Generalized chaos synchronization of a weighted complex network with different nodes*

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This paper proposes a method of realizing generalized chaos synchronization of a weighted complex network with different nodes. Chaotic systems with diverse structures are taken as the nodes of the complex dynamical network, the nonlinear terms of the systems are taken as coupling functions, and the relations among the nodes are built through weighted connections. The structure of the coupling functions between the connected nodes is obtained based on Lyapunov stability theory. A complex network with nodes of Lorenz system, Coullet system, Rössler system and the New system is taken as an example for simulation study and the results show that generalized chaos synchronization exists in the whole weighted complex network with different nodes when the coupling strength among the nodes is given with any weight value. The method can be used in realizing generalized chaos synchronization of a weighted complex network with different nodes. Furthermore, both the weight value of the coupling strength among the nodes and the number of the nodes have no effect on the stability of synchronization in the whole complex network.

Keywords: chaos synchronization, weighted network, diverse structure, Lyapunov stability theory

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1. Introduction

A lot of complex systems which exist in nature have interaction with each other, and such associated systems can be described by a complex network. Each node of the network stands for a single natural system, while the interaction among the systems is described by the links among the nodes of the network. Since the most famous random graph model was proposed by Erdös, the complex network has attracted much attention in many fields of research, such as biology, physics, computer networks, the World Wide Web (WWW), and so on. The study is mainly focused on three aspects, i.e., the establishment of complex network models, the property and the dynamics of networks, and much important progress has been made. The most famous models of complex networks at present include the random graph model,[1] the small-world network,[2] scale-free network,[3] and generalized dynamic complex network model, etc. The property of spreading, clustering and distribution of node degree,[4−10] etc., are the emphases in network property studies. While in dynamics of network, the study is mainly on the spread of epidemics, phase changing, and chaos synchronization of the network, and so on. The chaos synchronization in networks has become a focal point in the study of dynamics of network due to the fact that network synchronization has many applications in various fields, such as the synchronous information exchange in the WWW internet, the synchronous transfer of digital or analog signals in the communication networks, etc. Since network synchronization has obvious advantages, it has great application value in practice. Therefore, Atay *et al.* studied synchronization of complex network when delays exist among the nodes;[11] Motter *et al.* studied the influence of coupling strength on the synchronizing ability of a complex network;[12] Timme *et al.* studied the web synchronization law of pulse-coupled dynamical systems;[13] Checco *et al.* studied the synchronization of random web;[14] Lü *et al.* constructed general complex dynamical networks, and studied the synchronization;[15] Lu and Chen studied synchronization analysis of linearly coupled networks of discrete time systems;[16] Han and Lu studied the changes of synchronization ability of coupled networks from ring networks to chain networks;[17] He and Yang studied adaptive synchronization in nonlinearly coupled

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dynamical networks;[18] Hung *et al.* studied globally generalized synchronization in scale-free networks.[19] These important research results have laid a good foundation for further theoretical research and practical application of complicated network synchronization.

The study on the method of synchronization in complex network is still at the exploratory stage, there is no sound theoretical system and the existing synchronization method basically used ideal network model and ideal synchronization programme, which cannot match the actual nature and is not practical. So far, three key issues are still unresolved in establishing a sound, practical and universal theoretical system. One is that, in the reported articles on network synchronization, an important and necessary condition is that all network nodes must have the same structure, that is, the chaotic systems of network nodes must be identical and the only difference is the initial conditions of the systems. Therefore, to realize synchronization of these identical chaotic systems is to restrain track separation caused by the sensitivity to the initial value rather than that caused by the different characteristics of bifurcation as well as the different domains of attraction. However, though the structures of the complex network nodes are often different, the study on it has not been reported. Secondly, most existing synchronization researches in complex networks are of unweighted networks, in which only connections between nodes are considered and their intensity difference and the effects are ignored. In reality, the intensity difference exists and it requires us to give a different weight value when we study complex networks, which then is called weighted network. Some correlative physical quantities are often regarded as the link weight of network in many practical physical processes, for example, the resistance value in resistance network, the bandwidth in Internet network, the successive amplified signal in laser network composed of emitting system, relay system and receiving system, and so on. Recently, it has aroused great interest of scholars both domestically and abroad to build weighted network models and study the characteristics of them. Barrat *et al.* presented a weighted network model, and studied its dynamic character;[20] Li and Cai made statistical analysis of a class of real networks;[21] Fang *et al.* presented a harmonious unifying hybrid preferential model, and applied the design to a typical analysis of weighted network models;[22] Xiang *et al.* proposed feedback controllers and added effective control on weighted complex networks with heterogeneous delays.[23] However, universal synchronization methods of weighted network still need further exploration and study. Thirdly, for the research of existing network synchronization, an important premise is that the structure of the complex network is known, that is, when we study network synchronization, we use definite network nodes as well as definite coupling connection. While in practice, the structure of a complex network is often partly known or totally unknown. At this point, most existing program above are limited or even failures.

A method of realizing generalized chaos synchronization of a weighted complex network with different nodes is proposed in this paper. Chaotic systems with diverse structures are taken as the nodes of the complex dynamical network, the nonlinear terms of the systems are taken as coupling functions, and the relations among the nodes are built through a weighted connection. The structure of the coupling functions among the connected nodes is obtained by Lyapunov stability theory. Complex network with nodes of Lorenz system, Coullet system, Rössler system and the New system is taken as an example in simulation study, and simulation results show that generalized chaos synchronization can be realized for the whole complex network when the coupling strength among the nodes is given with any weight value. The method can be used in realizing generalized chaos synchronization of a weighted complex network with different nodes. Furthermore, it is found that both the weight value of the coupling strength among the nodes and the number of the nodes have no effect on the stability of synchronization in the whole complex network.

2. A theorem of synchronization

Consider a complex dynamical network consisting of m nodes which are different chaotic systems, in which the n-dimensional variable of node i is $\boldsymbol{x}_i$, with $\boldsymbol{x}_i = (x_{i1}, x_{i2}, \ldots, x_{in}) \in \boldsymbol{R}^n$. When the coupling action is not considered, each node can be described by the following equation of state:

$$\dot{\boldsymbol{x}}_i = \boldsymbol{F}_i(\boldsymbol{x}_i), \tag{1}$$

where $\boldsymbol{F}_i : \boldsymbol{R}^n \to \boldsymbol{R}^n$.

The $\boldsymbol{F}_i(\boldsymbol{x}_i)$ can be properly separated as follows:

$$\dot{\boldsymbol{x}}_i = \boldsymbol{F}_i(x_i) = \boldsymbol{Q}_i(\boldsymbol{x}_i) + \boldsymbol{D}_i(\boldsymbol{x}_i), \tag{2}$$

where $\boldsymbol{Q}_i(\boldsymbol{x}_i)$ is a linear term. And $\boldsymbol{Q}_i(\boldsymbol{x}_i)$ can be written in the following form:

$$\boldsymbol{Q}_i(\boldsymbol{x}_i) = (\boldsymbol{A}_i - \boldsymbol{G}_i)\boldsymbol{x}_i, \tag{3}$$

where $\boldsymbol{A}_i$ is the linear coefficient matrix of the system, $\boldsymbol{G}_i$ is the coefficient matrix of control gain. Obviously it is

$$\boldsymbol{D}_i(\boldsymbol{x}_i) = \boldsymbol{F}_i(\boldsymbol{x}_i) - \boldsymbol{Q}_i(\boldsymbol{x}_i). \tag{4}$$

When coupling is considered, the equation of state of node i can be described by

$$\begin{aligned}\dot{\boldsymbol{x}}_i &= \boldsymbol{F}_i(\boldsymbol{x}_i) + \boldsymbol{J}_i(\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_m)\\ &= (\boldsymbol{A}_i - \boldsymbol{G}_i)\boldsymbol{x}_i + \boldsymbol{D}_i(\boldsymbol{x}_i) + \boldsymbol{J}_i(\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_m),\\ &\quad (i = 1, 2, \ldots, m),\end{aligned} \tag{5}$$

where $\boldsymbol{J}_i(\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_m)$ is the coupling function of node i.

Theorem When the coupling functions in Eq. (5) is

$$\begin{aligned}\boldsymbol{J}_k &= -\boldsymbol{D}_k(\boldsymbol{x}_k) + \sigma_k[\boldsymbol{D}_1(\boldsymbol{x}_1) + \boldsymbol{J}_1],\\ &\quad (k = 2, 3, \ldots, m),\end{aligned} \tag{6}$$

where σ_k is the relative weight of the coupling strength among the nodes of the network, then no matter what value of σ_k is given, generalized synchronization of the complex network can be fulfilled with a scaling factor $\beta_k = -\sigma_k/\sigma_{k+1}$ (with $\sigma_1 = 1$).

Proof The errors between the state variables of the network are defined as

$$\boldsymbol{e}_i = \boldsymbol{x}_i + \beta_i \boldsymbol{x}_{i+1}, \quad (i = 1, 2, \ldots, m-1), \tag{7}$$

we obtain from Eq. (7)

$$\begin{aligned}&\dot{\boldsymbol{e}}_i\\ &= \dot{\boldsymbol{x}}_i + \beta_i \dot{\boldsymbol{x}}_{i+1}\\ &= (\boldsymbol{A}_i - \boldsymbol{G}_i)\boldsymbol{e}_i + [(\boldsymbol{A}_{i+1} - \boldsymbol{G}_{i+1}) - (\boldsymbol{A}_i - \boldsymbol{G}_i)]\beta_i \boldsymbol{x}_{i+1}\\ &\quad + \Delta\boldsymbol{D}(\boldsymbol{x}_i, \boldsymbol{x}_{i+1}) + \Delta\boldsymbol{J}_i,\end{aligned} \tag{8}$$

where $\Delta\boldsymbol{D}(\boldsymbol{x}_i, \boldsymbol{x}_{i+1}) = \boldsymbol{D}_i(\boldsymbol{x}_i) + \beta_i \boldsymbol{D}_{i+1}(\boldsymbol{x}_{i+1})$, $\Delta\boldsymbol{J}_i = \boldsymbol{J}_i(\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_m) + \beta_i \boldsymbol{J}_{i+1}(\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_m)$.

Taking

$$\Delta\boldsymbol{J}_i = -\Delta\boldsymbol{D}(\boldsymbol{x}_i, \boldsymbol{x}_{i+1}), \quad (i = 1, 2, \ldots, m-1), \tag{9}$$

we have the following recurrence formulas for the coupling function

$$\begin{aligned}&\boldsymbol{J}_k\\ &= \left(-\frac{1}{\beta_{k-1}}\right)[\boldsymbol{J}_{k-1} + \Delta\boldsymbol{D}(\boldsymbol{x}_{k-1}, \boldsymbol{x}_k)]\\ &= \sum_{i=1}^{k-1}\prod_{j=i}^{k-1}\left(-\frac{1}{\beta_j}\right)\Delta\boldsymbol{D}(\boldsymbol{x}_i, \boldsymbol{x}_{i+1}) + \prod_{i=1}^{k-1}\left(-\frac{1}{\beta_i}\right)J_1\\ &= -\boldsymbol{D}_k(\boldsymbol{x}_k) + \prod_{i=1}^{k-1}\left(-\frac{1}{\beta_i}\right)[\boldsymbol{D}_1(x_1) + \boldsymbol{J}_1]\\ &= -\boldsymbol{D}_k(\boldsymbol{x}_k) + \sigma_k[\boldsymbol{D}_1(\boldsymbol{x}_1) + \boldsymbol{J}_1],\\ &\quad (k = 2, 3, \ldots, m).\end{aligned} \tag{10}$$

Where we take

$$\sigma_k = \prod_{i=1}^{k-1}\left(-\frac{1}{\beta_i}\right), \tag{11}$$

that is,

$$\beta_k = -\frac{\sigma_k}{\sigma_{k+1}}, \quad (\text{with} \quad \sigma_1 = 1). \tag{12}$$

From Eq. (10), we see σ_k is the relative weight of the coupling strength among the nodes of the network.

Constructing the Lyapunov function according to the weighted complex network with different nodes

$$V = \frac{1}{2}\sum_{i=1}^{m-1}\boldsymbol{e}_i^2, \tag{13}$$

and considering Eqs. (8) and (9), we can obtain the derivative form of V as

$$\begin{aligned}\dot{V} &= \sum_{i=1}^{m-1}\boldsymbol{e}_i\dot{\boldsymbol{e}}_i\\ &= \sum_{i=1}^{m-1}\Big\{(\boldsymbol{A}_i - \boldsymbol{G}_i)\boldsymbol{e}_i^2 + [(\boldsymbol{A}_{i+1} - \boldsymbol{G}_{i+1})\\ &\quad - (\boldsymbol{A}_i - \boldsymbol{G}_i)]\beta_i \boldsymbol{x}_{i+1}\boldsymbol{e}_i\Big\}.\end{aligned} \tag{14}$$

From Eq. (14), we can easily see that if

$$\begin{aligned}&\boldsymbol{G}_i \geq \boldsymbol{A}_i,\\ &\boldsymbol{G}_{i+1} = \boldsymbol{A}_{i+1} - \boldsymbol{A}_i + \boldsymbol{G}_i,\end{aligned} \tag{15}$$

then

$$\dot{V} \leq 0. \tag{16}$$

According to Lyapunov stability theory,[24] generalized synchronization can be realized for the whole network. From Eq. (12), we can see that the relationship between the scaling factor β_i and the relative weight of the coupling strength among the nodes of the network σ_k is built. Therefore, no matter what value σ_k is given, generalized synchronization of any complex network can be fulfilled with the value $\beta_k = -\sigma_k/\sigma_{k+1}$.

3. Simulation results

Lorenz system, Rössler system, Coullet system and the New system are taken as nodes of the network to show the synchronization mechanism mentioned above. Simulation is made as the number of the nodes is $m = 4$.

The dynamic equation of Lorenz system is described as follows:[25]

$$\begin{aligned} \dot{x}_1 &= a_1(y_1 - x_1), \\ \dot{y}_1 &= x_1(b_1 - z_1) - y_1, \\ \dot{z}_1 &= x_1 y_1 - c_1 z_1, \end{aligned} \tag{17}$$

where a_1, b_1, c_1 are parameters of the system. When the parameters are given as $a_1 = 10$, $b_1 = 28$, $c_1 = 8/3$, the chaos attractor of Eq. (17) is shown in Fig. 1.

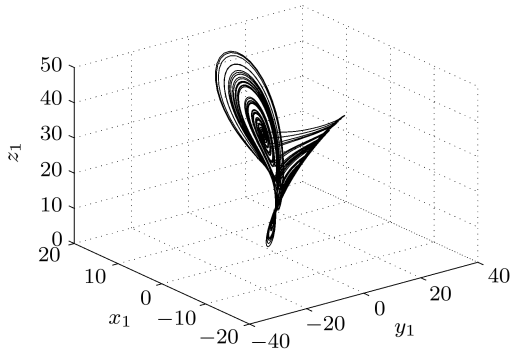


Fig. 1. The chaos attractor of Lorenz system.

The dynamic equation of Coullet system is described as follows:[26]

$$\begin{aligned} \dot{x}_2 &= y_2, \\ \dot{y}_2 &= z_2, \\ \dot{z}_2 &= a_2 z_2 + b_2 y_2 + c_2 x_2 - x_2^3, \end{aligned} \tag{18}$$

where a_2, b_2, c_2 are parameters of the system. When the parameters are given as $a_2 = -0.45$, $b_2 = -1.1$, $c_2 = 0.8$, the chaos attractor of Eq. (18) is shown in Fig. 2.

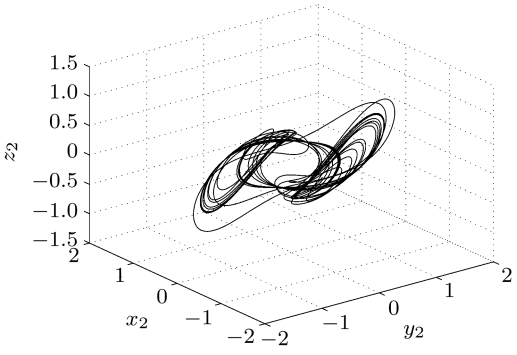


Fig. 2. The chaos attractor of Coullet system.

The dynamic equation of Rössler system is described as follows:[27]

$$\begin{aligned} \dot{x}_3 &= -y_3 - z_3, \\ \dot{y}_3 &= x_3 + a_3 y_3, \\ \dot{z}_3 &= b_3 + x_3 z_3 - c_3 z_3, \end{aligned} \tag{19}$$

where a_3, b_3, c_3 are parameters of the system. When the parameters are given as $a_3 = 0.2$, $b_3 = 0.2$, $c_3 = 5.7$, the chaos attractor of Eq. (19) is shown in Fig. 3.

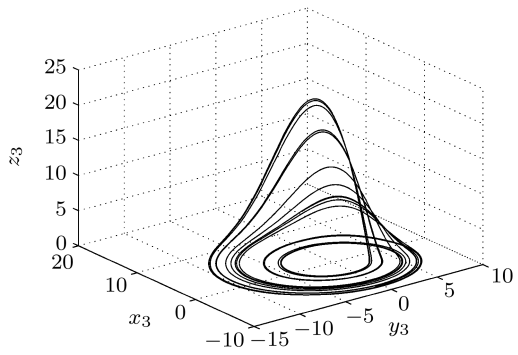


Fig. 3. The chaos attractor of Rössler system.

The dynamic equation of the New system is described as follows:[28]

$$\begin{aligned} \dot{x}_4 &= a_4 x_4 - y_4 z_4, \\ \dot{y}_4 &= -b_4 y_4 + x_4 z_4, \\ \dot{z}_4 &= x_4 y_4 - c_4 z_4, \end{aligned} \tag{20}$$

where a_4, b_4, c_4 are parameters of the system. When the parameters are given as $a_4 = 5$, $b_4 = 15$, $c_4 = 3.4$, the chaos attractor of Eq. (20) is shown in Fig. 4.

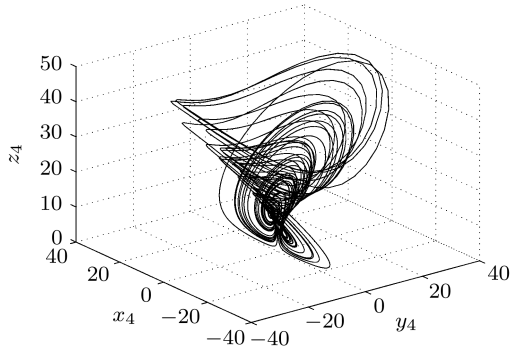


Fig. 4. The chaos attractor of the New system.

When the simulation is made for the network synchronization, the coupling function $\boldsymbol{J}_1$ is taken to be zero, i.e. $\boldsymbol{J}_1 = 0$, which means that the first node of

the network, Lorenz system, is taken as the target system and the others are response systems. The linear coefficient matrix and the coefficient matrix of control gain of the first nodes are taken as

$$\boldsymbol{A}_1 = \begin{bmatrix} -a_1 & a_1 & 0 \\ b_1 & -1 & 0 \\ 0 & 0 & -c_1 \end{bmatrix}, \tag{21}$$

$$\boldsymbol{G}_1 = \begin{bmatrix} 0 & a_1 & 0 \\ b_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{22}$$

The complex network of the four different chaotic systems is constructed according to Eq. (5)

$$\begin{aligned} \dot{x}_1 &= a_1(y_1 - x_1), \\ \dot{y}_1 &= x_1(b_1 - z_1) - y_1, \\ \dot{z}_1 &= x_1 y_1 - c_1 z_1; \end{aligned} \tag{23}$$

$$\begin{aligned} \dot{x}_2 &= y_2 + J_{21}, \\ \dot{y}_2 &= z_2 + J_{22}, \\ \dot{z}_2 &= a_2 z_2 + b_2 y_2 + c_2 x_2 - x_2^3 + J_{23}; \end{aligned} \tag{24}$$

$$\begin{aligned} \dot{x}_3 &= -y_3 - z_3 + J_{31}, \\ \dot{y}_3 &= x_3 + a_3 y_3 + J_{32}, \\ \dot{z}_3 &= b_3 + x_3 z_3 - c_3 z_3 + J_{33}; \end{aligned} \tag{25}$$

$$\begin{aligned} \dot{x}_4 &= a_4 x_4 - y_4 z_4 + J_{41}, \\ \dot{y}_4 &= -b_4 y_4 + x_4 z_4 + J_{42}, \\ \dot{z}_4 &= x_4 y_4 - c_4 z_4 + J_{43}; \end{aligned} \tag{26}$$

where $\boldsymbol{J}_k$ $(k = 2, 3, 4)$ fulfills Eq. (6)

$$\boldsymbol{J}_2 = \begin{bmatrix} J_{21} \\ J_{22} \\ J_{23} \end{bmatrix} = - \begin{bmatrix} 0 \\ 0 \\ -x_2^3 \end{bmatrix} + \sigma_2 \begin{bmatrix} 0 \\ -x_1 z_1 \\ x_1 y_1 \end{bmatrix}, \tag{27}$$

$$\boldsymbol{J}_3 = \begin{bmatrix} J_{31} \\ J_{32} \\ J_{33} \end{bmatrix} = - \begin{bmatrix} 0 \\ 0 \\ x_3 z_3 \end{bmatrix} + \sigma_3 \begin{bmatrix} 0 \\ -x_1 z_1 \\ x_1 y_1 \end{bmatrix}, \tag{28}$$

$$\boldsymbol{J}_4 = \begin{bmatrix} J_{41} \\ J_{42} \\ J_{43} \end{bmatrix} = - \begin{bmatrix} -y_4 z_4 \\ x_4 z_4 \\ x_4 y_4 \end{bmatrix} + \sigma_4 \begin{bmatrix} 0 \\ -x_1 z_1 \\ x_1 y_1 \end{bmatrix}, \tag{29}$$

and $\boldsymbol{G}_i$ $(i = 2, 3, 4)$ fulfills Eq. (15).

The relative weights of the coupling strength among the nodes of the network are taken as $\sigma_2 = 2$, $\sigma_3 = -2$, $\sigma_4 = 0.5$. From the relation in Eq. (11), we have the values of the scaling factors, $\beta_1 = -1/2$, $\beta_2 = 1$,$\beta_3 = 4$. The network coupling is added at 5 seconds in the simulation. Simulation results are shown in Figs. 5–10. From Figs. 5–7 we can see that all the error signals of the systems in the network approach to zero smoothly and rapidly after adding network coupling. Figure 8 shows that the amplitude of node 2 is two times larger than that of node 1, but both with the same sign. In Fig. 9 the amplitude of node 3 is the same as that of node 2, but with an opposite sign. The amplitude of node 4 is a quarter of node 3 with an opposite sign as shown in Fig. 10.

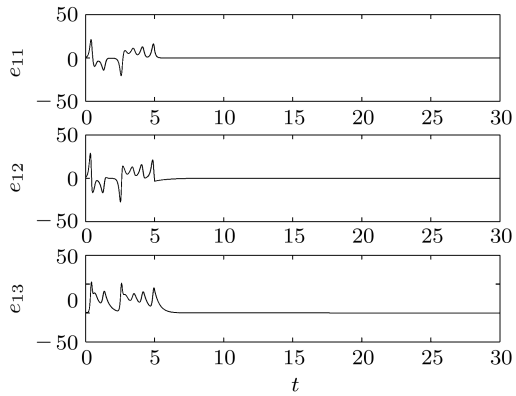


Fig. 5. The temporal evolution of error variables of nodes 1 and 2.

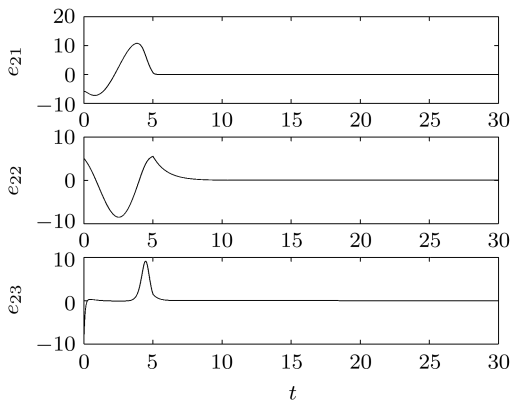


Fig. 6. The temporal evolution of error variables of nodes 2 and 3.

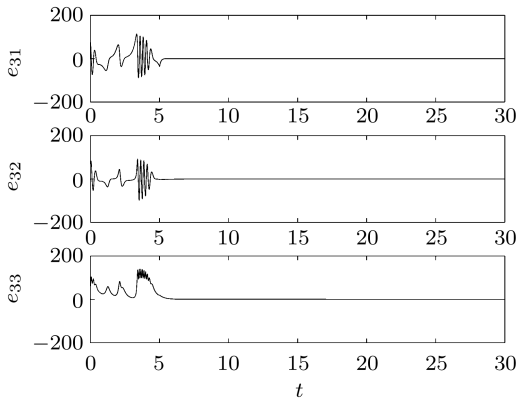


Fig. 7. The temporal evolution of error variables of nodes 3 and 4.

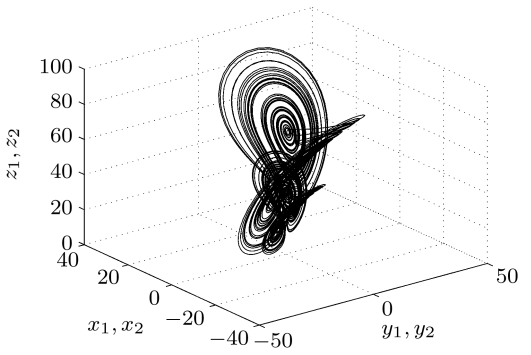


Fig. 8. The chaos attractors of nodes 1 and 2 in synchronization.

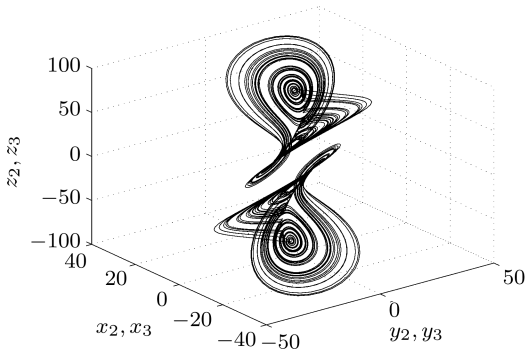


Fig. 9. The chaos attractors of nodes 2 and 3 in synchronization.

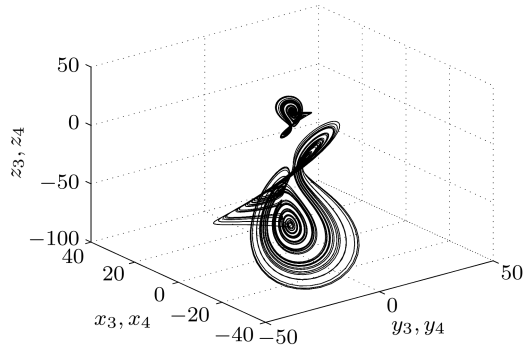


Fig. 10. The chaos attractors of nodes 3 and 4 in synchronization.

Furthermore, in the condition that the linear coefficient matrices of the chaotic systems and the coefficient matrices of control gain fulfill Eq. (15), generalized chaos synchronization exists when the coupling strength among the nodes is given with any weight value. When the weight value of the coupling strength is changed in the simulation and keeping the other parameters and simulation data unchanged, we find that the change of the relative weight value has no obvious effect on the stability of the generalized synchronization. Meanwhile, the number of the nodes has no effect on the stability of synchronization in the whole complex network.

4. Conclusion

Generalized chaos synchronization of a weighted complex network with different nodes is studied analytically and numerically. In the study chaotic systems with diverse structures are taken as the nodes of the complex dynamical network, the nonlinear terms of the systems are taken as coupling functions, and the complex network is built through a weighted connection. The structure of the coupling functions among the connected nodes is obtained based on Lyapunov stability theory. When the Lorenz system, Coullet system, Rössler system and the New system are taken as nodes of the complex network in simulation, our results show that in the condition that the linear coefficient matrices of the chaos systems and the coefficient matrices of control gain fulfill Eq. (15), generalized chaos synchronization exists when the coupling strength among the nodes is given with any weight value. Both the weight value of the coupling strength between the nodes and the number of the nodes have no effect on the stability of synchronization in the whole complex network.

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