Quantum coherence in Λ-three-level system driven by bichromatic coupling field*

Yang Li-Jun(杨丽君), Zhao Min(赵 敏)†, Zhang Lian-Shui(张连水),
Feng Xiao-Min(冯晓敏), Li Xiao-Li(李晓莉), and Wei Chang-Jiang(尉长江)

College of Physical Science and Technology, Hebei University, Baoding 071002, *China*

(Received 24 July 2009; revised manuscript received 16 December 2009)

We present a theoretical study of quantum coherent effects in a Λ-three-level system with a strong bichromatic coupling field and a weak probe field. When one component of the strong bichromatic coupling field is resonant with a corresponding transition and the other is detuning with an integer fraction of the Rabi frequency of the resonant field, the absorption spectrum exhibits a series of symmetrical doublets. While two frequencies of the strong bichromatic coupling field are symmetrically detuned from the transition, the position and the relative intensity of the absorption peak are both affected by the coupling field intensity and detuning. An explanation of the spectrum is given in term of the dressed-state formalism.

Keywords: three-level system, bichromatic coupling field, dressed state, quantum coherence

PACC: 4250, 3270

1. Introduction

Quantum coherence and interference is one of the most interesting topics in laser physics and quantum optics. It has been shown that the quantum coherence can induce many new effects, such as lasing without inversion (LWI),[1] enhancement of the nonlinear process,[2,3] electromagnetically induced absorption (EIA)[4,5] and electromagnetically induced transparency (EIT).[6−9] The EIT has attracted considerable attention both theoretically and experimentally. In an EIT system, the absorption and the dispersion can be significantly modified by a coupling field, so the EIT can be used for controlling optical characteristics of materials.[10] In addition, it also possesses potential applications in quantum information storage,[11] ultraslow light,[12,13] susceptibility enhancement,[6,14,15] and quantum communication.[16−18] The simplest case of EIT is the Λ-three-level scheme interacting with a coupling field and a probe field, and such a Λ-three-level scheme has been studied extensively.[19−21] Recently there has been interest in controling the EIT by using an additional electromagnetic field.[22−28] The remarkable feature revealed by these studies is the multipeak spectrum.

In this paper, we consider a Λ-three-level scheme interacting with a bichromatic coupling field and a probe field. The scheme is studied for the case where one of the coupling fields is weak, and the other is strong, in which the absorption spectrum exhibits an EIT and an EIA. In this paper we study the case where the bichromatic coupling fields can be strong. We consider two particular cases. In the first case, one of the strong coupling fields is resonant with the $|2\rangle \rightarrow |3\rangle$ transition, and the second strong coupling field is detuned from the $|2\rangle \rightarrow |3\rangle$ transition with chosen detunings as shown in Fig. 1(a). The detuning of the second field is close to an integer fraction of the Rabi frequency of the first field. The spectrum obtained by probing a third level is dominated by a series of doublets. Greentree *et at.*[29] reported on an experimental observation on a V-three-level scheme driven by two coupling fields and probed by a weak probe field. Although their configuration is different from ours, the fundamental principle and the interpretation of dressed state are the same. The general feature of our calculated spectrum is consistent with experimental observation.[29] In the second case, two frequencies of the strong bichromatic coupling fields are symmetrically detuned from the $|2\rangle \rightarrow |3\rangle$ transition as shown in Fig. 1(b). It is found that the spectrum exhibits a series of symmetrical peaks. There is a relevant experimental study by Wang *et al.*[30] where a three-peak spectrum was obtained. Due to experimental difficulties, there is no study on the general pattern of absorp-

*Project supported by the Natural Science Foundation of Hebei Province, China (Grant No. A2009000140).
†Corresponding author. E-mail: zhaomin1212@126.com
© 2010 Chinese Physical Society and IOP Publishing Ltd

tion spectrum under various Rabi frequencies of the coupling field. In both of the cases mentioned above, our calculated absorption spectra can be conveniently explained in terms of dressed-atom description.

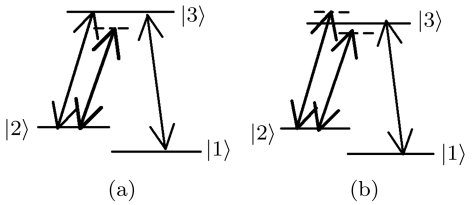


Fig. 1. Energy level diagram for a Λ-three-level quantum system interacting with a bichromatic coupling field and a probe field.

2. Theory

We consider a Λ-three-level scheme with a couple of ground states ($|1\rangle$ and $|2\rangle$) and a common excited state ($|3\rangle$) interacting with two bichromatic coupling fields and a probe field as shown in Fig. 1. Both the coupling fields, separately, with frequencies ω_{c1} and ω_{c2} interact with the transition between $|2\rangle \to |3\rangle$ and the probe field with frequency ω_p interacts with the transition between $|1\rangle \to |3\rangle$. The effects of the coupled fields are monitored by the probe field. The Rabi frequencies of the bichromatic coupling fields and the probe field are denoted as Ω_{c1}, Ω_{c2} and Ω_p respectively.

The density matrix equation of motion in the rotating-wave approximation can be written as follows:

$$
\begin{aligned}
\dot{\rho}_{11} &= \mathrm{i}\chi_p(\rho_{31}-\rho_{13}) + \Gamma_{31}\rho_{33} + \Gamma_{21}\rho_{22} - \Gamma_{12}\rho_{11},\\
\dot{\rho}_{22} &= \mathrm{i}\chi_{c1}(\rho_{32}-\rho_{23}) + \mathrm{i}\chi_{c2}\left(\rho_{32}\,\mathrm{e}^{\mathrm{i}\delta t} - \rho_{23}\,\mathrm{e}^{-\mathrm{i}\delta t}\right)\\
&\quad + \Gamma_{32}\rho_{33} + \Gamma_{12}\rho_{11} - \Gamma_{21}\rho_{22},\\
\dot{\rho}_{33} &= -\mathrm{i}\chi_p(\rho_{31}-\rho_{13}) - \mathrm{i}\chi_{c1}(\rho_{32}-\rho_{23})\\
&\quad - \mathrm{i}\chi_{c2}\left(\rho_{32}\,\mathrm{e}^{\mathrm{i}\delta t} - \rho_{23}\,\mathrm{e}^{-\mathrm{i}\delta t}\right) - \Gamma_3\rho_{33},\\
\dot{\rho}_{31} &= -d_{31}\rho_{31} + \mathrm{i}\left(\chi_{c1} + \chi_{c2}\,\mathrm{e}^{-\mathrm{i}\delta t}\right)\rho_{21}\\
&\quad - \mathrm{i}\chi_p(\rho_{33}-\rho_{11}),\\
\dot{\rho}_{32} &= -d_{32}\rho_{32} - \mathrm{i}\left(\chi_{c1} + \chi_{c2}\,\mathrm{e}^{-\mathrm{i}\delta t}\right)(\rho_{33}-\rho_{22})\\
&\quad + \mathrm{i}\chi_p\rho_{12},\\
\dot{\rho}_{21} &= -d_{21}\rho_{21} + \mathrm{i}\left(\chi_{c1} + \chi_{c2}\,\mathrm{e}^{\mathrm{i}\delta t}\right)\rho_{31} - \mathrm{i}\chi_p\rho_{23},
\end{aligned}
$$

where χ_{c1}, χ_{c2} and χ_p are the parameters introduced for convenience and they are equal to half the Rabi frequencies ($\chi_{c1} = \Omega_{c1}/2$, $\chi_{c2} = \Omega_{c2}/2$ and $\chi_p = \Omega_p/2$) correspondingly; d_{31}, d_{32} and d_{21} are the complex detunings defined as $d_{31} = \mathrm{i}\Delta_p + \gamma_{31}$, $d_{32} = \mathrm{i}\Delta_{c1} + \gamma_{32}$ and $d_{21} = \mathrm{i}(\Delta_{c1} - \Delta_p) + \gamma_{21}$ respectively; Δ_{c1}, Δ_{c2} and Δ_p are coupling detunings that are defined as $\Delta_{c1} = \omega_{32} - \omega_{c1}$, $\Delta_{c2} = \omega_{32} - \omega_{c2}$ and $\Delta_p = \omega_{31} - \omega_p$ respectively; δ denotes the probe detuning which is defined as $\delta = \omega_{c2} - \omega_{c1}$; Γ_{ij} is the population relaxation rate from level $|i\rangle$ to level $|j\rangle$; γ_{ij} is the coherent relaxation rate from level $|i\rangle$ to level $|j\rangle$. An ideal system is considered here, so it has $\Gamma_{32} = \Gamma_{31}$, $\Gamma_{21} = \Gamma_{12} = \Gamma$, $\gamma_{31} = (\Gamma_{31} + \Gamma_{32} + \Gamma_{12})/2$, $\gamma_{32} = (\Gamma_{31} + \Gamma_{32} + \Gamma_{12})/2$, and $\gamma_{21} = (\Gamma_{12} + \Gamma_{21})/2$. For simplicity we assume that $\Gamma_3 = \Gamma_{31} + \Gamma_{32} = 1$ and $\Gamma = 10^{-4}$. All parameters are normalized to Γ_3. The absorption of the probe laser is proportional to the imaginary part of ρ_{31} and the dispersion is proportional to the real part of ρ_{31}. In this paper we are interested primarily in the absorption spectrum of probe laser.

3. Result and discussion

First, we consider the case where one bichromatic coupling field has frequency ω_{c1} and exactly resonates with the $|2\rangle \to |3\rangle$ transition and another one has frequency ω_{c2} and detunes from the $|2\rangle \to |3\rangle$ transition.

In the absence of coupling fields, the absorption spectrum of a weak field probing the $|1\rangle \to |3\rangle$ transition is shown in Fig. 2(a), which is the absorption spectrum of a two-level scheme scanned by a weak field. In the presence of a resonant coupling field only, it can be seen that the resonant coupling field induces an Autler–Townes doublet. The splitting degree is determined by Ω_{c1} as shown in Fig. 2(b). This is similar to the scenario of a Λ-three-level scheme. When the off-resonant components of the bichromatic coupling field are applied, the results are shown in Figs. 2(c)–2(f). The first coupling field with Rabi frequency $\Omega_{c1} = 30$ is resonant with the $|2\rangle \to |3\rangle$ transition, and the second coupling field with Rabi frequency $\Omega_{c2} = 15$ is detuned from the $|2\rangle \to |3\rangle$ transition with detunings corresponding to the first to fourth subharmonic resonances of $\Omega_{c1}(\Delta_{c2} = \pm\Omega_{c1}/k)$.

It can be found that there are two pairs of Autler–Townes doublets in Fig. 2(c) and there are three pairs of Autler–Townes doublets in Fig. 2(b). In Fig. 2(b) there appears one more pair of peaks than that in Fig. 2(c), which is because the number of Autler–Townes doublets increases with k. This situation can be interpreted well by dressed states[29] as shown in Fig. 3.

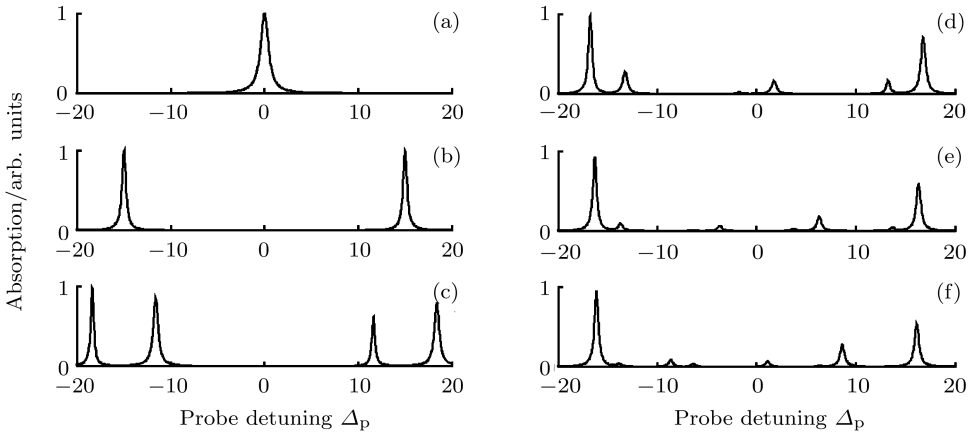


Fig. 2. Absorption spectra of weak probe field: (a) absorption spectrum in the absence of coupling field, (b) absorption spectrum with only resonant coupling field, and (c)–(f) absorption profiles in the presence of bichromatic coupling field with $\Omega_{c1} = 30$, $\Omega_{c2} = 15$, and $\Delta_{c2} = -30$, –15, –10 and –7.5, respectively.

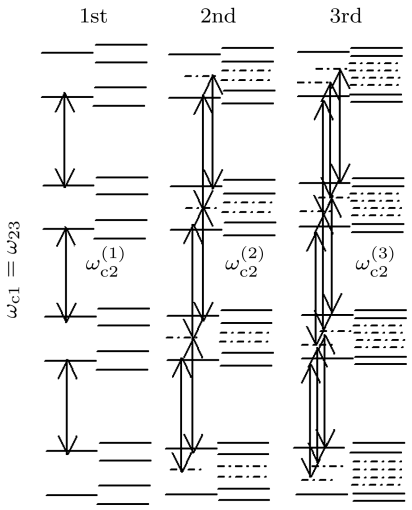


Fig. 3. Dressed state levels for the first, second and third subharmonic resonances. In the first column $\omega_{c2}^{(1)}$ is resonant with a transition between the single field dressed state levels, and a limited number of the doubly dressed states are shown on the right. The equivalent situations for the second and third subharmonic resonances are also shown. $\omega_{c2}^{(2)}$ is resonant with a two-photon transition between the single field dressed states and $\omega_{c2}^{(3)}$ is resonant with a three-photon transition between single field dressed states.

Secondly, we consider the case where both components of the bichromatic coupling fields have the same Rabi frequencies and the same detunings from the $|2\rangle \rightarrow |3\rangle$ transition but with the opposite signs $\Delta_{c1} = -\Delta_{c2} = \Delta$ as shown in Fig. 1(b). In this case, the spectral characteristic of the probe field is examined as a function of coupling field strength and the result can be interpreted well by dressed states[30] as shown in Fig. 5.

It can be found that the number of peaks increases in pairs correspondingly when the coupling field strength increases from Fig. 4(a) to Fig. 4(e). The positions of peaks are determined by Δ. The dressed states associated with the coupling field consist of an infinite ladder which results in multiple peak absorption spectrum corresponding to the dressed transition $|1\rangle \rightarrow |m\rangle$ as shown in Fig. 5.

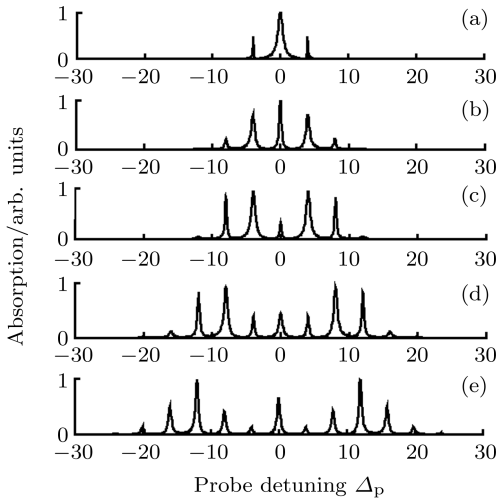


Fig. 4. Absorption spectra of a weak field probing the $|1\rangle \rightarrow |3\rangle$ transition, with off-resonant bichromatic coupling field applied to the $|2\rangle \rightarrow |3\rangle$ transition. Panels (a)–(e) show absorption profiles for different coupling field intensities, Rabi frequencies $\Omega_{c1} = \Omega_{c2} = 2.0$, 6.0, 8.0, 13.0, 17.0, and $\Delta = 4$.

In Fig. 4, trace (a) exhibits three peaks, corresponding to the dressed state transitions $|1\rangle$ to $|m = 0\rangle$ and $|m = \pm 1\rangle$, and trace (b) exhibits five peaks, corresponding to the dressed state transition $|1\rangle$ to $|m = 0\rangle$, $|m = \pm 1\rangle$ and $|m = \pm 2\rangle$, etc., respectively. The neighbouring levels of the dressed states have an

equal separation $|\Delta|$, which determines the interval between two peaks, that is, the interval equals $|\Delta|$.

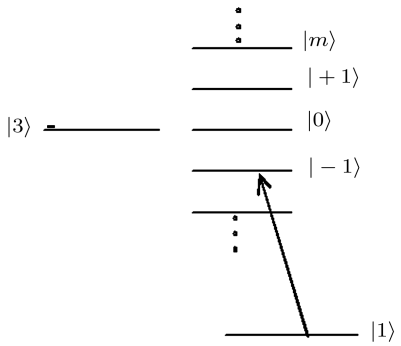


Fig. 5. Dressed state picture of the three-level system driven by strong bichromatic coupling field.

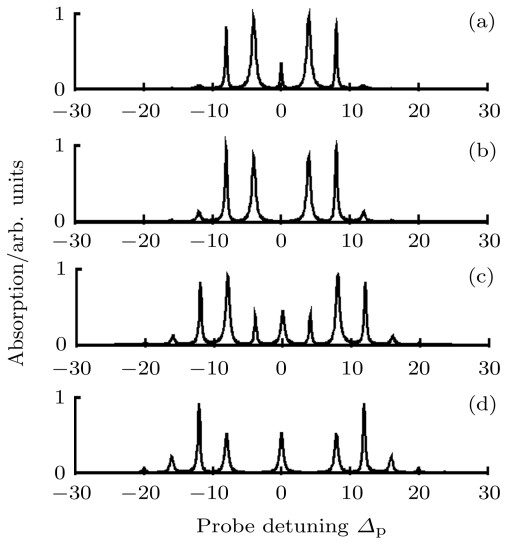


Fig. 6. Absorption spectrum of a weak field probing the $|1\rangle \rightarrow |3\rangle$ transition, with strong bichromatic coupling field applied to the $|2\rangle \rightarrow |3\rangle$ transition. Traces (a)–(d) are for a given field intensity and Rabi frequencies $\Omega_{c1} = \Omega_{c2} = 8.0$, 9.6, 13.0, 15.4.

Figure 6 shows that the variation in intensity of absorption peak depends on the Rabi frequency of coupling field. With the increase of the Rabi frequency of coupling field the peaks disappear alternately. Firstly, by comparison with Fig. 6(a), the central peak disappears in Fig. 6(b). Then with the further increase of the Rabi frequency, the intensity of the central peak begins to rise, and the intensities of the second-order peaks near the centre peak begin to decrease. In Fig. 6(d), the second-order peaks disappear, etc. We believe that these new spectral absorption features result from the overlap of dressed states.

4. Conclusion

In this paper, we presented a theoretical study on a Λ-three-level scheme driven by a bichromatic coupling field and provided the absorption profiles of a weak probe field in special circumstances. The spectrum is characterized by a series of doublets and an explanation is presented in terms of dressed states. When one bichromatic coupling field is exactly resonant with the transition and the other is detuning with Δ_{c2} ($\Delta_{c2} = \pm\Omega_{c1}/k$ for integer k), the spectrum displays a series of doublets and the number of peaks changes regularly with detuning Δ_{c2} varying. When the bichromatic coupling fields are both symmetrically detuned from the transition, that is, $\Delta_{c1} = -\Delta_{c2}$, the spectrum also displays a series of doublets, but the number of peaks changes regularly with intensity of the coupling field increasing. The results are found to be in excellent agreement with experimental expectations for a doubly driven system.

References

[1] Harris S E 1989 *Phys. Rev. Lett.* **62** 1033

[2] Tu X H, Wang J, Jiang K J, He M, Li K, Zhong J Q and Zhan M S 2003 *Chin. Phys. Lett.* **20** 1954

[3] Xiao F, Guo R M, Chen S, Zhang Y, Li L M and Chen X Z 2003 *Chin. Phys. Lett.* **20** 1257

[4] Taichenachev A V, Tumaikin A M and Yudin V I 1999 *Phys . Rev.* A **61** 011802

[5] Zhang L S, Li X L, W J, Yang L J, Feng X M, Li X W and Fu G S 2008 *Acta Phys. Sin.* **57** 4921 (in Chinese)

[6] Harris S E, Field J E and Imamoglu A 1990 *Phys. Rev. Lett.* **64** 1107

[7] Boller K J, Imamolu A and Harrise S E 1991 *Phys. Rev. Lett.* **66** 2593

[8] Yang L J, Zhang L S, Zhuang Z H, Guo Q L and Fu G S 2008 *Chin. Phys.* B **17** 2147

[9] Li Y F and Sun J F 2003 *Acta Phys. Sin.* **52** 547 (in Chinese)

[10] Wei C J and Manson 1999 *J. Opt.* B: *Quantum Semiclass. Opt.* **1** 464

[11] Ham B S, Shahriar M S, Kim M K and Hemmer P R 1997 *J. Opt. Lett.* **22** 1849

[12] Phillips D F, Fleischhauer A, Maie A, Walsworth R L and Lukin M D 2001 *Phys. Rev. Lett.* **86** 783

[13] Schmidt Q, Wynands R, Hussein Z and Meschede D 1996 *Phys. Rev.* A **53** R27

[14] Scully M O 1991 *Phys. Rev. Lett.* **67** 1855

[15] Julio G B, Li Y Q and Xiao M 1995 *Phys. Rev.* A **51** 576

[16] Payne M G and Deng L 2003 *Phys. Rev. Lett.* **91** 123602

[17] Paternostro M, Kim M S and Ham B S 2003 *Phys. Rev.* A **67** 023811
[18] Xiao H L, Ou Y S and Nie Z P 2009 *Acta Phys. Sin.* **58** 3685 (in Chinese)
[19] Narducci M L, Scully M O, Oppo G L, Ru P and Tredicce J R 1990 *Phys. Rev.* A **42** 1630
[20] Ichimura K, Yamamoto K and Gemma N 1998 *Phys. Rev.* A **58** 4116
[21] Yu Y C, Wang D L and Ding J W 2009 *Acta Phys. Sin.* **58** 3198 (in Chinese)
[22] Wilson E A, Manson N B and Wei C 2005 *Phys. Rev.* A **72** 063813
[23] Sunish Menon and Agarwal G S 1999 *Phys. Rev.* A **59** 1
[24] Berman P R and Dubetsky B 2000 *Phys. Rev.* A **62** 053412
[25] Sadeghi S M and Meyer J 1998 *Phys. Rev.* A **59** 5
[26] Wallis H 1995 *Phys. Rev.* A **52** 2
[27] Zhang J P, Xu J, Gessler Hernandez, Hu X M and Zhu Y 2007 *Phys. Rev.* A **75** 043810
[28] Li P, Takashi Nakajima and Ning X J 2006 *Phys. Rev.* A **74** 043408
[29] Greentree A D, Wei C, Holmstrom S A, Martin P D, Manson N B, Catchpole K R and Savage C 1999 *J. Opt.* B: *Quantum Semiclass. Opt.* **1** 240
[30] Wang J, Zhu Y, Jiang K J and Zhan M S 2003 *Phys. Rev.* A **68** 063810