Chaos and chaotic control in a relative rotation nonlinear dynamical system under parametric excitation*

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This paper studies the chaotic behaviours of a relative rotation nonlinear dynamical system under parametric excitation and its control. The dynamical equation of relative rotation nonlinear dynamical system under parametric excitation is deduced by using the dissipation Lagrange equation. The criterion of existence of chaos under parametric excitation is given by using the Melnikov theory. The chaotic behaviours are detected by numerical simulations including bifurcation diagrams, Poincaré map and maximal Lyapunov exponent. Furthermore, it implements chaotic control using non-feedback method. It obtains the parameter condition of chaotic control by the Melnikov theory. Numerical simulation results show the consistence with the theoretical analysis. The chaotic motions can be controlled to period-motions by adding an excitation term.

Keywords: relative rotation, nonlinear dynamical system, parametric excitation, chaotic control
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1. Introduction

Since the theory of rotational relativistic mechanics was first established by Carmeli in 1985,[1,2] the rotational relativistic system has been an active research topic. In 1996, Luo set up the theory of relativistic analytical mechanics of the rotational systems and extended the Hamilton principle, Lagrange equation and Hamilton canonical equation on rotational relativistic system.[3,4] Recently, the algebraic construction, Noether symmetry and Lie symmetry of rotational relativistic system have been researched.[5,6] The theory, the time and space construction and the geometric character of rotation are known. In recent years, more attentions have been paid to the research field on the rotational relativistic system. Many achievements have been obtained in this popular research field, which include the dynamical characteristic of relativistic variable mass systems,[7−9] the basic integration theory of the dynamics of a rotational relativistic system,[10] the theory of relativistic analytical statics of rotational systems,[11] the basic theory of relativistic Birkhoffian dynamics of rotational system,[12−21] etc. The stability, integral solution and series solution for the relative-rotation motion equation have been reported in Refs. [22]–[25]. The dynamic behaviours of relative rotation nonlinear dynamic system have been analysed in Refs. [26]–[34]. The chaotic behaviours of a relative rotation nonlinear dynamic system under nonlinear stiffness and nonlinear damping have been investigated in our previously works.[35]

In this paper, the chaos and its control in a relative rotation nonlinear dynamical system under parametric excitation are studied by theoretical analysis and numerical simulation method. The dynamic equation of system under parametric excitation is deduced according to the dissipation Lagrange equation of analytical mechanics. The criterion of existence of chaos under parametric excitation is obtained by using the Melnikov theory. The chaotic behaviours are further verified by the numerical simulations including bifurcation diagrams, Poincaré map and the maximal Lyapunov exponent. Furthermore, we implement chaotic control by adding an excitation term. The chaotic control is illustrated by theoretical analysis and numerical simulations.

The paper is organised as follows. The novel relative rotation nonlinear dynamical system under

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parametric excitation is presented in Section 2. The chaotic behaviours of a relative rotation nonlinear dynamical system under parametric excitation are analysed in Section 3. The chaotic control is illustrated in Section 4. Finally, conclusions are presented in Section 5.

2. Dynamical system

Based on our previously works,[30,35] we study the system of two inertial relative rotation under nonliearity parametric excitation torsional rigidity and nonlinear damping force. The total kinetic and potential energy of the system are given by

$$E = \sum_{i=1}^{2} \frac{1}{2} I_i \dot{\theta}_i^2 = \frac{1}{2} I_1 \dot{\theta}_1^2 + \frac{1}{2} I_2 \dot{\theta}_2^2, \tag{1}$$

$$U = \frac{1}{2} K \left[1 + f_1 \cos(\Omega t)\right] (\theta_1 - \theta_2)^2 + \frac{1}{4} K_1 (\theta_1 - \theta_2)^4, \tag{2}$$

where I_i $(i = 1, 2)$ is the moment of inertia; K is the linearity torsional rigidity. $f_1 \cos(\Omega t)$ represents the nonlinear parametric excitation item; K_1 denotes the cubic nonlinearity torsional rigidity of the system; θ_i $(i = 1, 2)$ and $\dot{\theta}_i$ $(i = 1, 2)$ are angle of rotation and rotational speed respectively.

The generalised force Q_i $(i = 1, 2)$ is given by

$$Q_j = \sum_{i=1}^{2} F_i^i \frac{\partial \theta_i}{\partial q_j}, \quad (j = 1, 2), \tag{3}$$

where $F_i^i = T_i + F_i^{\rm c}$; T_i is generalised force(generalised torque); $F_i^{\rm c}$ is generalised damping force; q_j $(j = 1, 2)$ is generalised coordinates.

Suppose

$$F_1^{\rm c} = -D(\dot{\theta}_1 - \dot{\theta}_2), \tag{4}$$

$$F_2^{\rm c} = -D(\dot{\theta}_2 - \dot{\theta}_1) + f(\dot{\theta}_1 - \dot{\theta}_2), \tag{5}$$

then

$$F_1^1 = T_1 - D(\dot{\theta}_1 - \dot{\theta}_2), \tag{6}$$

$$F_2^2 = T_2 - D(\dot{\theta}_2 - \dot{\theta}_1) + f(\dot{\theta}_1 - \dot{\theta}_2), \tag{7}$$

where D denotes linearity damping coefficient; $f(\dot{\theta}_1 - \dot{\theta}_2)$ is nonlinearity damping force.

Using the dissipation Lagrange equation

$$\frac{\rm d}{{\rm d}t} \frac{\partial E}{\partial \dot{q}_j} - \frac{\partial E}{\partial q_j} + \frac{\partial U}{\partial q_j} = Q_j \quad (j = 1, 2), \tag{8}$$

we obtain the following Lagrange equations of motion:

$$I_1 \ddot{\theta}_1 + K \left[1 + f_1 \cos(\Omega t)\right] (\theta_1 - \theta_2) + K_1 (\theta_1 - \theta_2)^3 + D(\dot{\theta}_1 - \dot{\theta}_2) = T_1, \tag{9}$$

$$I_2 \ddot{\theta}_2 + K \left[1 + f_1 \cos(\Omega t)\right] (\theta_2 - \theta_1) + K_1 (\theta_2 - \theta_1)^3 + D(\dot{\theta}_2 - \dot{\theta}_1) - f(\dot{\theta}_1 - \dot{\theta}_2) = T_2, \tag{10}$$

where $\ddot{\theta}_i (i = 1, 2)$ is the angular acceleration. From Eqs. (9) and (10) we obtain the following equation of relative angle:

$$(\ddot{\theta}_1 - \ddot{\theta}_2) + \frac{(I_1 + I_2)}{I_1 I_2} K \left[1 + f_1 \cos(\Omega t)\right] (\theta_1 - \theta_2) + \frac{(I_1 + I_2)}{I_1 I_2} K_1 (\theta_1 - \theta_2)^3 + \frac{(I_1 + I_2)}{I_1 I_2} D(\dot{\theta}_1 - \dot{\theta}_2) + \frac{I_1}{I_1 I_2} f(\dot{\theta}_1 - \dot{\theta}_2) = \frac{1}{I_1 I_2} (I_2 T_1 - I_1 T_2). \tag{11}$$

Let $\theta_\Delta = \theta_1 - \theta_2$, $\dot{\theta}_\Delta = \dot{\theta}_1 - \dot{\theta}_2$, $\ddot{\theta}_\Delta = \ddot{\theta}_1 - \ddot{\theta}_2$, $a_0 = (I_1 + I_2)/I_1 I_2$, $f(\dot{\theta}_1 - \dot{\theta}_2) = -\alpha(\dot{\theta}_1 - \dot{\theta}_2) + \beta(\dot{\theta}_1 - \dot{\theta}_2)^3$, $a_1 = K a_0$, $a_2 = K_1 a_0$, $b_0 = I_2/I_1 I_2$, $b_1 = \alpha b_0 - a_0 D$, $b_2 = \beta b_0$, $F(t) = b_0 T_1 + (b_0 - a_0) T_2$, we can rewrite Eq. (10)

$$\ddot{\theta}_\Delta + a_1 \left[1 + f_1 \cos(\Omega t)\right] \theta_\Delta + a_2 \theta_\Delta^3 - b_1 \dot{\theta}_\Delta + b_2 \dot{\theta}_\Delta^3 = F(t). \tag{12}$$

3. Chaos for parametric excitation

Let $a_1 = \omega_0^2$, $\omega_0 t = \tau$, $\theta_\Delta / A = \varphi$, $a = b_1/\omega_0$, $b = b_2 \omega_0 A^2$, $c = A^2/\omega_0^2 a_2$, $\omega = \Omega/\omega_0$, $F = 1/\omega_0^2$, $A = b_0 f$, $T_1 = T_2 = 0$, Eq. (12) is rewritten as

$$\ddot{\varphi} + \left[1 + f_1 \cos(\omega \tau)\right] \varphi + c\varphi^3 - a\dot{\varphi} + b\dot{\varphi}^3 = 0. \tag{13}$$

Let $\varphi = \phi$, $\dot{\varphi} = \dot{\phi} = \psi$, Eq. (13) can be rewritten

as

$$\begin{cases} \dot{\phi} = \psi, \\ \dot{\psi} = -\left[1 + f_1 \cos(\omega\tau)\right]\phi - c\phi^3 + a\psi - b\psi^3. \end{cases} \tag{14}$$

Consider the small parameter ε, Eq. (14) is rewritien as

$$\begin{cases} \dot{\phi} = \psi, \\ \dot{\psi} = -\phi - c\phi^3 + \varepsilon\left(a\psi - b\psi^3 - f_1 \cos(\omega\tau)\,\phi\right). \end{cases} \tag{15}$$

If $\varepsilon = 0$, then Eq. (15) is considered as an unperturbed system and can be written as

$$\begin{cases} \dot{\phi} = \psi, \\ \dot{\psi} = -\phi - c\phi^3. \end{cases} \tag{16}$$

The unperturbed system has two heteroclinic orbits

$$\begin{cases} \phi^0(\tau) = \pm\sqrt{\dfrac{1}{-c}}\tanh\left(\sqrt{\dfrac{1}{2}}\tau\right), \\ \psi^0(\tau) = \pm\sqrt{\dfrac{1}{-2c}}\,\mathrm{sech}^2\left(\sqrt{\dfrac{1}{2}}\tau\right). \end{cases} \tag{17}$$

The heteroclinic orbits break when the perturbation is added. According to the Smale–Birkhoff homoclinic theorem, the existence of such orbits results in chaotic dynamics. To study the chaotic behaviours of Eq. (15), we use the Melnikov method.[36–39] The Melnikov function for Eq. (15) is

$$\begin{aligned} M(\tau_0) &= \int_{-\infty}^{\infty}\left[a\psi^0(\tau) - b\left[\psi^0(\tau)\right]^3 - f_1\cos\left(\omega\left(\tau+\tau_0\right)\right)\phi^0(\tau)\right]\psi^0(\tau)\,\mathrm{d}\tau \\ &= \int_{-\infty}^{\infty} a\left[\psi^0(\tau)\right]^2\mathrm{d}\tau + \int_{-\infty}^{\infty} -b\left[\psi^0(\tau)\right]^4\mathrm{d}\tau - \int_{-\infty}^{\infty}\psi^0(\tau)\,\phi^0(\tau)\,f_1\cos\left(\omega\left(\tau+\tau_0\right)\right)\mathrm{d}\tau \\ &= \int_{-\infty}^{\infty} a\left[\psi^0(\tau)\right]^2\mathrm{d}\tau + \int_{-\infty}^{\infty} -b\left[\psi^0(\tau)\right]^4\mathrm{d}\tau + f_1\sin(\omega\tau_0)\int_{-\infty}^{\infty}\psi^0(\tau)\,\phi^0(\tau)\sin(\omega\tau)\,\mathrm{d}\tau \\ &= I_1 + I_2 + f_1 I_3\sin(\omega\tau_0), \end{aligned} \tag{18}$$

where τ_0 is the cross section time of the Poincaré map and can be interpreted as the initial time of the forcing term,

$$\begin{aligned} I_1 &= \int_{-\infty}^{\infty} a\left[\psi^0(\tau)\right]^2\mathrm{d}\tau = -\frac{2\sqrt{2}}{3c}a, \\ I_2 &= \int_{-\infty}^{\infty} -b\left[\psi^0(\tau)\right]^4\mathrm{d}\tau = -\frac{8\sqrt{2}}{35c^2}b, \\ I_3 &= \int_{-\infty}^{\infty}\psi^0(\tau)\,\phi^0(\tau)\sin(\omega\tau)\,\mathrm{d}\tau \\ &= \frac{\pi\omega^2}{c}\,\mathrm{csch}\left(\frac{\sqrt{2}}{2}\pi\omega\right). \end{aligned}$$

Obviously, when f_1, a, b, c, ω satisfy $f_1 > \tilde{f} = I_1 + I_2/I_3$, $M(\tau_0)$ has simple zeros, namely

$$f_1 > \frac{\dfrac{2\sqrt{2}}{3c}a + \dfrac{8\sqrt{2}}{35c^2}b}{\dfrac{\pi\omega^2}{c}\,\mathrm{csch}\left(\dfrac{\sqrt{2}}{2}\pi\omega\right)}. \tag{19}$$

This implies that if ε is sufficiently small, the transverse heteroclinic orbits exist and Eq. (15) may be chaotic.

Take $c = -1$, $\varepsilon a = -0.7$, $\varepsilon b = 0.7$, $\omega = 2$, and let εf_1 change in a wide range. The bifurcation diagram of Eq. (15) in (εf_1, ψ) plane is shown in Fig. 1.

The maximal Lyapunov exponents corresponding to Fig. 1 is shown in Fig. 2. The chaotic theory valve of εf_1 is 2.9976 according Eq. (19). From Fig. 1, we can see that the nonlinear dynamic equation (15) exhibits chaotic behaviours when the parameter εf_1 is [3.0, 3.3]. The largest Lyapunov exponent given by Fig. 2 can be convinced from the occurrence of chaotic motion. Figure 3 describes the phase trajectory of a chaotic state in Fig. 1 with $\varepsilon f_1 = 0.327$, Fig. 4 shows chaotic attractor for Poincaré map corresponding to Fig. 3. The largest Lyapunov exponent is found to be positive, and indicates chaotic behaviours.

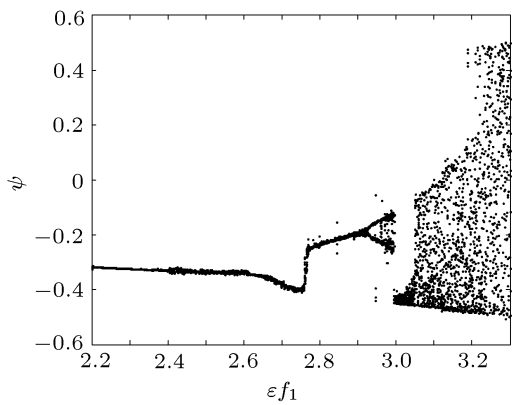


Fig. 1. Bifurcation diagram of system (16) in (εf_1, ψ) plane for $c = -1$, $\varepsilon a = -0.7$, $\varepsilon b = 0.7$, $\omega = 2$.

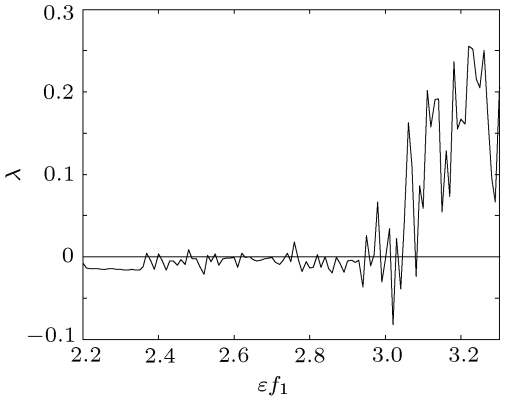


Fig. 2. The largest Lyapunov exponents corresponding to Fig. 1.

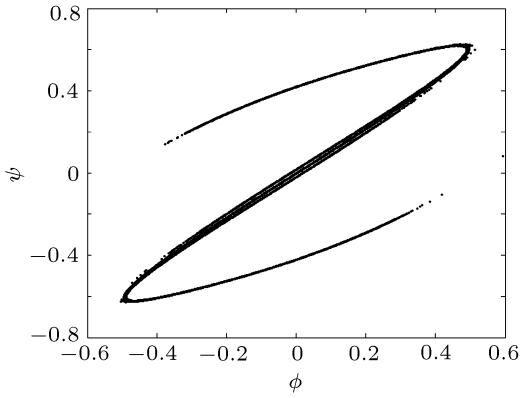


Fig. 4. Strange chaotic attractor for Poincaré map corresponding to Fig. 3.

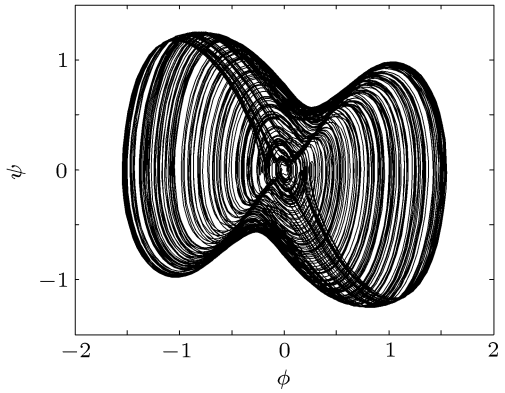


Fig. 3. Phase trajectory of a chaotic state in Fig. 1 with $\varepsilon f_1 = 0.327$.

4. Chaotic control

To suppress chaos, we use non-feedback method to make the Melnikov function of the new system having no simple zeros. The method is realized by adding the excitation term. Now the form of the new system is

$$\begin{aligned}\dot{\phi} &= \psi,\\ \dot{\psi} &= -\phi - c\phi^3 + \varepsilon(a\psi - b\psi^3 - f_1\cos(\omega\tau)\,\phi\\ &\quad -\mu f_1\cos(\omega\tau+\varphi)),\end{aligned} \tag{20}$$

where $\mu f_1\cos(\omega\tau+\varphi)$ represents an added excitation term; μ, ω, φ are amplitude, frequency and phase of the excitation term respectively. The Melnikov function of system Eq. (20) is

$$\begin{aligned}M(\tau_0) &= \int_{-\infty}^{\infty}\left[a\psi^0(\tau) - b\left[\psi^0(\tau)\right]^3 - f_1\cos(\omega(\tau+\tau_0))\,\phi^0(\tau) - \mu f_1\cos(\omega(\tau+\tau_0+\varphi))\right]\psi^0(\tau)\,\mathrm{d}\tau\\ &= I_1 + I_2 + f_1 I_3\sin(\omega\tau_0) - \int_{-\infty}^{\infty}\psi^0(\tau)\,\mu f_1\cos(\omega(\tau+\tau_0+\varphi))\,\mathrm{d}\tau\\ &= I_1 + I_2 + f_1 I_3\sin(\omega\tau_0) - \mu f_1 I_4\cos(\omega\tau_0+\varphi)\\ &= I_1 + I_2 + f_1 I_3\sin(\omega\tau_0) - \mu f_1 I_4\sin(\omega\tau_0+\varphi')\\ &= I_1 + I_2 + f_1\left[(I_3 - \mu I_4\cos\varphi')\sin(\omega\tau_0) - \mu I_4\sin\varphi'\cos(\omega\tau_0)\right]\\ &= I_1 + I_2 + f_1 A\sin(\omega\tau_0+\psi),\end{aligned} \tag{21}$$

where

$$I_4 = \int_{-\infty}^{\infty}\psi^0(\tau)\sin(\omega\tau)\,\mathrm{d}\tau = \sqrt{\frac{1}{-2\mathrm{c}}}\int_{-\infty}^{\infty}\mathrm{sech}^2\left(\sqrt{\frac{1}{2}}\tau\right)\cos(\omega\tau)\,\mathrm{d}\tau = \sqrt{\frac{2}{-\mathrm{c}}}\pi\omega\,\mathrm{csch}\left(\frac{\sqrt{2}}{2}\pi\omega\right),$$

$$A = \sqrt{I_3^2 - 2\mu I_3 I_4\cos\varphi' + \mu^2 I_4^2},\quad \tan\psi = \frac{-2\mu I_4\sin\varphi'}{I_3 - \mu I_4\cos\varphi'}.$$

Introducing a new variable: $\mu' = \mu I_4/I_3$, we can write Eq. (21) as follows:

$$M(\tau_0) = I_1 + I_2 + f_1 I_3 A'\sin(\omega\tau_0+\psi), \tag{22}$$

where

$$A' = \sqrt{1 - 2\mu'\cos\varphi' + \mu'^2},\qquad \varphi' = \varphi + \frac{\pi}{2},$$

when

$$f_1 I_3 A' < I_1 + I_2. \tag{23}$$

That is

$$A' < \frac{I_1 + I_2}{f_1 I_3}, \tag{24}$$

$M(\tau_0)$ has no zeros thus the chaos is suppressed.

Obviously, when the parameters f_1, a, b, c, ω, μ, ϕ satisfy $A' < I_1 + I_2/f_1 I_3$, $M(\tau_0)$ has simple zeros. The suppression of chaos can be fulfilled by adjusting the two parameters μ and ϕ In Section 3, the parametric excitation system expresses chaotic behaviours when we fix $c = -1$, $\varepsilon a = -0.7$, $\varepsilon b = 0.7$, $\omega = 2$, $f_1 = 3.27$. The chaotic behaviours can be controlled when A' is less than 0.9167 according the calculation of Eq. (24). Take $\mu = -1.0$, $\phi = 0.5$ with the initial condition $(x,\ y)$=(–0.3, 0.5), and let εf_1 to change in the range of chaos. The bifurcation diagram after control is shown in Fig. 5. The maximal Lyapunov exponents corresponding to Fig. 5 is shown in Fig. 6.

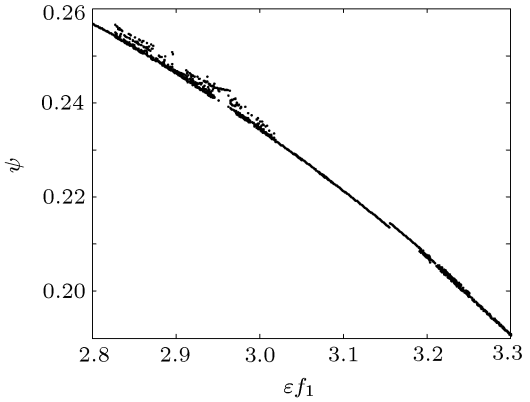


Fig. 5. Bifurcation diagram of Eq. (21) in (εf_1, ψ) plane for $c = -1$, $\varepsilon a = -0.7$, $\varepsilon b = 0.7$, $\omega = 2$, $\mu = -1.0$, $\phi = 0.5$.

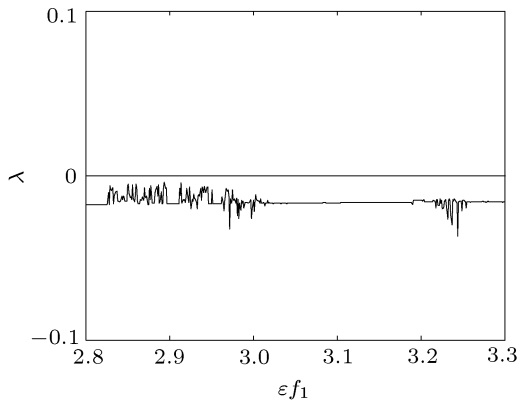


Fig. 6. The largest Lyapunov exponents corresponding to Fig. 5.

The largest Lyapunov exponent is found to be negative, and it indicates chaotic behaviour to be controlled. Figure 7 describes the phase trajectory with $\varepsilon f_1 = 0.327$ in Fig. 5. Figure 8 shows Poincaré map corresponding to Fig. 7. The largest Lyapunov exponent is found to be negative, and it indicates chaotic behaviours to be controlled in periodic motion, which illustrates the validity of this method.

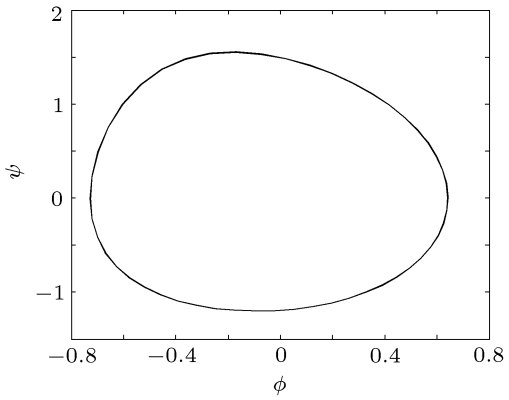


Fig. 7. A closed orbit in phase plane in Fig. 5 with $\varepsilon f_1 = 0.327$ for $\mu = -1.0$, $\phi = 0.5$.

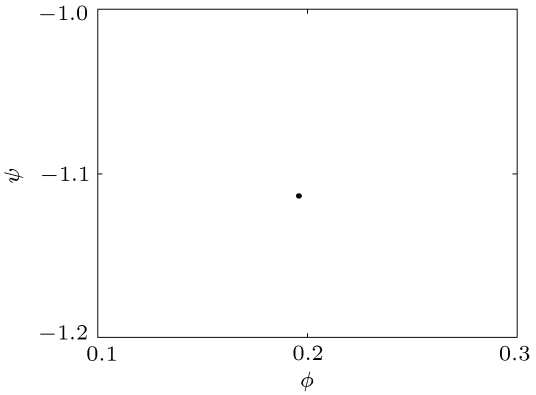


Fig. 8. Poincaré map corresponding to Fig. 7.

5. Conclusions

In this paper we study chaos and its control for a relative rotation nonlinear dynamical system under parametric excitation. First, the criterion of existence of chaos under parametric excitation is obtained by using the Melnikov theory. Secondly, the suppressing chaos can be realized based on the Melnikov function. The chaotic behaviours are suppressed by adding an excitation term. Also the top Lyapunov exponent, Poincaré cross-section and phase portrait are carried out to confirm the obtained results. Thus excellent agreement between these results is found.

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