One-dimensional nonparaxial bright solitons and their coherent interactions in Kerr media*

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The existence of one-dimensional bright Kerr solitons is investigated in Kerr media beyond the paraxial approximation. It is found that a nonparaxial soliton with no less than a minimum dimensionless width of about 0.76 can exist, which corresponds to the real width about a wavelength. Besides, the coherent interactions between two nonparaxial bright solitons in Kerr media are also investigated in detail. It is found that their separation and intensity ratio have great influence on the coherent interaction between these two solitons. Furthermore, the effect of the relative phase difference on the nonparaxial interaction is quite different from that on the paraxial interaction. Periodical breath, merging, repulsion, and energy transferring can be realized separately by choosing an appropriate initial relative phase between the coherent solitons.

Keywords: spatial soliton, nonparaxial soliton, coherent interaction

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1. Introduction

There has been increasing interest in optical spatial solitons since they are found to be potentially useful for serving as all-optical switching, beam steering and optical interconnects.[1−4] In most of the past researches, the spatial size of the optical beam is usually assumed to be much greater than the wavelength of the carrying electromagnetic wave. In those circumstances, the scalar parabolic equation, i.e., the nonlinear Schrödinger equation, is widely used in the self-focusing theories. This scalar equation, derived from the Helmholtz equation in the slowly varying amplitude and the paraxial approximations, provides an excellent description for the soliton dynamics. However, if the beam width is comparable to the wavelength, these approximations become invalid and many limitations begin to emerge in realistic applications,[5−8] e.g., catastrophic collapse will appear at the focus. Eventually, nonparaxial effects should be employed to avoid the nonphysical behaviours in the beam evolution. In the past years, many researchers contributed their efforts in this field. Early in 1975, Lax *et al.*[7] presented a systematic approach to nonparaxial propagation. Their approximation scheme is based on a power series expansion of the electric field vector. Later, Feit and Fleck[8] introduced higher-order diffraction terms into the Helmholtz equation and showed that these diffraction terms limit self-focusing indeed and caused the beam intensity to oscillate along the axis of the propagation. Recently, there have been some attempts to incorporate the vectorial nature of optical propagation.[9−11] De la Fuente *et al.*[12] presented a vector model derived by appropriately simplifying the exact equations in the spatial frequency domain. Ciattoni *et al.*[13] derived a propagation equation that describes the forward propagation of highly focused beams in Kerr media to all order nonparaxial corrections. Soon after, they[14,15] investigated this nonlinear propagation behaviour and predicted the existence of both bright and dark solitons in the (1+1)-dimensional case. Furthermore, 2-dimensional (2D) azimuthally polarized spatial dark solitons were also proved to exist as exact solutions of Maxwell's equation in the presence of the vectorial Kerr effect.[16] Very recently, we[17,18] proved that circularly polarized solitons and radially polarized vortex solitons can also exist in Kerr effect beyond the paraxial approximations.

On the other hand, the interaction behaviour of two optical spatial solitons in various types of media has been extensively studied theoretically and experimentally.[1,19] There are two possible types of soliton beam interactions: coherent and incoherent. Incoherent interaction occurs if the relative phase be-

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tween the beams varies much faster than the response time of the medium,[20] while coherent interaction occurs when the nonlinear medium can respond to the interference effect between the overlapping beams.[21] In the present paper, we consider the Kerr medium with an instantaneous time response and thus coherent interaction can occur. It is well known that in the paraxial case the coherent beams always attract each other when they are in phase, while they repel each other when they are out of phase.[20] Our question naturally arises: what will happen if the nonparaxial effect cannot be ignored? That is just the topic of this paper. In this paper, we first seek the bright soliton solution in the self-focusing medium beyond the nonparaxial approximation, and then discuss the effect of the separation, the relative phase difference, and the intensity ratio of these two solitons on the interaction behaviour.

2. Nonparaxial bright solitons

To investigate the coherent interaction behaviours of nonparaxial optical beams in self-focusing Kerr media in the (1+1)-dimensional case, we first concentrate our attention on seeking the nonparaxial bright soliton solutions. We assume that the optical field, propagating along the z direction, is initially polarized along the x direction and the y component remains negligible over the distance of propagation. Then, the x component of the electric field $A_x = A(x,z)$ obeys the following equation:[12,13]

$$\left(\mathrm{i}\frac{\partial}{\partial z}+\frac{1}{2k}\frac{\partial^2}{\partial x^2}\right)A(x,z)=-k\frac{n_2}{n_0}\left(|A|^2A+\frac{1}{3k^2}|A|^2\frac{\partial^2 A}{\partial x^2}+\frac{8}{3k^2}A\left|\frac{\partial A}{\partial x}\right|^2+\frac{5}{6k^2}A^2\left|\frac{\partial^2 A^*}{\partial x^2}\right|\right), \tag{1}$$

where n_2 is the nonlinear refractive-index coefficient, $k = n_0\omega/c$, n_0 is the linear refractive index of the nonlinear medium, ω is the angular frequency, and c is the light speed in vacuum. Since the defocusing medium ($n_2 < 0$) cannot support bright solitons, we consider only the self-focusing medium ($n_2 > 0$) hereafter.

For the sake of convenience, we adopt the following dimensionless coordinates and variables: $\zeta = z/(kx_0^2)$, $\xi = x/x_0$ and $u = kx_0\sqrt{n_2/n_0}A$, where x_0 is the soliton width parameter. It should be noted that the normalized coordinates used here are different from those in Ref. [12]. The normalized coordinates used here are more convenient to show the non-paraxial degree as well as the competing relation between the wavelength and the beam width. Employing these transformations, we find that the normalized planar envelope u satisfies

$$\mathrm{i}\frac{\partial u}{\partial \zeta}+\frac{1}{2}\frac{\partial^2 u}{\partial \xi^2}+|u|^2u+a\left(\frac{1}{3}|u|^2\frac{\partial^2 u}{\partial \xi^2}+\frac{8}{3}u\left|\frac{\partial u}{\partial \xi}\right|^2+\frac{5}{6}u^2\frac{\partial^2 u^*}{\partial \xi^2}\right)=0, \tag{2}$$

where $a = 1/(kx_0)^2$ is the non-paraxial parameter. If $a = 0$, equation (2) reduces to the standard paraxial nonlinear Schrödinger equation. On the other hand, when a is large (for example, $x_0 = \lambda$ results in $a = 0.025$), the non-paraxial effect becomes important. In this paper, we fix $a = 0.015$, which corresponds to $x_0 \approx 1.3\lambda$.

In the following, we look for soliton solutions in the form $u(\xi,\zeta) = y(\xi)\exp(\mathrm{i}\beta\zeta)$ of Eq. (2), where β is the propagation constant and $y(\xi)$ is a real function of ξ. After a few lines of algebras, equation (2) can be reduced to

$$\frac{\mathrm{d}f}{\mathrm{d}y}+\frac{32ay}{3+7ay^2}f=\frac{12\left(\beta-y^2\right)}{3+7ay^2}y, \tag{3}$$

where $f = (\mathrm{d}y/\mathrm{d}\xi)^2$. Let y_0 be the maximum value of the field envelope, and then we have $f(y_0) = 0$ and $f(0) = 0$ for a bright soliton. The substitution of these boundary conditions into Eq. (3) directly leads to

$$f=\frac{3}{8a}\left(\beta-y^2\right)-\frac{3}{8a}\left(\beta-y_0^2\right)\left(\frac{3+7ay_0^2}{3+7ay^2}\right)^{16/7}, \tag{4}$$

where $\beta = \int_0^{y_0^2} I\left(3+7aI\right)^{9/7}\mathrm{d}I/\int_0^{y_0^2}\left(3+7aI\right)^{9/7}\mathrm{d}I$. Obviously, it is very difficult to obtain the analytic solution of Eq. (4), and thus we numerically integrate this equation to obtain the non-paraxial spatial soliton envelopes. Figure 1(a) shows the normalized intensities of non-paraxial solitons for different values of y_0 when the non-paraxial parameter $a = 0.015$. From this figure, one can easily see that the solitons become narrow when the intensity increases. This feature can also be understood explicitly in the existence curve of the nonparaxial solitons. To obtain the existence curve, we express the full width at half maximum (FWHM), from Eq. (4), as

$$\Delta\xi_{\rm FWHM} = 2\int_{y_0}^{y_0/\sqrt{2}} \left[\frac{12}{(3+7ay^2)^{16/7}}\int_{y_0}^{y} \tilde{y}\left(\beta-\tilde{y}^2\right)\left(3+7a\tilde{y}^2\right)^{9/7}{\rm d}\tilde{y}\right]^{-1/2}{\rm d}y. \quad (5)$$

Then, we can plot an existence curve of these non-paraxial solitons when $a = 0.015$, as shown in Fig. 1(b). Obviously, the soliton FWHM is a monotonically decreasing function of intensity. For low intensity, the non-paraxial soliton behaves as its paraxial counterpart, and the width decreases quickly with the increase of the light intensity. When the intensity is extremely high, however, the FHWM presents saturation against the increase of intensity. Interestingly, for a large value of y_0, the width of the soliton is seen to exhibit a very weak dependence on y_0^2, and the soliton width is indeed independent of y_0^2 when the value of y_0 is extremely large. The soliton has a limited minimum dimensionless width about 0.76, which corresponds to the real width is about a wavelength.

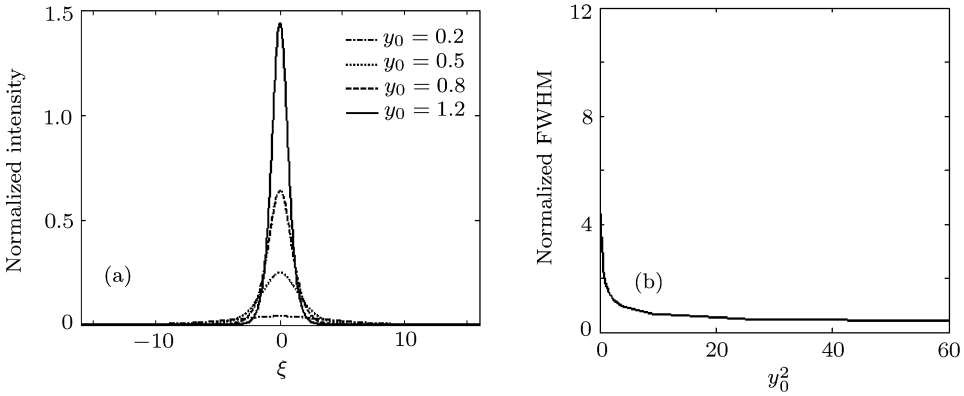


Fig. 1. (a) Normalized intensity profiles of non-paraxial bright Kerr solitons for various values of y_0 when the non-paraxial parameter $a = 0.015$; (b) the existence curve.

3. Coherent interaction between nonparaxial bright solitons

In order to investigate the effects of the parameters on the coherent interaction, we assume that two mutually coherent bright solitons propagates together in a Kerr medium. Also, we let the optical field at the face of nonlinear medium be $U(s,0) = r_1 y(\xi + \Delta\xi/2) + r_2(\xi - \Delta\xi/2)\exp(\mathrm{i}\,\Delta\varphi)$, where r_1 and r_2 are, respectively, the optical amplitudes of these two coherent solitons, $\Delta\xi$ is the separation space, and $\Delta\varphi$ is the phase difference, and $y(\xi)$ is the soliton solution of Eq. (4). Choosing different values of $\Delta\xi$, $\Delta\varphi$, r_1 and r_2, we numerically solve Eq. (2) to simulate the coherent interactions between two nonparaxial bright solitons. For the conveniences of description, we refer to the beam centre corresponding to positive ξ as beam A, and the beam centre corresponding to negative ξ as beam B.

3.1. Effect of phase difference on coherent interaction

Figure 2 shows the interaction behaviour of two beams with different phase differences when the initial separation is set as $\Delta\xi = 2$. From these figures, one can easily see that the interaction behaviours when $0 \le \Delta\varphi \le \pi/2$ are different from those when $\pi/2 < \Delta\varphi \le \pi$. When the phase difference is between $(0, \pi/2)$, the initial phase of soliton A is smaller than that of soliton B. Because the initial phase difference is smaller than $\pi/2$, the refractive index perturbations increase in overlap regions. As a result, they first attract each other once they enter the nonlinear medium. However, their transverse phases also change with the propagation because of the third and fourth terms occurring on the left-hand side of Eq. (2). So, the beams cannot merge into a single soliton, which is roughly different from the paraxial case. When $\Delta\varphi = 0$, the interaction presents the breath behaviour. When $0 \le \Delta\varphi < \pi/2$, they repel each other eventually after a short distance of propagation. If the initial phase difference is between $(\pi/2, \pi)$, how-

ever, the case is roughly different. In this case, the total intensity decreases in the overlap region when the beams are injected into the medium, and so the refractive index in the overlap region is smaller. Therefore, the beams repel each other once they enter the nonlinear medium. In other words, the critical case is that $\Delta\varphi = \pi/2$. In this case, the two incident beams first propagate a short distance along the initially incident direction, then repel each other as their phases change (see Fig. 2(b)). It can also be pointed out that there exists the energy transfer between the two solitons during their propagation. When $0 \leq \Delta\varphi \leq \pi$, the energy transfers from soliton B to soliton A. On the contrary, the energy transfers from soliton A to soliton B while the phase difference is between $(-\pi, 0)$ (not shown in this figure).

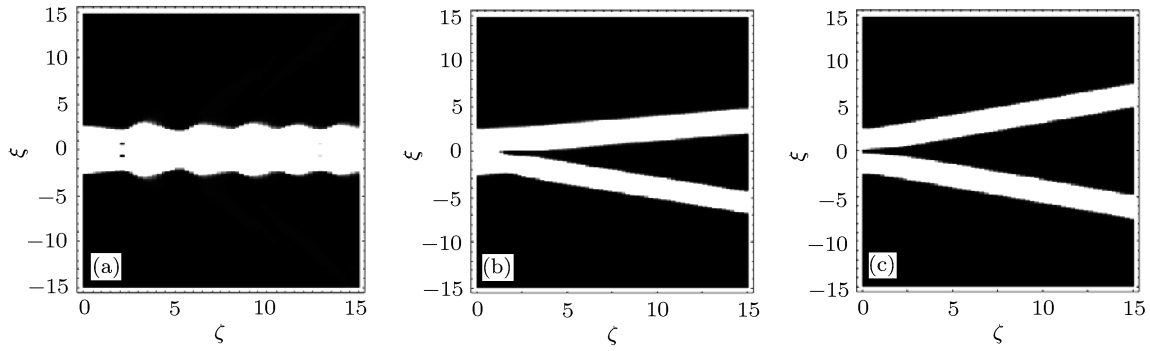


Fig. 2. Interaction behaviours of two coherent nonparaxial solitons with equal intensities but different values of phase difference $\Delta\varphi = 0$ (a), $\pi/2$ (b), and π (c).

3.2. Effect of separation on interaction

If the separation between the two coherent solitons changes, the interaction behaviour changes greatly. Figure 3 shows the interactions between two in-phase nonparaxial solitons with equal intensities but different separations. From this figure, we can see that the interaction behaviour is periodical and it presents a breath structure. In other words, they attract each other when they enter the nonlinear medium. After short distance propagation, they begin to repel because their phase difference changes. Interestingly, they attract each other again quickly. This phenomenon repeats until the beams leave out of the medium. The separation just influences the position of interaction. The larger the separation is, the longer the interaction position will be. However, if the injected beams are out of phase, the influence of separation distance on the interaction will be different. Figure 4 shows the interactions between two out-of-phase nonparaxial solitons for different separations. From these figures, we can see that the out-of-phase beams always repel each other no matter what value the separation is. When the separation is very small (see Fig. 4(a)), the repelling force between these two beams is very strong, and they repel each other shortly. As the separation increases, the repelling force decreases and therefore the separation between these two beams at the output face becomes small (Fig. 4(b)). When the separation distance is large enough, there is nearly no interaction between them (see Fig. 4(c)).

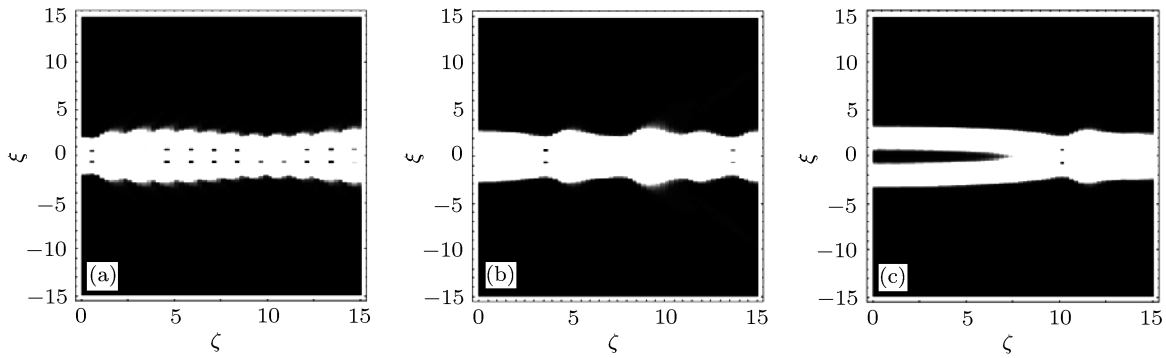


Fig. 3. Interaction behaviours of two in-phase non-paraxial solitons with equal intensities but different values of separation $\Delta\xi = 1.5$ (a), 3 (b), and 4 (c).

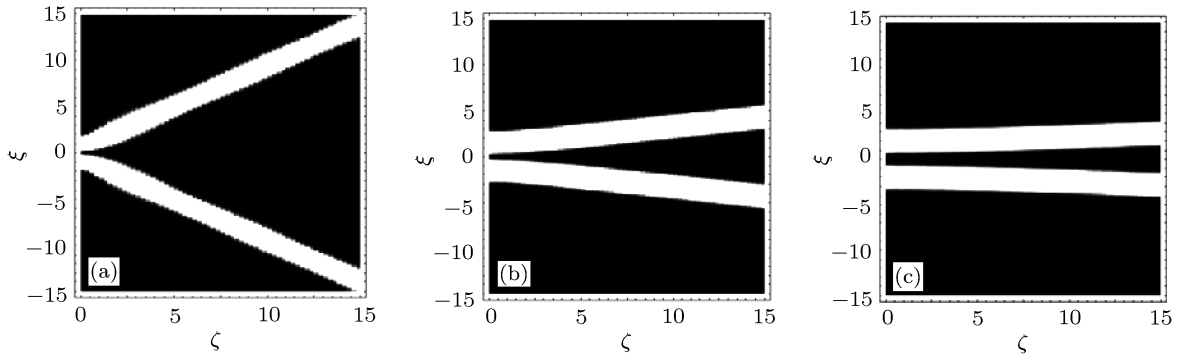


Fig. 4. Interaction behaviours of two out-of-phase non-paraxial solitons with equal intensities but different values of separation $\Delta\xi = 1.5$ (a), 3 (b), and 4 (c).

3.3. Effect of intensity ratio on interaction

Besides the separation distance and the phase difference between these solitons, the intensity ratio also has great influence on the interaction behaviour. Figure 5 shows the interaction behaviour of two solitons with different intensities when the separation is set to be 1. From this figure, one can see that the two beams sometimes attract each other and then repel when their intensities are equal as shown in Fig. 5(a). If the optical intensity of one beam is increased, the coherent interaction behaviour will be changed. The beams first attract each other after they have entered the nonlinear medium. After a short propagation distance, they begin to repel. Because the intensity of soliton B is smaller than that of soliton A, the attracting force will affect the propagation of soliton B greater. Therefore, the repelling angle of soliton B will be smaller than that of soliton A. As a result, the attracting force between them cannot bring them together again. Soliton A will propagate along a larger repelling direction while soliton B will propagate along a smaller angle in the remaining distance. If we increase the intensity of beam A further, this phenomenon will be more obvious.

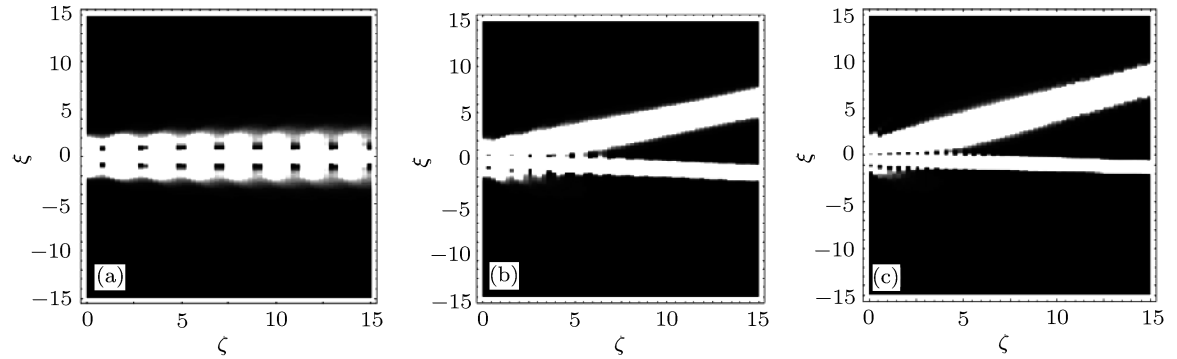


Fig. 5. Interaction behaviours of two in-phase nonparaxial solitons with different intensities. (a) $r_1 = 1$, $r_2 = 1$; (b) $r_1 = 2$, $r_2 = 1$; (c) $r_1 = 2\sqrt{2}$, $r_2 = 1$.

4. Conclusion

We prove that one-dimensional linearly polarized solitons can exist in Kerr medium beyond the nonparaxial approximation. The intensity profiles and their existence curve are also analysed in detail. It is found that the soliton FWHM monotonically decreases as the intensity increases. Furthermore, the interaction behaviours of two nonparaxial solitons are investigated in detail. It is found that the separation, the intensity ratio, and the relative phase difference of these two solitons have great effects on the coherent interation.

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