle_2 \langle 0| - |1\rangle_2 \langle 1|$.

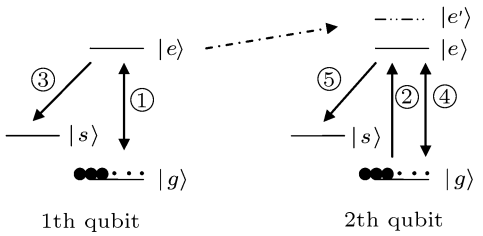


Fig. 5. The preparation of a two-qubit cluster state.

The preparation scheme of the two-qubit cluster state can be extended to generate an n-qubit cluster state. Suppose that the n-qubit is initially prepared in the state $|00\ldots0\rangle_n$. The operation $R^3_2 R^2_1 R^4_{1,2} R^3_1$ is applied on the 1th and 2th qubits. Thus the state of n-qubit evolves into

$$|\Psi\rangle_{n1} = \frac{1}{2}(|1\rangle_1 + |0\rangle_1 \ell^{(2)}_z)(|r\rangle_2 + |0\rangle_2) \bigotimes_{i=3}^{n} |0\rangle_i . \quad (15)$$

Then the manipulation $R^3_{i+1} R^2_i R^4_{i,i+1}$ ($i = 2, 3, \ldots, n-1$) is repeated $(n-2)$ times step by step, one obtains

$$\begin{aligned}|\Psi\rangle_{n2} = &\frac{1}{2^{n/2}} \bigotimes_{i=1}^{n-2}(|1\rangle_i + |0\rangle_i \sigma^{(i+1)}_z) \\ &\times(|1\rangle_{n-1} + |0\rangle_{n-1} \ell^{(n)}_z)(|r\rangle_n + |0\rangle_n). \end{aligned} \quad (16)$$

After the operation R^2_n with a π pulse is applied on the nth qubit, the state of the n qubits will become

$$|\Psi\rangle_{n3} = \frac{1}{2^{n/2}} \bigotimes_{i=1}^{n-1}(|1\rangle_i + |0\rangle_i \sigma^{(i+1)}_z)(|1\rangle_n + |0\rangle_n), \quad (17)$$

which is the n-qubit cluster state.

Similarly, we can prepare a two-dimensional (2D) cluster state, as illustrated in Fig. 6. Suppose that we have prepared a 1D n-qubit cluster state and the $(n+1)$th qubit in the state $|0\rangle_{n+1}$. First, the operation $R^4_{m,n+1} R^2_m$ with two pulses is applied on the mth and $(n+1)$th qubits. Then, the operation $R^2_{n+1} R^3_{n+1} R^2_m$ is performed on the mth and $(n+1)$th qubits. Thus we obtain the 2D cluster state. One can generate the 2D cluster state of an arbitrary configuration by taking a simple add-on strategy and repeating the above procedures.

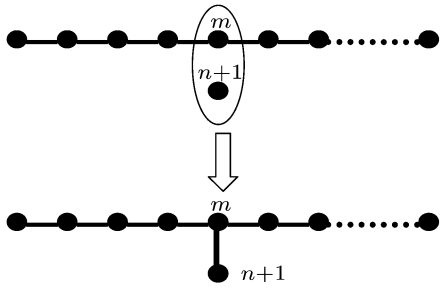


Fig. 6. The creation of 2D cluster state.

5. Discussion and conclusion

Now we have a brief discussion on the above schemes. We choose the $|5S_{1/2}, F = 1, m_F = -1\rangle \equiv |g\rangle$ and $|5S_{1/2}, F = 2, m_F = 1\rangle \equiv |s\rangle$ hyperfine levels of ^{87}Rb as the two ground states, whose approximately equal magnetic moments and the corresponding static Zeeman shifts result in the smallest magnetic induced decoherence,[33] while the high lying n's state ($n' \sim 70$) with a very long lifetime as the Rydberg state $|e\rangle$. In our schemes, the qubit is encoded by atomic ensemble, with a large number of atoms helping to suppress the negative impact of decoherence on an information stored in atomic ensemble.[17,34–37] The dipole–dipole potential energy curves are given by[38]

$$U = \frac{\delta}{2} \pm \sqrt{\left(\frac{\hbar\delta}{2}\right)^2 + \frac{4}{3}\frac{c_3^2}{r^6}},$$

with δ an effective average two-atom energy defect and

$$C_3 = \frac{\varepsilon^2}{4\pi\varepsilon_0} \times \langle n'p| |r| |n's\rangle \langle (n'-1)p| |r| |n's\rangle .$$

If $N(\mathrm{N} \sim 10^4)$ ^{87}Rb atoms are trapped in every single trap with a length scale of 5 μm, the distance from centre to centre between two adjacent atomic ensembles is also 5 μm. In this case, the energy shift is about 100 MHz when the atoms are 5 μm apart from each other and 2 MHz when the atoms are 10 μm apart from each other,[39] which is enough to blockade the excitation of all shifted atoms in the ensemble. The time for the generation of a three-qubit GHZ state is $\Delta t_1 \approx (5/2)\pi/\Omega_1 + 3\pi/\Omega_2$, where Ω_1 (Ω_2) is the Rabi frequency between the Rydberg state $|e\rangle$ and ground state $|g\rangle$ ($|s\rangle$). The time for the generation of a two-qubit cluster state is $\Delta t_2 \approx 2\pi/\Omega_1 + 2\pi/\Omega_2$. Due to the loss rate γ of the Rydberg state satisfies $\gamma \ll \Omega_i$, we can neglect the loss from the Rydberg state during the operation.

We note that Marcin Zwierz and Pieter Kok have proposed a scheme for generating the cluster-state with atomic ensembles via the dipole–blockade mechanism. However, there are some differences between our protocols and that in Ref. [20]. (a) Our schemes do not need single-photon source, which further reduces experimental difficulties. (b) The determinate creation of multiqubit entangled state in our schemes would be more efficient than the probabilistic generation in Ref. [20], and this efficiency would become even more significant with an increasing number of the qubits involved. Recently, an array of atomic assembles on an atom chip has been experimentally demonstrated.[40−42] All these show that our proposals open up a prospect to experimentally generate a large-scale multiqubit entangled state and experimentally implement large-scale quantum computation.

In conclusion, we have presented schemes for preparing the n-qubit GHZ state and the cluster state via the dipole blockade mechanism. In the ideal case, the successful probabilities of our protocols approach unity. Most importantly, our schemes are experimentally feasible based on the current technique and make it possible for scalable quantum communication and quantum computation.

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