Teleportation and thermal entanglement in two-qubit Heisenberg XYZ spin chain with the Dzyaloshinski–Moriya interaction and the inhomogeneous magnetic field*

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This paper studies the average fidelity of teleportation and thermal entanglement for a two-qubit Heisenberg XYZ chain in the presence of both an inhomogeneous magnetic field and a Dzyaloshinski–Moriya interaction. It shows that for a fixed D_z, the increase of b_z will broaden the critical temperature at the cost of decreasing the thermal entanglement. And it can modulate the inhomogeneous magnetic field and the Dzyaloshinski–Moriya interaction for the average fidelity of teleportation to be optimal.

Keywords: teleportation, thermal entanglement, Heisenberg XYZ spin chain
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In recent years, entanglement has been extensively studied because it has the fascinating features of quantum mechanics and it provides a fundamental resource in quantum information processing, such as quantum teleportation,[1,2] quantum dense coding,[3,4] quantum cryptographic key distribution,[5,6] quantum computation,[7,8] and so on. As one knows that quantum teleportation process allows the two distant parties, the sender Alice and the receiver Bob, to utilise the nonlocal correlations of the quantum channel initially shared by Einstin–Podolsky–Rosen (EPR)[9] pair, to teleport an unknown quantum state $|\varphi\rangle = a|\uparrow\rangle + b|\downarrow\rangle$, Alice makes a Bell-basis measurement on her EPR particle and the unknown quantum system, then Bob can reconstruct the state $|\varphi\rangle$ with a local unitary operation on his EPR particle according to the classical information published by Alice. Generally, for a perfect quantum teleportation scheme, the precondition is to find the quantum channel. Recently, the teleportation with some solid systems such as quantum chain becomes an attractive emerging field,[10−12] A quantum chain is usually referred as a spin chain which is a one-dimensional array of qubits coupled permanently by mutual interaction. Lately, great efforts have been taken to study the teleportation in the one-dimensional Heisenberg chain,[13−22] as it can be considered as an important source of entanglement. Yeo has studied the teleportation in thermal entangled state of a two-qubit Heisenberg XX chain,[23] Zhang studied thermal entanglement and teleportation in a two-qubit Heisenberg chain with Dzyaloshinski–Moriya (DM) interaction.[24] Very recently, the Heisenberg model, including Ising model,[25] XY model,[26] XXX model,[27] XXZ model,[28] and XYZ model,[29] have also been widely studied. In this paper, we study the quantum teleportation and thermal entanglement via a two-qubit Heisenberg XYZ spin chain by introducing the Z-component DM interaction and an inhomogeneous magnetic field along the Z-component in the presence of an external uniform magnetic field. By modulating the corresponding parameters, the thermal entanglement and the average fidelity can be efficiently controlled.

The model Hamiltonian of our system can be de-

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$$\begin{aligned} H &= J_x\sigma_1^x\sigma_2^x + J_y\sigma_1^y\sigma_2^y + J_z\sigma_1^z\sigma_2^z \\ &\quad + D_z(\sigma_1^x\sigma_2^y - \sigma_2^x\sigma_1^y) \\ &\quad + (B_z + b_z)\sigma_1^z + (B_z - b_z)\sigma_2^z, \end{aligned} \tag{1}$$

where J_i ($i = x, y, z$) are the real coupling coefficients with $J_i > 0$ corresponding to the antiferromagnetic case and $J_i < 0$ corresponding to the ferromagnetic case, D_z is the Z-component DM coupling parameter, B_z (uniform external magnetic field) and b_z (nonuniform external magnetic field) are the Z-component magnetic field parameters, and $\boldsymbol{\sigma}^i$ are the Pauli matrices. Under the standard bases $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$, the matrix form of Hamiltonian Eq. (1) can be expressed as

$$\boldsymbol{H} = \begin{pmatrix} J_z + 2B_z & 0 & 0 & J_x - J_y \\ 0 & -(J_z - 2b_z) & J_x + J_y + 2D_z\,\mathrm{i} & 0 \\ 0 & J_x + J_y - 2D_z\,\mathrm{i} & -(J_z + 2b_z) & 0 \\ J_x - J_y & 0 & 0 & J_z - 2B_z \end{pmatrix}. \tag{2}$$

Without loss of generality, after some straightforward calculations, the eigenvectors of this channel Hamiltonian is

$$\begin{aligned} |\Psi_{12}\rangle &= \sin\theta_{12}|00\rangle + \cos\theta_{12}|11\rangle, \\ |\Psi_{34}\rangle &= \sin\theta_{34}|01\rangle + \chi\cos\theta_{34}|10\rangle, \end{aligned} \tag{3}$$

with the corresponding eigenvalues

$$\begin{aligned} E_{12} &= J_z \pm \omega_1, \\ E_{34} &= -J_z \pm \omega_2, \end{aligned} \tag{4}$$

where

$$\begin{aligned} \omega_1 &= \sqrt{4B_z^2 + (J_x - J_y)^2}, \\ \omega_2 &= \sqrt{4b_z^2 + 4D_z^2 + (J_x + J_y)^2}\ , \\ \chi &= \frac{J_x + J_y - 2D_z\,\mathrm{i}}{\sqrt{(J_x + J_y)^2 + 4D_z^2}}, \\ \theta_{12} &= \arctan\frac{J_x - J_y}{\pm\omega_1 - 2B_z}, \\ \theta_{34} &= \arctan\frac{\sqrt{(J_x + J_y)^2 + 4D_z^2}}{\pm\omega_2 - 2b_z}. \end{aligned}$$

The state of the system at thermal equilibrium (thermal state) is $\boldsymbol{\rho}(T) = (1/Z)\mathrm{e}^{(-\beta H)}$, where $Z = \mathrm{Tr}[\mathrm{e}^{(-\beta H)}]$ is the partition function of the system, H is the Hamiltonian of the system, $\beta = 1/k_{\mathrm{B}}T$, T is the temperature, and k_{B} is the Boltzman constant which is taken to be 1 for simplicity. Therefore, the density matrix of the system discussed above can be written as

$$\boldsymbol{\rho}(T) = \frac{1}{Z}\begin{pmatrix} U_1 & 0 & 0 & \delta \\ 0 & U_2 & U_3 & 0 \\ 0 & U_4 & U_5 & 0 \\ \delta & 0 & 0 & U_6 \end{pmatrix}, \tag{5}$$

where

$$\begin{aligned} U_1 &= \mathrm{e}^{-\beta J_z}\left[\cosh(\beta\omega_1) - \frac{2B_z}{\omega_1}\sinh(\beta\omega_1)\right], \\ U_2 &= \mathrm{e}^{\beta J_z}\left[\cosh(\beta\omega_2) - \frac{2b_z}{\omega_2}\sinh(\beta\omega_2)\right], \\ U_3 &= -\chi\mathrm{e}^{\beta J_z}\frac{\sqrt{4D_z^2 + (J_x + J_y)^2}}{\omega_2}\sinh(\beta\omega_2), \\ U_4 &= -\chi^*\mathrm{e}^{\beta J_z}\frac{\sqrt{4D_z^2 + (J_x + J_y)^2}}{\omega_2}\sinh(\beta\omega_2), \\ U_5 &= \mathrm{e}^{\beta J_z}\left[\cosh(\beta\omega_2) + \frac{2b_z}{\omega_2}\sinh(\beta\omega_2)\right], \\ U_6 &= \mathrm{e}^{-\beta J_z}\left[\cosh(\beta\omega_1) + \frac{2B_z}{\omega_1}\sinh(\beta\omega_1)\right], \\ \delta &= -\mathrm{e}^{-\beta J_z}\frac{J_x - J_y}{\omega_1}\sinh(\beta\omega_1), \end{aligned}$$

where the partition function Z is given by $Z = 2[\mathrm{e}^{-\beta J_z}\cosh(\beta\omega_1) + \mathrm{e}^{\beta J_z}\cosh(\beta\omega_1)]$. We now study the quantum teleportation through a two-qubit Heisenberg XYZ chain by using the standard teleportation protocol.[30] Assume that the input state is an arbitrary two-qubit pure state, such as

$$\begin{aligned} |\Psi_{\mathrm{in}}\rangle &= \cos\frac{\theta}{2}|10\rangle + \mathrm{e}^{\mathrm{i}\,\phi}\sin\frac{\theta}{2}|01\rangle, \\ &(0 \le \theta \le \pi \ \text{ and } \ 0 \le \phi \le 2\pi). \end{aligned} \tag{6}$$

Then the output state is given by[31]

$$\boldsymbol{\rho}_{\mathrm{out}} = \sum_{i,j=0,x,y,z} P_{ij}(\boldsymbol{\sigma}_i \otimes \boldsymbol{\sigma}_j)\rho_{\mathrm{in}}(\boldsymbol{\sigma}_i \otimes \boldsymbol{\sigma}_j), \tag{7}$$

where $\boldsymbol{\sigma}_0$ is the identity matrix and $\boldsymbol{\sigma}_i$ ($i = x, y, z$) are the Pauli matrices, $\boldsymbol{\rho}_{\mathrm{in}}$ is the density matrix of the input state, $P_{ij} = \mathrm{Tr}\left[E^i\boldsymbol{\rho}(T)\right]\mathrm{Tr}[E^j\boldsymbol{\rho}(T)]$, $\sum_{ij} P_{ij} =$

1, and $E^0 = |\Psi^-\rangle\langle\Psi^-|$, $E^1 = |\Phi^-\rangle\langle\Phi^-|$, $E^2 = |\Phi^+\rangle\langle\Phi^+|$, $E^3 = |\Psi^+\rangle\langle\Psi^+|$, where $|\Phi^\pm\rangle = (|00\rangle \pm |11\rangle)/\sqrt{2}$, $|\Psi^\pm\rangle = (|01\rangle \pm |10\rangle)/\sqrt{2}$. After the calculations the output density matrix can be expressed as

$$\boldsymbol{\rho}_{\rm out} = \frac{1}{Z^2}\begin{pmatrix} a & 0 & 0 & \omega \\ 0 & b & \varepsilon & 0 \\ 0 & \varepsilon^* & c & 0 \\ \omega^* & 0 & 0 & d \end{pmatrix}, \tag{8}$$

where

$$\begin{aligned}
a &= (U_1+U_6)(U_2+U_5),\\
b &= (U_2+U_5)^2\left(\sin\frac{\theta}{2}\right)^2 + (U_1+U_6)^2\left(\cos\frac{\theta}{2}\right)^2,\\
c &= (U_1+U_6)^2\left(\sin\frac{\theta}{2}\right)^2 + (U_2+U_5)^2\left(\cos\frac{\theta}{2}\right)^2,\\
d &= (U_1+U_6)(U_2+U_5),\\
\omega &= (U_1+U_6)(U_2+U_5)\sin\theta(\mathrm{e}^{\mathrm{i}\,\phi}+\mathrm{e}^{-\mathrm{i}\,\phi}),\\
\omega^* &= (U_1+U_6)(U_2+U_5)\sin\theta(\mathrm{e}^{\mathrm{i}\,\phi}+\mathrm{e}^{-\mathrm{i}\,\phi}),\\
\varepsilon &= \frac{1}{2}(U_3+U_4)^2\mathrm{e}^{\mathrm{i}\,\phi}\sin\theta + 2\delta^2\mathrm{e}^{-\mathrm{i}\,\phi}\sin\theta,\\
\varepsilon^* &= \frac{1}{2}(U_3+U_4)^2\mathrm{e}^{-\mathrm{i}\,\phi}\sin\theta + 2\delta^2\mathrm{e}^{\mathrm{i}\,\phi}\sin\theta.
\end{aligned}$$

In order to describe the output quantum entanglement, we use Wooters' concurrence[32] which varies from $C = 0$ for a separable state to $C = 1$ for a maximally entangled state. Since the density matrix in this paper only contains non-zero elements along the main diagonal and anti-diagonal, (i.e., in an 'X' formation), the concurrence can be calculated as[33]

$$C(\boldsymbol{\rho}_{\rm out}) = 2\max\left\{0, |\epsilon| - \sqrt{ad}, |\omega| - \sqrt{bc}\right\}. \tag{9}$$

Figure 1 demonstrates the $C_{\rm out}$ versus temperature T for different inhomogeneous magnetic field b_z when the input state is a maximally entangled state. We can see that the $C_{\rm out}$ first maintains a stable value at a short range of temperature T, then it decreases monotonically with the increase of T. Let $T_{\rm o}^k$ $(k = 1, 2, \ldots, 6)$ be the intersection points for arbitrary two curves shown in Fig. 1. At these points, the $C_{\rm out}$ values corresponding to different b_z are equal. It can be easily seen that when $T < T_{\rm o}^k$, the $C_{\rm out}$ decreases with increasing b_z. While when $T > T_{\rm o}^k$, the $C_{\rm out}$ increases with increasing b_z, meanwhile, we obtain a higher critical temperature. Therefore, the inhomogeneous magnetic field is beneficial to broaden the critical temperature at the cost of decreasing the concurrence.

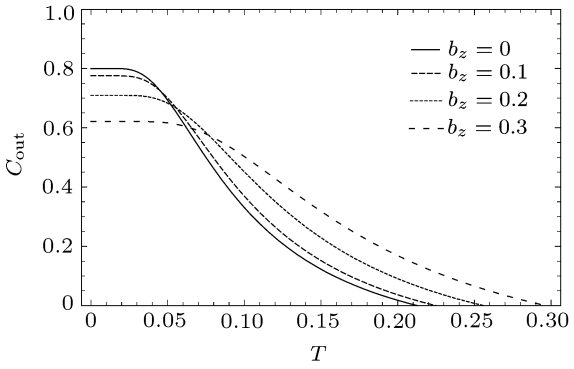


Fig. 1. Teleported thermal concurrence $C_{\rm out}$ as a function of T for different inhomogeneous magnetic field b_z, with the coupling constant $J_x = 0.95$, $J_y = 0.05$, $J_z = 0.2$, for a finite value of uniform magnetic field $B_z = 0.5$ and the DM interaction $D_z = 0.25$.

The quality of the teleported state $\boldsymbol{\rho}_{\rm out}$ is characterised by the fidelity $F(\boldsymbol{\rho}_{\rm in}, \boldsymbol{\rho}_{\rm out})$, defined by[34]

$$F(\boldsymbol{\rho}_{\rm in}, \boldsymbol{\rho}_{\rm out}) = \left[\mathrm{Tr}(\sqrt{(\boldsymbol{\rho}_{\rm in})^{1/2}\boldsymbol{\rho}_{\rm out}(\boldsymbol{\rho}_{\rm in})^{1/2}})\right]^2. \tag{10}$$

For $F(\boldsymbol{\rho}_{\rm in}, \boldsymbol{\rho}_{\rm out}) = 0$, it means that the information is completely destroyed in the transmission process, and the teleportation fails. While for $F(\boldsymbol{\rho}_{\rm in}, \boldsymbol{\rho}_{\rm out}) = 1$, it means that the final state is identical to the initial state. In common situation $0 < F < 1$, it results in distortions to the quantum information at a certain extent after being transmitted. After a straightforward calculation, the fidelity is given by

$$\begin{aligned}
F(\boldsymbol{\rho}_{\rm in}, \boldsymbol{\rho}_{\rm out}) = {} & (U_2+U_5)^2\left(\sin^4\frac{\theta}{2} + \cos^4\frac{\theta}{2}\right)\\
& + 4\sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2}\left[\frac{1}{2}(U_3+U_4)^2\right.\\
& \left. + 2\delta^2\cos(2\phi)\right]\\
& + 2(U_1+U_6)^2\sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2}.
\end{aligned} \tag{11}$$

As is well known that the fidelity can be regarded as a useful indicator of teleportation quality. When the input state is a pure state, the efficiency of quantum communication is characterised by the average fidelity. The average fidelity $F_{\rm A}$ of teleportation can be formulated as[35]

$$F_{\rm A} = \frac{\int_0^{2\pi}\mathrm{d}\phi\int_0^{\pi}F\sin\theta\mathrm{d}\theta}{4\pi}. \tag{12}$$

Through some straightforward algebra, we can obtain the average fidelity as

$$F_{\rm A}=\frac{4\left[2{\rm e}^{2\beta J_z}\cosh^2(\beta\omega_2)+{\rm e}^{-2\beta J_z}\cosh^2(\beta\omega_1)+{\rm e}^{2\beta J_z}\frac{(J_x+J_y)^2}{\omega_2}\sinh^2(\beta\omega_2)\right]}{3Z^2}. \tag{13}$$

In Fig. 2, the average fidelity $F_{\rm A}$ is plotted as a function of the temperature T under different inhomogeneous magnetic field b_z. From the figure we can see that the average fidelity $F_{\rm A}$ first maintains a fixed value at a short range of temperature T then it decreases monotonically with the increase of T, at last T, these four lines superpose. Let $T_{\rm o}^i$ $(i=1,2,\ldots,6)$ be the intersection points for arbitrary two curves shown in Fig. 2. When $T<T_{\rm o}^i$, the $F_{\rm A}$ decreases with increasing b_z, when $T>T_{\rm o}^i$, the $F_{\rm A}$ increases with increasing b_z. Meanwhile, increasing b_z can broaden the region where $F_{\rm A}$ is larger than 2/3 for the classical case.

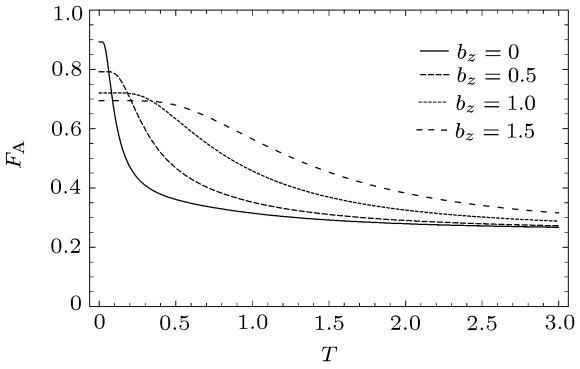


Fig. 2. The average fidelity $F_{\rm A}$ as a function of the temperature T with different inhomogeneous magnetic field b_z, where we set $J_x=0.95$, $J_y=0.05$, $J_z=0.2$, $B_z=0.5$, $D_z=0.25$.

In Fig. 3 we plot the average fidelity $F_{\rm A}$ as a function of the DM interaction D_z with different values of the inhomogeneous magnetic field b_z at a finite temperature $T=0.1$, where we set $J_x=0.95$, $J_y=0.05$, $J_z=0.2$ and $B_z=0.5$. From this figure we can see clearly that for small b_z ($b_z=0$), we can get the maximum value of the fidelity $F_{\rm A}$ by increasing the D_z. Then the average fidelity $F_{\rm A}$ tends to the limited value of fidelity 2/3 for classical communication following the further increase of D_z. While for relatively large value of b_z ($b_z=0.6$), the average fidelity $F_{\rm A}$ decreases monotonically with the increase of D_z to the limited value of fidelity 2/3 for classical communication. Therefore, in order to illustrate the superiority of quantum communication, one should take D_z less than 1.5 in the presence of the inhomogeneous magnetic field.

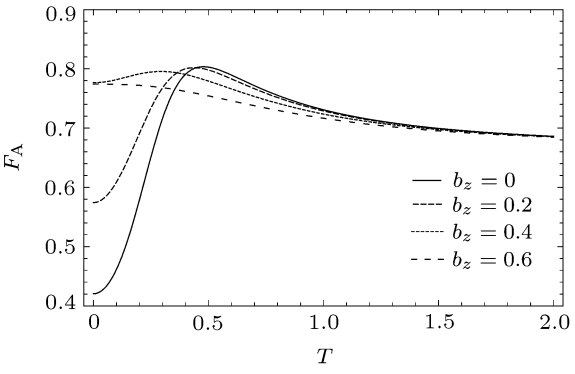


Fig. 3. The average fidelity $F_{\rm A}$ as a function of the DM interaction with the different inhomogeneous magnetic field b_z at a finite temperature $T=0.1$. We work in units where D_z and b_z are dimensionless.

In conclusion, we have studied the quantum teleportation via a two-qubit Heisenberg XYZ chain under the DM coupling parameter and the inhomogeneous magnetic field. Our study shows that both the concurrence and the fidelity decrease with the increase of T, and we know that the inhomogeneous magnetic field not only increases the output entanglement when $T>T_{\rm o}^k$, but also enhances the critical temperature. We can see that the average fidelity increases with the increase of the inhomogeneous magnetic field when the DM coupling parameter is set to zero. The influence of D_z on the model resembles the effects of b_z when b_z is modulative. In addition, the effects of the inhomogeneous magnetic field on the average fidelity and output entanglement are studied for a fixed D_z. The inhomogeneous magnetic field not only can improve the critical temperature, but also can enhance the average fidelity.

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