Nonlinear two-mode squeezing obtained by analysing two-mode exponential quadrature operators in entangled state representation*

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By analysing the properties of two-mode quadratures in an entangled state representation (ESR) we derive from ESR some complicated exponential quadrature operators for nonlinear two-mode squeezing, which directly leads to wave function of the nonlinear squeezed state in ESR.

Keywords: nonlinear two-mode squeezing, two-mode exponential quadrature operators, entangled state representation

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1. Introduction

Because the quantum fluctuation of one quadrature of the light field in the squeezed state is less than in a coherent state, squeezed states is a very important topic in quantum optics[1−7] and has potential applications in quantum communication.[8−10] Besides, it has been recognised that the two-mode squeezed state (idle mode and signal mode) produced from a parametric down conversion is an entangled state in the frequency domain.[11] The conception of entangled state, has received more and more attention as it can be employed in quantum teleportation, quantum cryptography, and quantum superdense coding.[12] For analysing two-mode light field, the two-mode quadratures that are characteristic of the quantum fluctuation of various states are

$$\frac{a_1+a_2+a_1^{\dagger}+a_2^{\dagger}}{2^{3/2}}=(Q_1+Q_2)/2, \quad \frac{a_1+a_2-a_1^{\dagger}-a_2^{\dagger}}{2^{3/2}i}=(P_1+P_2)/2, \tag{1}$$

with $[(Q_1+Q_2)/2,(P_1+P_2)/2]=i$. It has been known that the exponential quadrature operator $\mathrm{e}^{\mathrm{i}\lambda(P_2Q_1+Q_2P_1)}=\mathrm{e}^{\mathrm{i}\lambda[(P_1+P_2)(Q_1+Q_2)-(Q_1-Q_2)(P_1-P_2)]/2-\lambda}$ is a standard two-mode squeezing operator, an interesting question thus challenges us, i.e. how a complicated exponential quadrature operator, say, $\mathrm{e}^{\mathrm{i}\lambda(P_1+P_2)^2(Q_1+Q_2)/2}$, affects squeezing. This problem seems tough. In this work we show that by analysing the properties of two-mode quadratures in an entangled state representation[13,14] we can derive some generalised two-mode squeezing operators in the entangled state representation (ESR), these complicated operators generate nonlinear two-mode squeezing. The merit of the ESR of these operators lies in that they can directly lead to the wave function of the nonlinear squeezed state in ESR.

2. Brief review of two bipartite entangled states

In Refs. [13]–[15] it has been shown that operators P_1+P_2 and Q_1-Q_2 possess a common eigenvector, that is,

$$|\eta\rangle=\exp\left[-\frac{1}{2}|\eta|^2+\eta a_1^{\dagger}-\eta^* a_2^{\dagger}+a_1^{\dagger}a_2^{\dagger}\right]|00\rangle\,,\ \eta=\eta_1+\mathrm{i}\eta_2, \tag{2}$$

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with the eigenvector equations

$$(Q_1 - Q_2)\,|\eta\rangle = \sqrt{2}\eta_1\,|\eta\rangle\,, \quad (P_1 + P_2)\,|\eta\rangle = \sqrt{2}\eta_2\,|\eta\rangle\,, \tag{3}$$

where $a_i^\dagger$ $(i = 1, 2)$ are the bosonic operators obeying $\left[a_i, a_j^\dagger\right] = \delta_{ij}$, and $|00\rangle$ is the vacuum state in Fock space. It is Einstein–Podolsky–Rosen (EPR)[16] who first used the commutative relation $[Q_1 - Q_2, P_1 + P_2] = 0$ to propose the concept of quantum entanglement and argued the incompleteness of quantum mechanics. With using the technique of integration within an ordered product (IWOP) of operators,[17−19] $|\eta\rangle$ state spans a complete and orthonormal space

$$\int \frac{\mathrm{d}^2\eta}{\pi}\,|\eta\rangle\,\langle\eta| = 1, \tag{4}$$

$$\langle\xi|\xi'\rangle = \pi\delta(\eta - \eta')\delta(\eta^* - \eta'^*). \tag{5}$$

Thus we name $|\eta\rangle$ the EPR entangled state. The state conjugate to $|\eta\rangle$ is $|\xi\rangle$, which is the common eigenvector of another pair of EPR commutative operators $Q_1 + Q_2$ and $P_1 - P_2$, the explicit form of $|\xi\rangle$ is

$$|\xi\rangle = \exp\left[-\frac{1}{2}\,|\xi|^2 + \xi a_1^\dagger + \xi^* a_2^\dagger - a_1^\dagger a_2^\dagger\right]|00\rangle\,, \tag{6}$$

which satisfies the eigenvalue equations

$$\begin{aligned}(Q_1 + Q_2)\,|\xi\rangle &= \sqrt{2}\xi_1\,|\xi\rangle\,,\\ (P_1 - P_2)\,|\xi\rangle &= \sqrt{2}\xi_2\,|\xi\rangle\,,\end{aligned} \tag{7}$$

and the completeness relation

$$\int \frac{\mathrm{d}^2\xi}{\pi}\,|\xi\rangle\,\langle\xi| = 1, \quad \xi = \xi_1 + \mathrm{i}\xi_2. \tag{8}$$

$$\langle\xi|\xi'\rangle = \pi\delta(\xi - \xi')\delta(\xi^* - \xi'^*). \tag{9}$$

Then in Refs. [20] and [21] it has been shown that the two-mode squeezing operator $\exp\left[\lambda\left(a_1^\dagger a_2^\dagger - a_1 a_2\right)\right]$ is neatly expressed as

$$\exp\left[\lambda\left(a_1^\dagger a_2^\dagger - a_1 a_2\right)\right] = \int \frac{\mathrm{d}^2\eta}{\mu\pi}\,|\eta/\mu\rangle\,\langle\eta|\,, \mu = \mathrm{e}^\lambda, \tag{10}$$

or

$$S_2 = \int \frac{\mu\mathrm{d}^2\xi}{\pi}\,|\mu\xi\rangle\,\langle\xi|\,. \tag{11}$$

The overlap between these two mutually conjugate states is

$$\langle\eta|\,\xi\rangle = \frac{1}{2}\exp\left[\left(\eta^*\xi - \eta\xi^*\right)/2\right]. \tag{12}$$

Thus an interesting question that naturally arises is whether we can derive the standard squeezing operator directly from the properties of two-mode quadratures in the entangled state representation ($|\eta\rangle$ or $|\xi\rangle$). The answer is affirmative.

3. Exponential quadrature operators for standard linear squeezing

We begin with elucidating the properties of quadrature operators in $|\eta\rangle$ state in more detail. From Eq. (2) we see

$$\left(a_1 - a_2^\dagger\right)|\eta\rangle = \eta\,|\eta\rangle\,, \quad \left(a_2 - a_1^\dagger\right)|\eta\rangle = -\eta^*\,|\eta\rangle\,, \tag{13}$$

and using

$$P_i = \frac{1}{\sqrt{2}\mathrm{i}}\left(a_i - a_i^\dagger\right), \quad Q_i = \frac{1}{\sqrt{2}}\left(a_i + a_i^\dagger\right), \tag{14}$$

we have the quadrature operators behaviour in $\langle\eta|$ representation

$$\langle\eta|\,(P_1 - P_2) = -\mathrm{i}\sqrt{2}\frac{\partial}{\partial\eta_1}\,\langle\eta|\,, \tag{15}$$

$$\langle\eta|\,(Q_1 + Q_2) = \mathrm{i}\sqrt{2}\frac{\partial}{\partial\eta_2}\,\langle\eta|\,. \tag{16}$$

One can check Eq. (15) by using Eq. (12),

$$\begin{aligned}&\langle\eta|\,(P_1 - P_2)\,|\xi\rangle\\ &= \sqrt{2}\xi_2\,\langle\eta|\,\xi\rangle\\ &= \frac{1}{\sqrt{2}}\xi_2\exp\left[\mathrm{i}\left(\eta_1\xi_2 - \xi_1\eta_2\right)\right],\end{aligned} \tag{17}$$

$$\begin{aligned}&\langle\eta|\,(Q_1 + Q_2)\,|\xi\rangle\\ &= \sqrt{2}\xi_1\,\langle\eta|\,\xi\rangle\\ &= \frac{1}{\sqrt{2}}\xi_1\exp\left[\mathrm{i}\left(\eta_1\xi_2 - \xi_1\eta_2\right)\right].\end{aligned} \tag{18}$$

So

$$\langle\eta|\,\frac{1}{2}\,(P_1 + P_2)\,(Q_1 + Q_2) = \mathrm{i}\eta_2\frac{\partial}{\partial\eta_2}\,\langle\eta|\,. \tag{19}$$

Letting $\eta_2 \equiv \mathrm{e}^y$ yields

$$\begin{aligned}&\langle\eta|\,\frac{1}{2}\,(P_1 + P_2)\,(Q_1 + Q_2)\\ &= \mathrm{i}\,\mathrm{e}^y\frac{\partial y}{\partial\eta_2}\frac{\partial}{\partial y}\,\langle\eta_1, \eta_2 = \mathrm{e}^y|\\ &= \mathrm{i}\frac{\partial}{\partial y}\,\langle\eta_1, \eta_2 = \mathrm{e}^y|\,.\end{aligned} \tag{20}$$

From $\mathrm{e}^{-\lambda\frac{\partial}{\partial y}}f(y) = f(y - \lambda)$ and Eq. (20) we see

$$\langle\eta_1,\eta_2|\,\mathrm{e}^{\mathrm{i}\lambda(P_1+P_2)(Q_1+Q_2)/2}=\mathrm{e}^{-\lambda\frac{\partial}{\partial y}}\langle\eta_1,\eta_2=\mathrm{e}^{y}|=\left\langle\eta_1,\,\mathrm{e}^{y-\lambda}\right|=\left\langle\eta_1,\,\mathrm{e}^{-\lambda}\eta_2\right|, \tag{21}$$

which indicates that unitary operator $\mathrm{e}^{[\mathrm{i}\lambda(P_1+P_2)(Q_1+Q_2)-\lambda]/2}$ plays a role of one-sided squeezing, it squeezes

$$\langle\eta|\,\mathrm{e}^{[\mathrm{i}\lambda(P_1+P_2)(Q_1+Q_2)-\lambda]/2}=\mathrm{e}^{-\lambda/2}\left\langle\eta_1,\,\mathrm{e}^{-\lambda}\eta_2\right|. \tag{22}$$

Using complete-orthonormal relation (4), we have

$$\mathrm{e}^{\mathrm{i}\lambda(P_1+P_2)(Q_1+Q_2)/2}=\int\frac{\mathrm{d}^2\eta}{\pi}\,|\eta\rangle\,\langle\eta|\,\mathrm{e}^{\mathrm{i}\lambda(P_1+P_2)(Q_1+Q_2)/2}=\int\frac{\mathrm{d}^2\eta}{\pi}\,|\eta\rangle\left\langle\eta_1,\,\mathrm{e}^{-\lambda}\eta_2\right|, \tag{23}$$

which is the natural representation of $\mathrm{e}^{\mathrm{i}\lambda(P_1+P_2)(Q_1+Q_2)/2}$ in $|\eta\rangle$. Similarly, from Eq. (15) we can prove

$$\langle\eta|\,\mathrm{e}^{-\mathrm{i}\lambda(Q_1-Q_2)(P_1-P_2)/2}=\left\langle\mathrm{e}^{-\lambda}\eta_1,\eta_2\right|. \tag{24}$$

Due to

$$[(P_1+P_2)(Q_1+Q_2),(Q_1-Q_2)(P_1-P_2)]=0, \tag{25}$$

we see

$$\langle\eta_1,\eta_2|\,\mathrm{e}^{\mathrm{i}\lambda[(P_1+P_2)(Q_1+Q_2)-(Q_1-Q_2)(P_1-P_2)]/2-\lambda}=\langle\eta_1,\eta_2|\,\mathrm{e}^{\mathrm{i}\lambda(P_2Q_1+Q_2P_1)}=\mathrm{e}^{-\lambda}\left\langle\mathrm{e}^{-\lambda}\eta_1,\,\mathrm{e}^{-\lambda}\eta_2\right| \tag{26}$$

or

$$\langle\eta_1,\eta_2|\,\mathrm{e}^{\mathrm{i}\lambda\left(a_1a_2-a_1^\dagger a_2^\dagger\right)}=\mathrm{e}^{-\lambda}\left\langle\mathrm{e}^{-\lambda}\eta_1,\,\mathrm{e}^{-\lambda}\eta_2\right|. \tag{27}$$

From completeness (4) we indeed have usual two-mode squeezing operator

$$\mathrm{e}^{-\lambda}\int\frac{\mathrm{d}^2\eta}{\pi}\,|\eta\rangle\left\langle\mathrm{e}^{-\lambda}\eta_1,\,\mathrm{e}^{-\lambda}\eta_2\right|=\mathrm{e}^{\lambda\left(a_1a_2-a_1^\dagger a_2^\dagger\right)}=\mathrm{e}^{\mathrm{i}\lambda(P_2Q_1+Q_2P_1)}. \tag{28}$$

We can make a similar discussion on the squeezing unitary operator in $\langle\xi|$ representation.

4. Entangled state representation of some complicated exponential quadrature operators for nonlinear squeezing

Now we turn to the nonlinear squeezing, in Eq. (2) letting $\eta_2\equiv 1/w$, so $\partial w/\partial\eta_2=-1/\eta_2^2=-w^2$, we consider the following product of quadrature operators in $\langle\eta|$ representation:

$$\langle\eta|\,\frac{1}{2}\left(P_1+P_2\right)^2(Q_1+Q_2)=\mathrm{i}\sqrt{2}\eta_2^2\frac{\partial w}{\partial\eta_2}\frac{\partial}{\partial w}\left\langle\eta_1,\frac{1}{w}\right|=-\mathrm{i}\sqrt{2}\frac{\partial}{\partial w}\left\langle\eta_1,\frac{1}{w}\right|, \tag{29}$$

then using $\mathrm{e}^{\lambda\frac{\partial}{\partial y}}f(y)=f(y+\lambda)$ we have

$$\langle\eta_1,\eta_2|\,\mathrm{e}^{\mathrm{i}\lambda(P_1+P_2)^2(Q_1+Q_2)/2}=\mathrm{e}^{\sqrt{2}\lambda\frac{\partial}{\partial w}}\left\langle\eta_1,\frac{1}{w}\right|=\left\langle\eta_1,\frac{1}{w+\sqrt{2}\lambda}\right|=\left\langle\eta_1,\frac{\eta_2}{1+\sqrt{2}\lambda\eta_2}\right|. \tag{30}$$

Noticing

$$\left[\left(P_1+P_2\right)^2(Q_1+Q_2),(P_1+P_2)(Q_1+Q_2)\right]=-2\mathrm{i}\left(P_1+P_2\right)^2(Q_1+Q_2), \tag{31}$$

and using the operator identity

$$\mathrm{e}^{A+B}=\mathrm{e}^{A\left(\mathrm{e}^{\lambda}-1\right)/\lambda}\mathrm{e}^{B},\qquad[A,B]=-\lambda A, \tag{32}$$

we have

$$\begin{aligned}&\mathrm{e}^{\mathrm{i}\lambda(P_1+P_2)^2(Q_1+Q_2)/2-\mathrm{i}f(P_1+P_2)(Q_1+Q_2)/2}\\&=\exp\left[\mathrm{i}\lambda\frac{\mathrm{e}^f-1}{f}\left(P_1+P_2\right)^2(Q_1+Q_2)/2\right]\mathrm{e}^{-\mathrm{i}f(P_1+P_2)(Q_1+Q_2)/2}.\end{aligned} \tag{33}$$

From Eqs. (33) and (30) we can deduce

$$\begin{aligned}&\langle\eta_1,\eta_2|\ \mathrm{e}^{\mathrm{i}\lambda(P_1+P_2)^2(Q_1+Q_2)/2-\mathrm{i}f(P_1+P_2)(Q_1+Q_2)/2}\\ =\ &\langle\eta_1,\eta_2|\exp\left[\mathrm{i}\lambda\frac{\mathrm{e}^f-1}{f}\left(P_1+P_2\right)^2\left(Q_1+Q_2\right)/2\right]\ \mathrm{e}^{-\mathrm{i}f(P_1+P_2)(Q_1+Q_2)/2}\\ =\ &\langle\eta_1,\tilde{\eta}_2|\ \mathrm{e}^{-\mathrm{i}f(P_1+P_2)(Q_1+Q_2)/2}=\left\langle\eta_1,\ \mathrm{e}^f\tilde{\eta}_2\right|,\end{aligned}\tag{34}$$

where

$$\tilde{\eta}_2=\frac{\eta_2}{1+\sqrt{2}\lambda\dfrac{\mathrm{e}^f-1}{f}\eta_2}.\tag{35}$$

Using completeness relation (4) yields the following entangled state representation of this exponential quadrature operator for nonlinear squeezing:

$$\mathrm{e}^{\mathrm{i}\lambda(P_1+P_2)^2(Q_1+Q_2)/2-\mathrm{i}f(P_1+P_2)(Q_1+Q_2)/2}=\int\frac{\mathrm{d}^2\eta}{\pi}\left|\eta\right\rangle\left\langle\eta_1,\tilde{\eta}_2\right|.\tag{36}$$

Similarly, let $\tau_j\equiv1/\eta_j^{n-1},(j=1,2)$, and $\partial\tau_j/\partial\eta_j=-(n-1)/\eta_j^n$, (where $n\geq2$), we see

$$\begin{aligned}&\langle\eta|\frac{1}{2}\left(P_1+P_2\right)^n\left(Q_1+Q_2\right)\\ =\ &\mathrm{i}\sqrt{2^{n-1}}\left\langle\eta\right|\eta_2^n\frac{\partial}{\partial\eta_2}\left\langle\eta_1,\eta_2\right|\\ =\ &-\mathrm{i}\left(n-1\right)\sqrt{2^{n-1}}\frac{\partial}{\partial\tau_2}\left\langle\eta_1,\frac{1}{\sqrt[n-1]{\tau_2}}\right|,\end{aligned}\tag{37}$$

and

$$\begin{aligned}&\langle\eta|\frac{1}{2}\left(Q_1-Q_2\right)^n\left(P_1-P_2\right)\\ =\ &\mathrm{i}\left(n-1\right)\sqrt{2^{n-1}}\frac{\partial}{\partial\tau_1}\left\langle\frac{1}{\sqrt[n-1]{\tau_1}},\eta_2\right|,\end{aligned}\tag{38}$$

then using $\mathrm{e}^{-\lambda\frac{\partial}{\partial y}}f\left(y\right)=f\left(y-\lambda\right)$ we have

$$\langle\eta_1,\eta_2|\ \mathrm{e}^{\mathrm{i}\lambda(P_1+P_2)^n(Q_1+Q_2)/2}=\left\langle\eta_1,\frac{\eta_2}{K_2}\right|,\tag{39}$$

$$\langle\eta_1,\eta_2|\ \mathrm{e}^{-\mathrm{i}\lambda(Q_1-Q_2)^n(P_1-P_2)/2}=\left\langle\frac{\eta_1}{K_1},\eta_2\right|,\tag{40}$$

where

$$\begin{aligned}K_j=\ &\sqrt[n-1]{1+\lambda\left(n-1\right)\sqrt{2^{n-1}}\eta_j^{n-1}},\\ &\left(j=1,2\right).\end{aligned}\tag{41}$$

In particular, when $n=2$, Eq. (39) reduces to Eq. (30). Noting that $[(P_1+P_2)^n(Q_1+Q_2),(Q_1-Q_2)^n\left(P_1-P_2\right)]=0$, we can further derive

$$\begin{aligned}&\mathrm{e}^{\mathrm{i}\lambda[(P_1+P_2)^n(Q_1+Q_2)-(Q_1-Q_2)^n(P_1-P_2)]/2}\\ =\ &\int\frac{\mathrm{d}^2\eta}{\pi}\left|\eta\right\rangle\left\langle\frac{\eta_1}{K_1},\frac{\eta_2}{K_2}\right|.\end{aligned}\tag{42}$$

Thus we obtain a new approach to deriving two-mode nonlinear squeezing operators.

5. Wave function of nonlinear squeezed state in ESR

The advantage of Eqs. (42) and (36) lies in the fact that it directly leads us to derive the wave function of the nonlinear squeezed state in ESR, i.e., it is very convenient for us to calculate the wave function (WF) by using the above results. For example, for two-mode nonlinear squeezing

$$\begin{aligned}&\mathrm{e}^{\mathrm{i}\lambda(P_1+P_2)^2(Q_1+Q_2)/2-\mathrm{i}f(P_1+P_2)(Q_1+Q_2)/2}\left|\psi\right\rangle\\ \equiv\ &S\left|\psi\right\rangle,\end{aligned}\tag{43}$$

whose wave function in $\langle\eta|$ representation is given by Eqs. (36) and (5) as

$$\begin{aligned}\left\langle\eta\right|S\left|\psi\right\rangle\ &=\ \left\langle\eta\right|\int\frac{\mathrm{d}^2\eta'}{\pi}\left|\eta'\right\rangle\left\langle\eta_1',\tilde{\eta}_2'\right|\ \psi\rangle\\ &=\ \left\langle\eta_1,\tilde{\eta}_2\,\mathrm{e}^f\ \right|\psi\rangle,\end{aligned}\tag{44}$$

the wave function of $S\left|\psi\right\rangle$ can be caluculated directly by $\langle\eta|\ \psi\rangle$. In particular, when $\left|\psi\right\rangle=\left|00\right\rangle$, then from

$$\left\langle\eta\right|\ 00\rangle=\exp\left[-\frac{1}{2}\left(\eta_1^2+\eta_2^2\right)\right],\tag{45}$$

we know that the wave function is

$$\begin{aligned}&\left\langle\eta\right|S\left|00\right\rangle=\exp\left[-\frac{1}{2}\left(\eta_1^2+\tilde{\eta}_2^2\,\mathrm{e}^{2f}\right)\right],\\ &\tilde{\eta}_2=\frac{\eta_2}{1+\sqrt{2}\lambda\dfrac{\mathrm{e}^f-1}{f}\eta_2}.\end{aligned}\tag{46}$$

Thus we provide a new approach to calculating wave functions of some two-mode nonlinear squeezed states.

The resolution of unity in differential form for EPR entangled state representation and its application in nonlinear operators normal ordering expansion are shown in Ref. [22].

In summary, after knowing the properties of two-mode quadratures in the entangled state representation, we derive some complicated exponential quadrature operators for nonlinear two-mode squeezing, which directly leads to the wave function of the nonlinear squeezed state.

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