Robust adaptive synchronization of uncertain and delayed dynamical complex networks with faulty network*

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This paper presents a new robust adaptive synchronization method for a class of uncertain dynamical complex networks with network failures and coupling time-varying delays. Adaptive schemes are proposed to adjust controller parameters for the faulty network compensations, as well as to estimate the upper and lower bounds of delayed state errors and perturbations to compensate the effects of delay and perturbation on-line without assuming symmetry or irreducibility of networks. It is shown that, through Lyapunov stability theory, distributed adaptive controllers constructed by the adaptive schemes are successful in ensuring the achievement of asymptotic synchronization of networks in the present of faulty and delayed networks, and perturbation inputs. A Chua's circuit network example is finally given to show the effectiveness of the proposed synchronization criteria.

Keywords: dynamical complex networks, synchronization, robust adaptive control, faulty network, time-varying delays

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1. Introduction

Over the past decades, complex networks have received considerable attention from physicists, biologists and industries. Various complicated problems in networks have been studied, such as topological structures,[1] small-world,[2] robustness and fragility.[3] The synchronization problem[4] of dynamical complex networks has also received significant attention over the last few years, which is involved in lots of practical control systems and many physical phenomena, such as synchronization of coupled oscillators, rendezvous in space, aircraft formation control, flocking theory, and so on.

In complex networked control systems, networks are of great importance to stability and performance of the whole system.[5] As we know, the synchronization behaviour of complex systems is reached by exchanging neighbour nodes' information through networks. Nowadays, dynamic network topology becomes a hot research topic because the links among nodes are always added or removed at an unknown time due to unreliable transmission or limited communication capability. Under investigation of consensus in networks, switching network topologies were considered in Ref. [6]. The studies of random topology were developed with random weighted directed graphs,[7] different link Bernoulli random probabilities,[8] temporally Markovian fault model,[9] and so on. Besides, adaptive control has also been developed as a useful strategy to design controller feedback gain for implementing synchronization with dynamical networks. Adaptive laws were developed to estimate unknown coupling strength[10−14] and controller parameter[15−22] for asymptotic synchronization of networks. The recent paper[23] concerned with network structure identification of unknown general complex dynamical networks by using adaptive model matching method. As we know, there exists another type of dynamical network caused by faulted networks, which means the network working negatively for synchronization. However, the above studied works are based on the networks working positively. Few efforts have been made to consider the synchronization behaviour of complex

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networked systems with network failure. Recently, a paper[8] proposed a discrete-time consensus protocol to deal with the convergence speed problem in the case of network failure. Thus, a novel adaptive method will be adopted to deal with a robust synchronization problem of an unknown directed dynamic network with nonuniform coupling strength and network failure in this paper.

The controlled synchronization of complex dynamical networks has become a rather significant topic in practical applications, since the networks may always be affected by time-delays and perturbations. The studies of delayed dynamical networks were developed with coupling delays,[13−16,23−25] delayed nodes,[20,26] decaying function,[27] distributed delays,[28,29] and so on. However, the delays considered under either constant status or some assumptions in the above papers such as the delay derivative is less than 1. The asymptotic synchronization problem is more challenging in the presence of unknown time-varying perturbations. References [21] and [22] considered the robust synchronization with uncertain dynamical networks under some assumptions. In Ref. [30], linear matrix inequality (LMI) method was proposed to optimize H_∞ performance of a linear complex dynamical network subject to external disturbances. Reference [31] utilized a sliding-mode adaptive observer method to synchronize chaotic systems with known bound perturbations. Recently, adaptive method was used in Ref. [32] to compensate the effect of disturbance, but the requirement of upper bound of disturbance is needed and asymptotic consensus cannot be guaranteed. It should be pointed out that there are many control approaches in existing large scale system literatures to deal with the perturbation rejection problem, such as high gain control,[33] adaptive compensation control,[34] and sliding mode control.[35] In this paper, a bounded perturbations as well as unknown time-varying coupling delays in communications are treated with dynamical network topology. Besides, a new adaptive method is proposed to compensate for the effect of imperfect communications without bound information of delays and perturbations.

In this paper, by using adaptive control techniques, an effective approach is proposed to solve the robust synchronization problem for a class of uncertain and delayed dynamical complex networks against network failures. Distributed adaptive control is developed to achieve synchronization when topological structure and coupling strength of networks, and the bounds of delayed state errors and perturbations are assumed to be unknown. For this purpose, some adaptation laws are proposed to estimate unknown parameters which include the information of topological structure and coupling strength of networks on-line first. Meanwhile, we also propose some adaptive laws to estimate the upper and lower bounds of delayed state errors and perturbations, respectively. Then, a class of distributed adaptive state feedback controllers is constructed relying on the updated values of these estimations. Based on Lyapunov stability theory, the adaptive closed-loop complex networked system can be guaranteed to be asymptotic synchronization in the presence of network failures, coupling time-delays and perturbations. Besides, it should be noted that the new proposed adaptive design method is not necessary for the estimations to give the exact information.

The robust synchronization problem formulation is described in Section 2. In Section 3 distributed adaptive state feedback controllers are developed. A Chua's circuit example and simulations are provided in Section 4. Finally, conclusion is given in Section 5.

2. Preliminaries

In this paper, we consider a general dynamical network G composed of N linear coupling identical nodes,[4] with each node G_i, $i = 1, 2, \ldots, N$ being an n-dimensional dynamical system, described by:

$$\dot{x}_i(t) = f(x_i(t), t) + c_i \sum_{j=1}^{N} a_{ij} \Gamma x_j(t - \tau_j(t)) + u_i(t) + d_i(t), \quad (1)$$

where $f : R^n \to R^n$ is continuously differentiable, $x_i(t) \in R^n$ is the state of node i, $u_i(t) \in R^n$ is the control input, $d_i(t) \in R^n$ represents the uncertainty in system dynamics that may include parameter perturbations, external disturbances, unmodeled dynamics, etc., and $\underline{d}_i \le d_i(t) \le \bar{d}_i$ where $\underline{d}_i$ and $\bar{d}_i$ denote the unknown lower and upper bounds of $d_i(t)$, respectively. a_{ij} represents the topological structure of the network. $a_{ij} = 1$ denotes that the link between the i-th node and the j-th node is connected, while $a_{ij} = 0$ denotes that the link is disconnected. It should be noted that if the network between nodes i and j is faulted, the topology parameter denotes by $a_{ij} = -1$. $c_i > 0$ is an unknown constant topology weighted parameter representing the coupling strength of node

i, which can be changed intermittently due to the effect of some uncertainty reasons, such as environment change and equipment corruption. $\tau_j(t)$ is a state time-delay with $0 \leq \tau_j(t) \leq \bar{\tau}_j < \infty$ for the system, and $\bar{\tau}_j$ is an unknown constant. The matrix $\Gamma \in R^{n\times n}$ is the inner connecting matrix of each node. Let $l_{ij} = -\sum_{k=1,k\neq i}^{N} a_{ik}$ $(j = i)$ and $l_{ij} = a_{ij}$ $(j \neq i)$, then we call $L = [l_{ij}]_{N\times N}$ as the graph Laplacian induced by the information flow G.[6]

Network (1) describes a general model of coupled dynamical units ranging from physical to technological, and to biological fields, where nodes represent physical units and links represent the interactions among them. The topology of such networks has been extensively studied and some common architectures have been discovered.[1] The model of many chaotic systems such as the Lorenz oscillator system, Chen's oscillator, Rösler oscillator and Chua's circuit can also be formulated by network (1). It should be noted that the nonlinearities $f_i(x,t)$, $i = 1,2,\ldots,N$ of such networks are always satisfying a Lipshitz condition, i.e., there exist unknown constants $q_i > 0$ for any two different constants $x_1, x_2 \in R^n$ such that[8−33]

$$\begin{aligned}&(x_1 - x_2)^{\mathrm{T}} P(f_i(x_1,t) - f_i(x_2,t))\\ &\leq q_i\|x_1 - x_2\|^2,\end{aligned} \tag{2}$$

where $P = \mathrm{diag}\{p_1, p_2, \ldots, p_n\}$ is an $(n \times n)$-dimensional diagonal matrix composed by positive constants p_l, $l = 1,2,\ldots,n$.

It is well known that a general physical phenomenon of complex network is to synchronize network with a given orbit. Similar to Ref. [1], let $s(t)$ be a solution of the isolate node of the network, which is assumed to exist and is unique, satisfying $\dot{s}(t) = f(s(t),t)$, where $s(t)$ can be an equilibrium point, a nontrivial periodic orbit, or even a chaotic orbit. The dynamical network (1) is said to achieve asymptotic synchronization if $x_1(t) \to x_2(t) \to \cdots \to x_N(t) \to s(t)$ as $t \to \infty$. Here, the main objective of control is to find some smooth controllers such that the solutions of network (1) synchronize with $s(t)$. And we shift the solutions to the origin through the transformation $e_i(t) = x_i(t) - s(t)$. Then, the stability of the synchronous solution $x_1(t) = x_2(t) = \cdots = x_N(t) = s(t)$ can be studied by the following variational equation:[4]

$$\begin{aligned}\dot{e}_i(t) =\ & f(x_i(t),t) - f(s(t),t) + c_i\sum_{j=1}^{N} a_{ij}\Gamma e_j(t)\\ &+ u_i(t) + z_i(t),\end{aligned} \tag{3}$$

where $z_i(t) = [z_{i1}(t), z_{i2}(t), \ldots, z_{in}(t)]^{\mathrm{T}} \in R^n$ is an external disturbed signal composed by the delayed state errors and perturbations of networks, described by $z_i(t) = c_i\sum_{j=1}^{N} a_{ij}\Gamma(e_j(t-\tau_j(t)) - e_j(t)) + d_i(t)$. Since $\tau_j(t)$ is a bounded time-delay, thus $z_i(t)$ is a bounded signal. Then we let $\underline{z}_i(t)$ and $\bar{z}_i(t)$ denote the unknown lower and upper bounds of $z_i(t)$, respectively.

In the following section, the main objective is to construct robust adaptive state feed-back controllers $u_i(t)$, $i = 1,2,\ldots,N$ such that the error system (3) can be guaranteed to be $\lim_{t\to\infty} e_i(t) = 0$ even in the cases of coupling delay and perturbation effects all the time and network deteriorated.

3. Adaptive synchronization controller design

As we know, a fault of network may occur at uncertain time and the deteriorated network is also unknown. Moreover, the fault can result in the change of network topology, but it remains a fixed topological characteristics of network under a period of time. Thus the adaptive method is suitable for adjusting controller parameters to compensate for the faults. On the other hand, as external disturbed signal $z_{il}(t)$ is a time-varying but bounded signal, we utilize the constants $\underline{z}_{il}$ and $\bar{z}_{il}$ to describe it with the following equality $e_i^{\mathrm{T}} P_l z_{il}(t) \leq e_i^{\mathrm{T}} P_l((1-\sigma_{il})\bar{z}_{il} + \underline{z}_{il})$, where $\sigma_{il} = 1$ if $e_i^{\mathrm{T}} P_l \geq 0$ and $\sigma_{il} = 0$ if $e_i^{\mathrm{T}} P_l < 0$, $z_{il}(t)$ is the l-th element of $z_i(t)$ and P_l is the l-th column of P defined in Eq. (2). Then adaptive method is adopted to estimate the constants $\underline{z}_{il}$ and $\bar{z}_{il}$ for constructing controllers and eliminating the effects of external disturbed signals. Thus, we consider linear state-feedback controller as follows:

$$\begin{aligned}u_i(t) =\ & -\hat{k}_{1i}(x_i(t) - s(t))\\ &- ((I - \sigma_i)\hat{\bar{z}}_i(t) + \hat{\underline{z}}_i(t)),\end{aligned} \tag{4}$$

where $i = 1,2,\ldots,N$, $\hat{k}_{1i}$ is the estimate of k_{1i}, which is an unknown constant including the information of network topology; $\hat{\bar{z}}_i(t)$ and $\hat{\underline{z}}_i(t)$ are the estimates of $\bar{z}_i(t)$ and $\underline{z}_i(t)$, respectively; and $\sigma_i = \mathrm{diag}[\sigma_{i1}, \sigma_{i2}, \ldots, \sigma_{in}]$ is an $n \times n$-dimensional diagonal matrix composed by constants $\sigma_{il} \in \{0,1\}$, $l = 1,2,\ldots,n$.

Then, by combining the Lyapunov function methods and the adaptive schemes, the following theorem can be obtained which shows the globally boundedness of the solutions of the adaptive closed-loop system.

Theorem 1 Under Lipshitz condition (2), using the control scheme described in Eq. (4) with updated adaptive laws

$$\begin{aligned}\frac{\mathrm{d}\hat{k}_{1i}(t)}{\mathrm{d}t} &= \kappa_i \underline{p}\|e_i\|^2,\\ \frac{\mathrm{d}\hat{\bar{z}}_{il}(t)}{\mathrm{d}t} &= s_{il}e_i^{\mathrm{T}}(t)P_l,\\ \frac{\mathrm{d}\hat{\underline{z}}_{il}(t)}{\mathrm{d}t} &= s_{il}e_i^{\mathrm{T}}(t)P_l,\end{aligned} \tag{5}$$

where κ_i, s_{il}, $i = 1, 2, \ldots, N$, $l = 1, 2, \ldots, n$ are positive constants, one can guarantee that all signals of complex networked system (3) are bounded and $\lim_{t\to\infty} x_i(t) = s(t)$ for any initial value $x_i(0)$.

Proof Please see Appendix.

Remark 1 Following Eq. (5), the adaptive laws to estimate upper and lower bounds of $z_i(t)$ are the same. However, the estimation values are different because of the different initial values.

Remark 2 Most of existing works consider the topology element a_{ij} as a nonnegative constant. Thus the coupling network plays a good effective role for synchronization of the whole networks, since the eigenvalues of Laplacian matrix L are always non-positive. However, the existing classical results on Laplacian matrices are no longer applicable if the topology matrix of a network may have negative elements, which means the network faulted.[8] The proposed method in this section can also guarantee synchronization of the networks in that case.

4. Numerical example

In this section, an example of robust synchronization design is given to demonstrate the proposed method. A Chua's circuit network composed of twenty dynamical nodes borrowed from Refs. [15] and [31] with perturbation inputs added is considered.

Consider the following target model described by Chua's circuit which is illustrated in Fig. 1. Following the knowledge of electrics, a single Chua's circuit can be described by the following differential equations:

$$\begin{aligned}C_1\dot{v}_1 &= \frac{1}{R}(-v_1 + v_2) - f(v_1),\\ C_2\dot{v}_2 &= \frac{1}{R}(v_1 - v_2) + i_3,\\ L\dot{i}_3 &= -v_2,\end{aligned} \tag{6}$$

where v_1 and v_2 are the voltages across the capacitors C_1 and C_2, i_3 denotes the current through the inductances L, R are linear resistors, the term $f(v_1)$ represents the current through the nonlinear resistor N_R which is a piecewise-linear function expressed as $f(v_1) = G_{b1}v_1 + 0.5(G_{a1} - G_{b1})(|v_1 + 1| - |v_1 - 1|)$ under Lipshitz condition.[15] Then the formulation of Chua's circuit (6) can be defined by the general model (1), that is:

$$\begin{aligned}\begin{bmatrix}\dot{x}_{i1}\\ \dot{x}_{i2}\\ \dot{x}_{i3}\end{bmatrix} &= \begin{bmatrix}-p & p & 0\\ q & -q & r\\ 0 & -v & 0\end{bmatrix}\begin{bmatrix}x_{i1}\\ x_{i2}\\ x_{i3}\end{bmatrix} + \begin{bmatrix}-pf(x_{i1})\\ 0\\ 0\end{bmatrix}\\ &+ c_i\sum_{j=1}^{N} a_{ij}\Gamma x_j(t-\tau_j) + u_i + d_i,\end{aligned} \tag{7}$$

where $x_{i1} = v_{i1}$, $x_{i2} = v_{i2}$, $x_{i3} = i_{i3}$, $p = 1/RC_1$, $q = 1/RC_2$, $r = 1/C_2$, $v = 1/L$, $i = 1, 2, \ldots, N$. In this simulation, the network composed of twenty Chua's circuits. Similar to Ref. [15], we choose $C_1 = 1$ F, $C_2 = 2$ F, $R = (1/7)\ \Omega$, $L = (1/7)$ H, and $G_{b1} = -1/7, G_{a1} = -40/7$. Then the desired synchronization manifold is shown in Fig. 2.

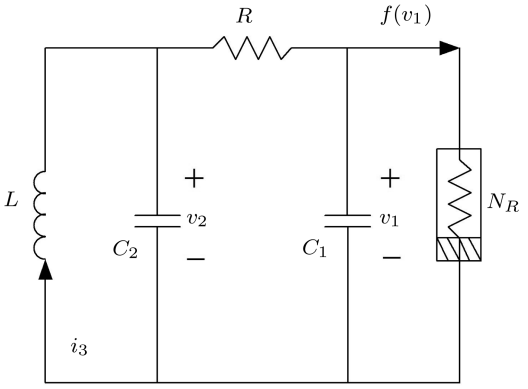


Fig. 1. Chua's circuit.

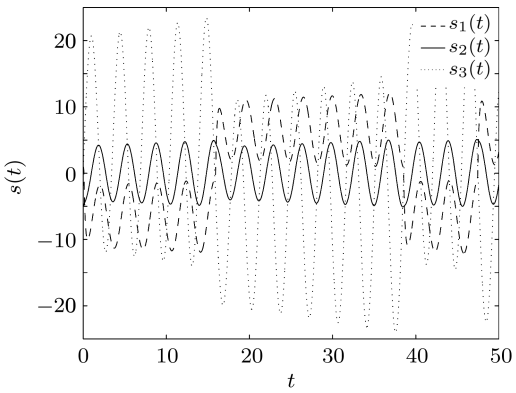


Fig. 2. The desired target orbit, where $s(t) = (s_1(t)), s_2(t)), s_3(t))^{\mathrm{T}}$.

Then, to verify the effectiveness of the proposed adaptive method, the simulations are given

with the following parameters and initial conditions: $\hat{k}_{1i}(0) = 0$, $\hat{\hat{z}}_{il}(0) = 5$, $\underline{\hat{z}}_{il}(0) = -5$, $\underline{p} = 1$, $\lambda = 2$, $\kappa_i = 0.05$, $s_{il} = 1$, $i = 1, 2, \ldots, 20$, $l = 1, 2, 3$. The following case is considered in the simulations, that is, the disturbances: $i = 1, 2, \ldots, 20$, $d_i = [0.3i, \sin(it), -0.2i\cos(0.1it)]$ enter into the networked system at the beginning ($t \geq 0$). Meanwhile, the delays $\tau_i(t) = |0.3\sin(0.3it)|$, $i = 1, 2, \ldots, 20$ exist in communications. The weight matrix $L = (a_{ij})_{20\times 20}$ is a Laplacian matrix with $a_{ij} = 0$ or $a_{ij} = 1$ at the beginning. But at $t = 2$ s, some coupling networks among nodes are faulted, i.e. $a_{ij} = -1$, and the coupling strength of network is also changed. Due to the space limitation, we omit the formulation of network changes here.

Figures 3 and 4 denote respectively the response curves of system states and synchronization errors, which illustrate the proposed distribute adaptive controllers can ensure asymptotic synchronization of networks in the above-mentioned cases. Figures 5 and 6 demonstrate the response curves of the estimations of controller parameters k_{1i} and upper and lower bounds of z_{il}, $i = 1, 2, \ldots, 20$, $l = 1, 2, 3$, respectively. It is easy to see that the estimations can converge and all signals are uniformly bounded.

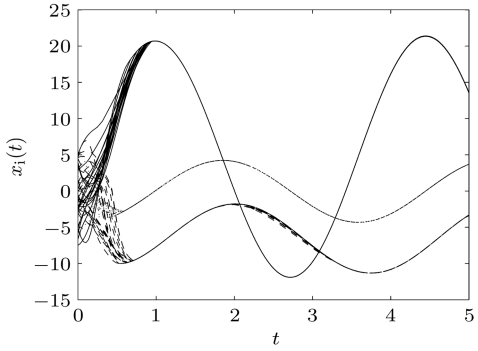


Fig. 3. Synchronization of Chua's network with 20 circuits with adaptive control (x_{i1}-dashed, x_{i2}-dotted, x_{i3}-solid, $i = 1, 2, \ldots, 20$).

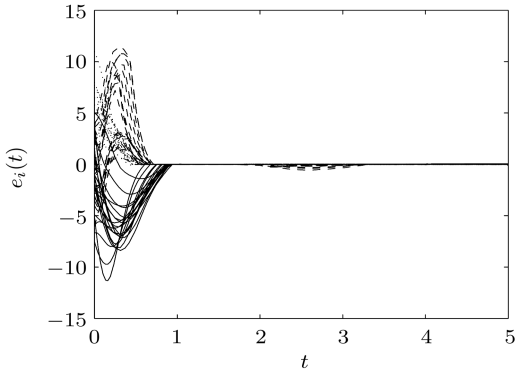


Fig. 4. Synchronization errors of Chua's network with 20 circuits with adaptive control (e_{i1}-dashed, e_{i2}-dotted, e_{i3}-solid, $i = 1, 2, \ldots, 20$).

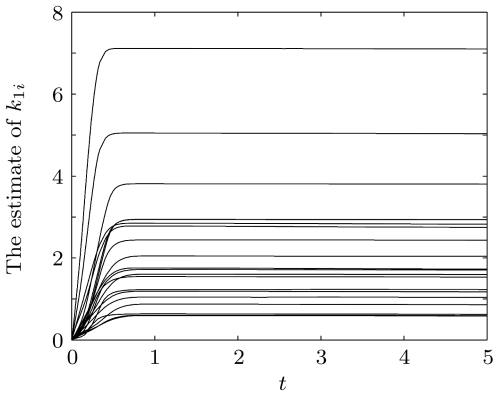


Fig. 5. Response curves of the estimates of $k_{1i}(t)$, $i = 1, 2, \ldots, 20$.

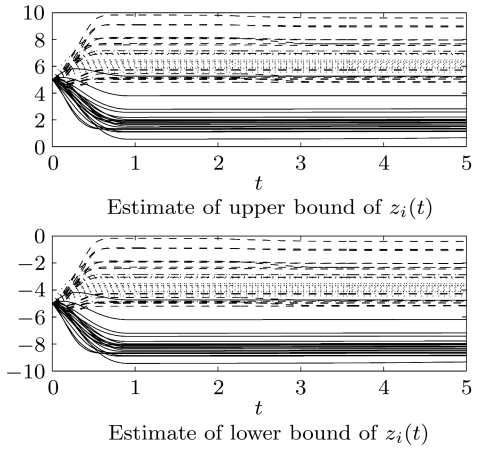


Fig. 6. Response curves of the estimates of upper and lower bounds of $z_{i1}(t)$ (dashed), $z_{i2}(t)$ (dotted), $z_{i3}(t)$ (solid), $i = 1, 2, \ldots, 20$.

5. Conclusion

In this paper, we have shown an adaptive design method to solve the robust synchronization problem for a class of complex networked control systems with coupling time-delays, perturbations and network failures. Adaptive schemes are proposed to estimate the unknown parameters which include the information of network topological structure and coupling strength for automatically compensating for the effects of network failures, as well as to estimate the upper and lower bounds of delayed state errors and perturbations for compensating for the effects of delays and perturbations. On the basis of Lyapunov stability theory, it has been shown that the adaptive closed-loop complex networked system can be guaranteed to be asymptotic synchronization even in the presence of network coupling delays, perturbations and faulted networks. A numerical example has been given to illustrate the effectiveness of the proposed methods.

Appendix: Proof of Theorem 1

Since $\hat{k}_{1i}(t)$, $\hat{\tilde{z}}_{il}(t)$ and $\hat{\underline{z}}_{il}(t)$ are the estimates of $k_{1i}(t)$, $\bar{z}_{il}(t)$ and $\underline{z}_{il}(t)$, respectively, we denote the errors of estimations and their real values as $\tilde{k}_{1i}(t) = \hat{k}_{1i}(t) - k_{1i}$, $\tilde{\tilde{z}}_{il}(t) = \hat{\tilde{z}}_{il}(t) - \bar{z}_{il}$, $\tilde{\underline{z}}_{il}(t) = \hat{\underline{z}}_{il}(t) - \underline{z}_{il}$. As k_{1i}, $\underline{z}_l$, $\bar{z}_l$ are unknown constants, it is obvious that $\dot{\tilde{k}}_{1i}(t) = \dot{\hat{k}}_{1i}(t)$, $\dot{\tilde{\tilde{z}}}_{il}(t) = \dot{\hat{\tilde{z}}}_{il}(t)$, $\dot{\tilde{\underline{z}}}_{il}(t) = \dot{\hat{\underline{z}}}_{il}(t)$, where $i = 1, 2, \ldots, N$, $l = 1, 2, \ldots, n$.

In the following, we use the well-known Lyapunov stability theory to prove that the solutions of network (1) synchronize with $s(t)$ by using control scheme (4) with updated adaptive laws (5). Thus, define a Lyapunov function candidate as:

$$V(t) = \sum_{i=1}^{N} e_i^{\mathrm{T}} P e_i + \sum_{i=1}^{N} \kappa_i^{-1} \tilde{k}_{1i}^2 + \sum_{i=1}^{N}\sum_{l=1}^{n} \frac{(1-\sigma_{il})\tilde{\tilde{z}}_{il}^2}{s_{il}} + \sum_{i=1}^{N}\sum_{l=1}^{n} \frac{\sigma_{il}\tilde{\underline{z}}_{il}^2}{s_{il}}. \tag{A1}$$

It is easily seen that $V(t) > 0$ for any $e_i \neq 0$. Then, from Eqs. (3) and (4), the time derivative of $V(t)$ for $t > 0$ can be given by

$$\begin{aligned}
&\dot{V}(t) \\
&= 2\sum_{i=1}^{N} e_i^{\mathrm{T}} P(f(x_i,t) - f(s,t)) \\
&\quad + 2\sum_{i=1}^{N} e_i^{\mathrm{T}} P c_i \sum_{j=1}^{N} a_{ij} \Gamma e_j \\
&\quad - 2\sum_{i=1}^{N} e_i^{\mathrm{T}} P \hat{k}_{1i} e_i + 2\sum_{i=1}^{N} e_i^{\mathrm{T}} P z_i \\
&\quad - 2\sum_{i=1}^{N} e_i^{\mathrm{T}} P((I-\sigma_i)\hat{\tilde{z}}_i + \hat{\underline{z}}_i) + \sum_{i=1}^{N} 2\kappa_i^{-1}\tilde{k}_{1i}\dot{\tilde{k}}_{1i} \\
&\quad + \sum_{i=1}^{N}\sum_{l=1}^{n} \frac{2(1-\sigma_{il})\tilde{\tilde{z}}_{il}\dot{\tilde{\tilde{z}}}_{il}}{s_{il}} + \sum_{i=1}^{N}\sum_{l=1}^{n} \frac{2\sigma_{il}\tilde{\underline{z}}_{il}\dot{\tilde{\underline{z}}}_{il}}{s_{il}}.
\end{aligned} \tag{A2}$$

Now, we denote that $\underline{p} = \min\{p_1, p_2, \ldots, p_n\}$ is the minimal number among p_l. Moreover, since the positive diagonal matrix P and inner connecting matrix Γ are constant matrices, there always exists a positive constant λ such that the Euclidean norm $\|P\Gamma\| \leq \lambda$. Then, according to the Lipshitz condition (2) and the fact that $b_1\theta^{\mathrm{T}}\zeta \pm b_2\zeta^{\mathrm{T}}\theta \leq \varepsilon_1 b_1 \theta^{\mathrm{T}}\theta + \varepsilon_2 b_2 \zeta^{\mathrm{T}}\zeta$ for any vectors $\theta \in R^n$, $\zeta \in R^n$, constants $b_1 > 0$, $b_2 > 0$, and $\varepsilon_1 \geq 1/2 + b_2/2b_1$, $\varepsilon_2 \geq 1/2 + b_1/2b_2$, equation (A2) can be rewritten by

$$\begin{aligned}
&\dot{V}(t) \\
&\leq \sum_{i=1}^{N} 2\left(q_i + \lambda c_i \sum_{j=1}^{N}(\delta_{ij}-1)a_{ij}\right)\underline{p}^{-1}\|e_i\|^2 \\
&\quad + 2\sum_{i=1}^{N}\sum_{l=1}^{n} e_i^{\mathrm{T}} P_l(z_{il} + \hat{\underline{z}}_{il}) + \sum_{i=1}^{N} 2\kappa_i^{-1}\tilde{k}_{1i}\dot{\tilde{k}}_{1i} \\
&\quad + 2\sum_{i=1}^{N}\sum_{l=1}^{n} e_i^{\mathrm{T}} P_l(1-\sigma_{il})\hat{\tilde{z}}_{il} - 2\sum_{i=1}^{N} e_i^{\mathrm{T}} P \hat{k}_{1i} e_i \\
&\quad + \sum_{i=1}^{N}\sum_{l=1}^{n} \frac{2(1-\sigma_{il})\tilde{\tilde{z}}_{il}\dot{\tilde{\tilde{z}}}_{il}}{s_{il}} + \sum_{i=1}^{N}\sum_{l=1}^{n} \frac{2\sigma_{il}\tilde{\underline{z}}_{il}\dot{\tilde{\underline{z}}}_{il}}{s_{il}},
\end{aligned} \tag{A3}$$

where constant δ_{ij} $i, j = 1, 2, \ldots, N$ satisfies $\delta_{ij} \geq 1/2 + c_j/2c_i$, $\delta_{ij} \neq 1$.

For the sake of eliminating time-varying signal $z_{il}(t)$ in Eq. (A3), we choose $\sigma_{il} = 1$ for $e_i^{\mathrm{T}} P_l \geq 0$ and $\sigma_{il} = 0$ for $e_i^{\mathrm{T}} P_l < 0$, thus the following inequality holds true

$$\sum_{i=1}^{N}\sum_{l=1}^{n} e_i^{\mathrm{T}} P_l z_{il} \leq \sum_{i=1}^{N}\sum_{l=1}^{n} e_i^{\mathrm{T}} P_l((1-\sigma_{il})\bar{z}_{il} + \underline{z}_{il}). \tag{A4}$$

On the other hand, when $k_{1i} = (q_i + \lambda c_i \sum_{j=1}^{N}(\delta_{ij}-1)a_{ij})\underline{p}^{-1}$, and we choose the updated adaptive laws as Eq. (5), we can rewrite Eq. (A3) as

$$\begin{aligned}
\dot{V}(t) &\leq \sum_{i=1}^{N} 2\left(q_i + \lambda c_i \sum_{j=1}^{N}(\delta_{ij}-1)a_{ij}\right)\underline{p}^{-1}\|e_i\|^2 \\
&\quad - \sum_{i=1}^{N} 2\hat{k}_{1i}\underline{p}\|e_i\|^2 + \sum_{i=1}^{N} 2\kappa_i^{-1}\tilde{k}_{1i}\dot{\tilde{k}}_{1i} \\
&= -\sum_{i=1}^{N} 2\tilde{k}_{1i}\underline{p}\|e_i\|^2 + \sum_{i=1}^{N} 2\kappa_i^{-1}\tilde{k}_{1i}\dot{\tilde{k}}_{1i} = 0.
\end{aligned} \tag{A5}$$

According to Eq. (A4), we have $\sum_{i=1}^{N}\sum_{l=1}^{n} e_i^{\mathrm{T}} P_l z_{il} = \sum_{i=1}^{N}\sum_{l=1}^{n} e_i^{\mathrm{T}} P_l((1-\sigma_{il})\bar{z}_{il}(t) + \underline{z}_{il}(t))$ if and only if $e_i = 0$. Thus, it is obvious that $\dot{V} = 0$ if and only if $e_i = 0$, $i = 1, 2, \ldots, N$, and a set E can be found as $E = \{(e, \hat{k}_1, \hat{\tilde{z}}, \hat{\underline{z}}) : \dot{V} = 0\} = \{(e, \hat{k}_1, \hat{\tilde{z}}, \hat{\underline{z}}) : e = 0\}$, where $e = (e_1, e_2, \ldots, e_N)^{\mathrm{T}}$, $\hat{k}_1 = (\hat{k}_{11}, \hat{k}_{12}, \ldots, \hat{k}_{1N})^{\mathrm{T}}$, $\hat{\tilde{z}} = (\hat{\tilde{z}}_1, \hat{\tilde{z}}_2, \ldots, \hat{\tilde{z}}_N)^{\mathrm{T}}$, $\hat{\underline{z}} = (\hat{\underline{z}}_1, \hat{\underline{z}}_2, \ldots, \hat{\underline{z}}_N)^{\mathrm{T}}$. Starting with arbitrary initial values $e_i(0)$, $\hat{k}_{1i}(0)$, $\hat{\tilde{z}}_i(0)$ and $\hat{\underline{z}}_i(0)$, the orbit converges asymptotically to $e_i = 0$, $\hat{k}_{1i} = k_{1i}$, $\hat{\tilde{z}}_i = \bar{z}_i$ and $\hat{\underline{z}}_i = \underline{z}_i$, where k_{1i}, $\bar{z}_i$ and $\underline{z}_i$ are constants. Based on LaSalle invariance principle, the system trajectories converge to the largest positively invariant subset $M = \{(e, \hat{k}_1, \hat{\tilde{z}}, \hat{\underline{z}}) : e = 0, \dot{\hat{k}}_1 = 0, \dot{\hat{\tilde{z}}} = 0, \dot{\hat{\underline{z}}} = 0\}$. Then, the solutions of closed-loop complex network are uniformly bounded, and the states $x_i(t)$, $i = 1, 2, \ldots, N$ converges asymptotically to $s(t)$. This ends the proof.

References

[1] Wang X F 2002 *Int. J. Bifurc. Chaos* **5** 885
[2] Barahona M and Pecora L M 2004 *Phys. Rev. Lett.* **89** 054101
[3] Callway D S, Newman M E J, Strogatz S H and Watts D J 2000 *Phys. Rev. Lett.* **85** 4626
[4] Pecora L M and Carroll T L 1990 *Phys. Rev. Lett.* **64** 821
[5] Duan Z S, Wang J Z, Chen G R and Huang L 2008 *Automatica* **44** 1028
[6] Olfati-Saber R, Fax J A and Murray R M 2007 *Procceedings of the IEEE* **95** 215
[7] Porfiri M and Stilwell D J 2007 *IEEE Trans. Autom. Control.* **52** 1767
[8] Zhou J and Wang Q 2009 *Automatica* **45** 1455
[9] Kar S and Moura J M F 2009 *IEEE Trans. Signal Process.* **57** 355
[10] Li Z, Jiao L and Lee J J 2008 *Physica* A **387** 1369
[11] Wang D F and Han P 2008 *Chin. Phys.* B **17** 3603
[12] Gou L X, Xu Z Y and Hu M F 2008 *Chin. Phys.* B **17** 4067
[13] Wu X 2008 *Physica* A **387** 997
[14] Zheng S, Bi Q and Cai G 2009 *Phys. Lett.* A **373** 1553
[15] Wang L, Dai H P, Dong H, Shen Y H and Sun Y X 2008 *Physica* A **372** 3632
[16] Lou X Y and Cui B T 2008 *Chin. Phys.* B **17** 520
[17] Xiao Y Z, Xu W, Li X C and Tang S F 2008 *Chin. Phys.* B **17** 80
[18] Li X C, Xu W and Xiao Y Z 2008 *Acta Phys. Sin.* **57** 4721 (in Chinese)
[19] Luo X H, Li H Q and Chen Q H 2009 *Acta Phys. Sin.* **58** 7532 (in Chinese)
[20] Zhang Q, Lu J A and Lü J 2008 *IEEE Trans. Circuits Syst. II* **55** 183
[21] Li Z and Chen G 2004 *Phys. Lett.* A **324** 166
[22] Zhou J, Lu J A and Lü J 2006 *IEEE Trans. Autom. Control* **51** 652
[23] Liu H, Lu J A, Lü J and Hill D J *Automatica* **45** 1799
[24] Qi W and Wang Y H 2009 *Chin. Phys.* B **18** 1404
[25] Gao H, Lam J and Chen G 2006 *Phys. Lett.* A **360** 267
[26] Wang Y, Wang Z and Liang J 2008 *Phys. Lett.* A **372** 6066
[27] Yu W, Cao J, Chen G, Lü J, Han J and Wei W 2009 *IEEE Trans. Syst., Man, Cybern.* B: *Cybern.* **39** 230
[28] Liu Y, Wang Z, Liang J and Liu X 2008 *IEEE Trans. Syst., Man, Cybern.* B: *Cybern.* **38** 1314
[29] Tang Y, Zhong H H and Fang J A 2008 *Chin. Phys.* B **17** 4080
[30] Li Z K, Duan Z S and Chen G R 2009 *Chin. Phys.* B **18** 5228
[31] Azemi A and Yaz E 2000 *Transactions of the ASME* **122** 758
[32] Hou Z G, Cheng L and Tan M 2009 *IEEE Trans. Syst., Man, Cybern.* B: *Cybern.* **39** 636
[33] Saberi A, Lin Z and Teel A R 1996 *IEEE Trans. Autom. Control* **41** 368
[34] Jin X Z and Yang G H 2009 *Acta Automatica Sin.* **35** 1114
[35] Chou C H and Cheng C C 2003 *IEEE Trans. Autom. Control* **48** 1213