Projective synchronization of spatiotemporal chaos in a weighted complex network*

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Projective synchronization of a weighted complex network is studied in which nodes are spatiotemporal chaos systems and all nodes are coupled not with the nonlinear terms of the system but through a weighted connection. The range of the linear coefficient matrix of separated configuration, when the synchronization is implemented, is determined according to Lyapunov stability theory. It is found that projective synchronization can be realized for unidirectional star-connection even if the coupling strength between the nodes is a given arbitrary weight value. The Gray–Scott models having spatiotemporal chaos behaviours are taken as nodes in the weighted complex network, and simulation results of spatiotemporal synchronization show the effectiveness of the method.

Keywords: weighted network, spatiotemporal chaos, projective synchronization, Lyapunov stability theory

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1. Introduction

Network dynamics as a cutting-edge issue, has aroused the great interest of scholars in recent years. Great progress has been made on the establishment of complex network models, the properties and the dynamics of networks[1−10] since Newman *et al.*,[11] Watts and Strogatz,[12] and Albert *et al.*[13] presented random graph, small-world network and scale-free network, respectively. Especially, the research of chaos synchronization in networks has become a focal point of network dynamics investigation. Henning and Schimansky-Geier[14] studied the synchronization property of complex network with nodes of Fitzhugh–Nagumo systems; Yu and Cao[15] realized the synchronization of stochastic delayed neural networks based on stability theory; Lu and Chen[16] reported on the synchronization analysis of linearly coupled networks of discrete time systems; Hung *et al.*[17] discussed the globally generalized synchronization in scale-free networks; Atay *et al.*[18] investigated the synchronization of complex network when delays existed among the nodes; Lü *et al.*[19] constructed general complex dynamical networks and simulated artificially the synchronization; Han and Lu[20] introduced the changes in synchronization ability of coupled networks from ring networks to chain networks. However, most existing synchronization researches involving complex networks are for unweighted networks, that is, only connections between nodes are considered but the intensity difference and their effects are ignored. It is known that most practical networks are weighted networks, and the difference in weight value between nodes should be considered to study complex networks. Recently, the theory and the application investigation on the weighted network has attracted great attention in many fields.[21−25] But chaos synchronization of weighted networks is less reported so far.

In this paper, the projective synchronization of spatiotemporal chaos in a weighted complex network is analysed. Spatiotemporal chaos systems are taken as nodes of the network, the nonlinear terms of the systems are treated as coupling functions, and the relations between the nodes are built through a weighted connection. The range of the linear coefficient matrix of separated configuration, when the synchronization is implemented, is determined based on Lyapunov stability theory. It is found that projective synchronization can be realized for unidirectional star-connection when the coupling strength between the nodes is given to be any weight value. The Gray–Scott models having spatiotemporal chaos behaviours are used as nodes in the weighted complex network, and simulation re-

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sults of spatiotemporal synchronization show the effectiveness of the method.

2. Synchronization principle

Suppose that a complex dynamical network consists of m nodes, and the state equations of the nodes are spatiotemporal chaos systems, in which the n-dimensional variable of node i at time t is $\boldsymbol{x}_i(r,t)$, and $\boldsymbol{x}_i(r,t) \in \boldsymbol{R}^n$. When the coupling action is not considered, each node can be described by the following equation:

$$\frac{\partial \boldsymbol{x}_i(r,t)}{\partial t} = \boldsymbol{F}(\boldsymbol{x}_i(r,t)), \tag{1}$$

where r and t are spatial and temporal variables, and $\boldsymbol{F} : \boldsymbol{R}^n \to \boldsymbol{R}^n$.

$\boldsymbol{F}(\boldsymbol{x}_i(r,t))$ can be properly separated as follows:

$$\begin{aligned}\frac{\partial \boldsymbol{x}_i(r,t)}{\partial t} &= \boldsymbol{F}(\boldsymbol{x}_i(r,t)) \\ &= \boldsymbol{A}\boldsymbol{x}_i(r,t) + \boldsymbol{H}(\boldsymbol{x}_i(r,t)),\end{aligned} \tag{2}$$

where $\boldsymbol{A}$ is the linear coefficient matrix of the system with proper separation configuration.

Coupling on other nodes is added by using the nonlinear part of Eq. (2), and the state equation (2) of the i-th node is rewritten as

$$\begin{aligned}\frac{\partial \boldsymbol{x}_i(r,t)}{\partial t} &= \boldsymbol{A}\boldsymbol{x}_i(r,t) + \boldsymbol{H}(\boldsymbol{x}_i(r,t)) + \boldsymbol{J}_i \\ &= \boldsymbol{A}\boldsymbol{x}_i(r,t) + \boldsymbol{H}(\boldsymbol{x}_i(r,t)) \\ &\quad + \sigma \sum_{j=1}^{m} \hat{G}_{ij}\boldsymbol{H}(\boldsymbol{x}_j(r,t)),\end{aligned} \tag{3}$$

where $\boldsymbol{J}_i$ are coupling functions of the connected nodes, $\boldsymbol{J}_i = \sigma \sum_{j=1}^{m} \hat{G}_{ij}\boldsymbol{H}(\boldsymbol{x}_j(r,t))$, with σ being the coupling strength between the nodes of the network, and selected to be $\sigma = 1$ in the work; $\hat{G}_{ij} = \alpha_{ij}G_{ij}$ with α_{ij} being the relative coupling weight, and G_{ij} being the matrix element of the coupling matrix, whose specific forms depend on the connection type of the network. Unidirectional star-connection is discussed in the following parts.

The error of the network is defined as

$$\begin{aligned}&\boldsymbol{e}_i(r,t) = \boldsymbol{x}_i(r,t) + \beta_i \boldsymbol{x}_{i+1}(r,t), \\ &(i = 1, 2, \ldots, m-1),\end{aligned} \tag{4}$$

where β_i are scale factors.

Theorem Any node can be taken as a target system according to network system (3). For example, the first node is taken as the target system. Then, the projective synchronization of the spatiotemporal chaos in a weighted complex network can be fulfilled if $\alpha_{ii} = 1$ and $\beta_i = -\alpha_{i1}/\alpha_{i+11}$.

Proof In a unidirectional star-connection, system (3) of the network can be written as

$$\begin{aligned}\frac{\partial \boldsymbol{x}_i(r,t)}{\partial t} &= \boldsymbol{A}\boldsymbol{x}_i(r,t) + \boldsymbol{H}(\boldsymbol{x}_i(r,t)) \\ &\quad + \alpha_{i1}\boldsymbol{H}(\boldsymbol{x}_1(r,t)) - \alpha_{ii}\boldsymbol{H}(\boldsymbol{x}_i(r,t)).\end{aligned} \tag{5}$$

The errors of the networks are defined as

$$\begin{aligned}&\boldsymbol{e}_i(r,t) = \boldsymbol{x}_i(r,t) + \beta_i \boldsymbol{x}_{i+1}(r,t), \\ &(i = 1, 2, \ldots, m-1),\end{aligned} \tag{6}$$

then

$$\begin{aligned}\frac{\partial \boldsymbol{e}_i(r,t)}{\partial t} &= \frac{\partial \boldsymbol{x}_i(r,t)}{\partial t} + \beta_i \frac{\partial \boldsymbol{x}_{i+1}(r,t)}{\partial t} \\ &= \boldsymbol{A}\boldsymbol{e}_i(r,t) + (1-\alpha_{ii})\boldsymbol{H}(\boldsymbol{x}_i(r,t)) \\ &\quad + (\alpha_{i1} + \beta_i\alpha_{i+11})\boldsymbol{H}(\boldsymbol{x}_1(r,t)) \\ &\quad + \beta_i(1-\alpha_{i+1,i+1})\boldsymbol{H}(\boldsymbol{x}_{i+1}(r,t)).\end{aligned} \tag{7}$$

If

$$\alpha_{ii} = 1, \tag{8}$$

$$\beta_i = -\frac{\alpha_{i1}}{\alpha_{i+11}}, \tag{9}$$

we have

$$\frac{\partial \boldsymbol{e}_i(r,t)}{\partial t} = \boldsymbol{A}\boldsymbol{e}_i(r,t). \tag{10}$$

Constructing Lyapunov function

$$V(r,t) = \frac{1}{2}\sum_{i=1}^{m-1} \boldsymbol{e}_i^2(r,t), \tag{11}$$

and considering Eq. (10), the derivative form of $V(r,t)$can be obtained as

$$\frac{\partial V(r,t)}{\partial t} = \sum_{i=1}^{m-1} \boldsymbol{e}_i(r,t)\frac{\partial \boldsymbol{e}_i(r,t)}{\partial t} = \boldsymbol{A}\sum_{i=1}^{m-1} \boldsymbol{e}_i^2(r,t). \tag{12}$$

From Eq. (12), it can be easily seen that if $\boldsymbol{A}$ is taken as

$$\boldsymbol{A} \le 0, \tag{13}$$

then

$$\frac{\partial V(r,t)}{\partial t} \le 0. \tag{14}$$

The whole network will realize the projective synchronization based on the Lapunov stability theory.[26]

3. Simulation examples

Gray–Scott model is described by the following dynamics equations:[27]

$$\begin{cases} \dfrac{\partial x_1(r,t)}{\partial t} = -x_1(r,t)x_2^2(r,t) + a(1-x_1(r,t)) + D_1\nabla^2 x_1(r,t), \\ \dfrac{\partial x_2(r,t)}{\partial t} = x_1(r,t)x_2^2(r,t) - (a+b)x_2(r,t) + D_2\nabla^2 x_2(r,t), \end{cases} \tag{15}$$

where a and b are the system parameters; $x_1(r,t)$ and $x_2(r,t)$ are the state variables of the system; $x_1(r,t) \in [0,1]$; $x_2(r,t) \in [0,1]$; D_1 and D_2 are diffusion coefficients. System size L is 2.5×2.5, and $r \in [0,L]$. The initial conditions are given as $x_1(r,0) = 1$, $x_2(r,0) = 0$, and time step $\Delta t = 1$, space step $\Delta l = 0.01$, diffusion coefficients $D_1 = 2 \times 10^{-5}$ and $D_2 = 10^{-5}$, periodic boundary conditions are $x_1(0,t) = x_1(L,t) = 1$ and $x_2(0,t) = x_2(L,t) = 0$. The propagation of the maximum Lyapunov exponent of the system according to parameters a and b is shown in Fig. 1.

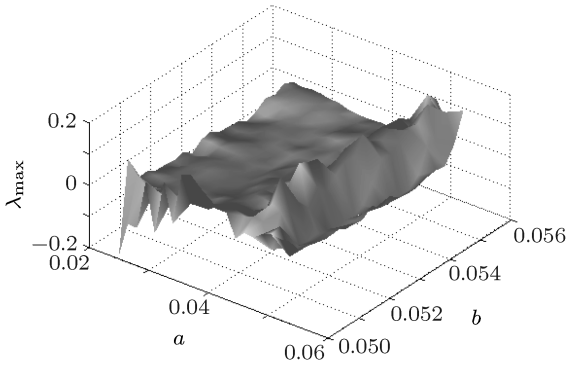


Fig. 1. Evolution of maximum Lyapunov exponent with parameters a and b.

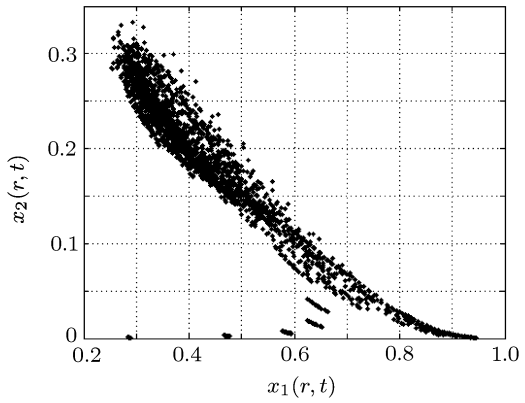


Fig. 2. Phase map of system (15).

It can be seen from Fig. 1 that the Lyapunov exponent is positive when parameters $a = 0.028$ and $b = 0.053$, meaning that the system is in chaos. The corresponding phase map of the system is given in Fig. 2, and the spatiotemporal evolutions of state variables $x_1(r,t)$ and $x_2(r,t)$ are shown in Fig. 3 and Fig. 4, respectively.

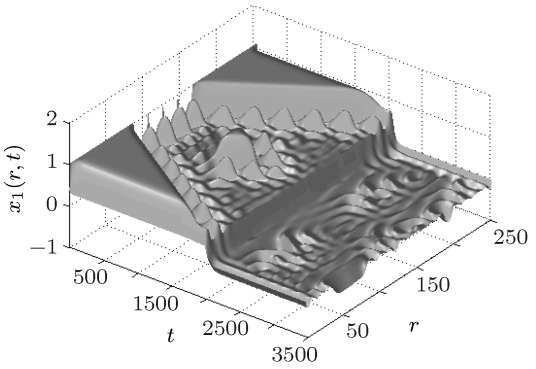


Fig. 3. Spatiotemporal evolution of variable $x_1(r,t)$.

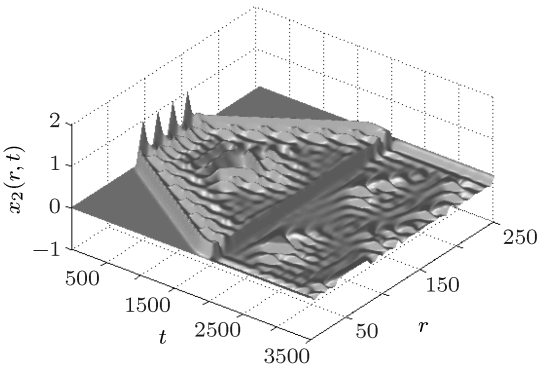


Fig. 4. Spatiotemporal evolution of variable $x_2(r,t)$.

Simulation is tested for the complex network with four nodes, i.e., $m = 4$. The state equation for each node is of Gray–Scott model described by Eq. (15), and state variables at four nodes are $(x_1(r,t), x_2(r,t))$, $(y_1(r,t), y_2(r,t))$, $(z_1(r,t), z_2(r,t))$, and $(w_1(r,t), w_2(r,t))$, respectively. The linear coefficient matrix of the node state equations is

$$\boldsymbol{A} = \begin{bmatrix} -a & 0 \\ 0 & -(a+b) \end{bmatrix}. \tag{16}$$

Coupling is made by using the other nonlinear terms in the Gray–Scott model according to Eq. (3)

$$\begin{cases} \dfrac{\partial x_1(r,t)}{\partial t} = -x_1(r,t)x_2^2(r,t) + a(1-x_1(r,t)) + D_1\nabla^2 x_1(r,t), \\ \dfrac{\partial x_2(r,t)}{\partial t} = x_1(r,t)x_2^2(r,t) - (a+b)x_2(r,t) + D_2\nabla^2 x_2(r,t), \end{cases} \tag{17}$$

$$\begin{cases} \dfrac{\partial y_1(r,t)}{\partial t} = -y_1(r,t)y_2^2(r,t) + a(1-y_1(r,t)) + D_1\nabla^2 y_1(r,t) + J_{21}, \\ \dfrac{\partial y_2(r,t)}{\partial t} = y_1(r,t)y_2^2(r,t) - (a+b)y_2(r,t) + D_2\nabla^2 y_2(r,t) + J_{22}, \end{cases} \tag{18}$$

$$\begin{cases} \dfrac{\partial z_1(r,t)}{\partial t} = -z_1(r,t)z_2^2(r,t) + a(1-z_1(r,t)) + D_1\nabla^2 z_1(r,t) + J_{31}, \\ \dfrac{\partial z_2(r,t)}{\partial t} = z_1(r,t)z_2^2(r,t) - (a+b)z_2(r,t) + D_2\nabla^2 z_2(r,t) + J_{32}, \end{cases} \tag{19}$$

$$\begin{cases} \dfrac{\partial w_1(r,t)}{\partial t} = -w_1(r,t)w_2^2(r,t) + a(1-w_1(r,t)) + D_1\nabla^2 w_1(r,t) + J_{41}, \\ \dfrac{\partial w_2(r,t)}{\partial t} = w_1(r,t)w_2^2(r,t) - (a+b)w_2(r,t) + D_2\nabla^2 w_2(r,t) + J_{42}. \end{cases} \tag{20}$$

The relative weight of the coupling strength between the nodes of the network is arbitrarily taken as $\alpha_{21} = -\frac{1}{2}$, $\alpha_{31} = -\frac{1}{4}$, and $\alpha_{41} = \frac{1}{4}$; the scale factors are $\beta_1 = 2$, $\beta_2 = -2$, and $\beta_3 = 1$ derived from Eq. (9). The network coupling is added at time series 1500, and simulation results are shown in Figs. 5–14.

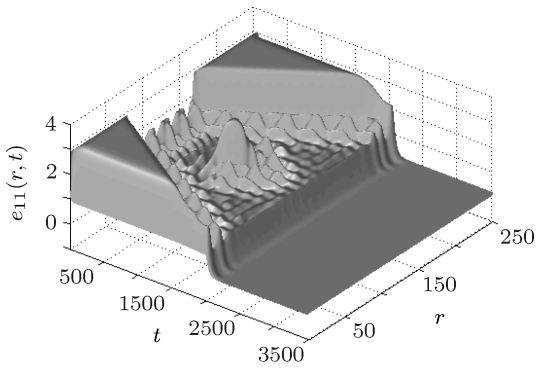


Fig. 5. Spatiotemporal evolution of error $e_{11}(r,t)$.

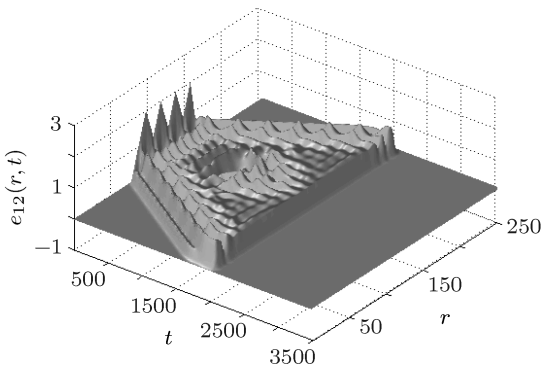


Fig. 6. Spatiotemporal evolution of error $e_{12}(r,t)$.

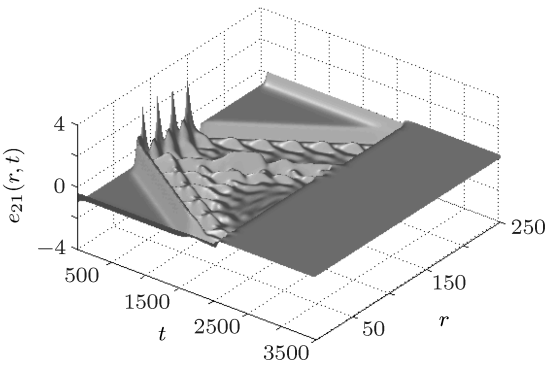


Fig. 7. Spatiotemporal evolution of error $e_{21}(r,t)$.

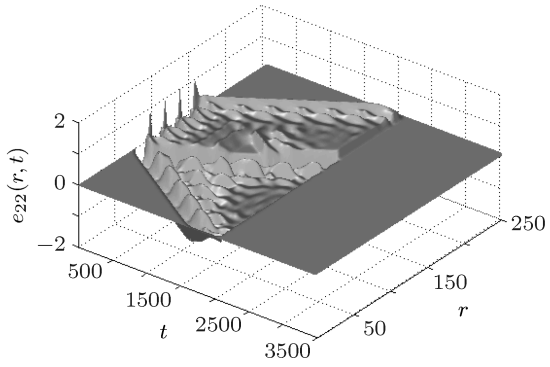


Fig. 8. Spatiotemporal evolution of error $e_{22}(r,t)$.

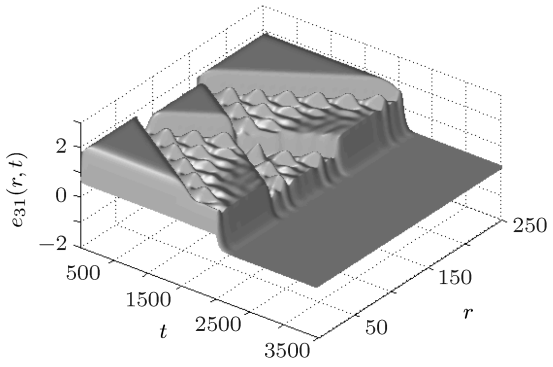


Fig. 9. Spatiotemporal evolution of error $e_{31}(r,t)$.

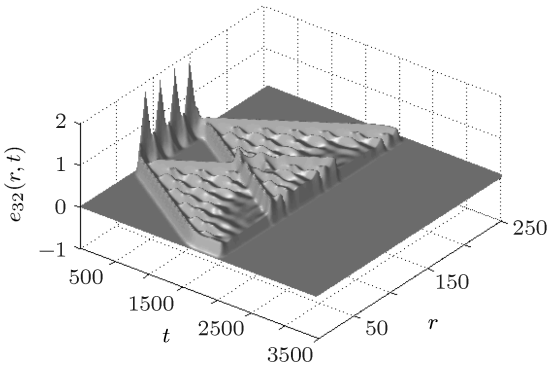


Fig. 10. Spatiotemporal evolution of error $e_{32}(r,t)$.

It can be seen from Figs. 5–10 that all the error signals $\boldsymbol{e}_1 = (e_{11}, e_{12})^{\mathrm{T}}$, $\boldsymbol{e}_2 = (e_{21}, e_{22})^{\mathrm{T}}$ and $\boldsymbol{e}_3 = (e_{31}, e_{32})^{\mathrm{T}}$ in the network approach to zero smoothly and rapidly in a short series of time once network coupling is added, which means that the projection synchronization is realized. Correspondingly, the amplitude of node 2 is half the amplitude of node 1 with an opposite sign, which is shown in Fig. 11; the amplitude of node 3 is half the amplitude of node 2 with the same sign, which exhibits in Fig. 12; the amplitude of node 4 is the same as that of node 3 with an opposite sign, which is displayed in Fig. 13; the amplitude of node 1 is four times as large as that of node 4 with the same sign, which is shown in Fig. 14.

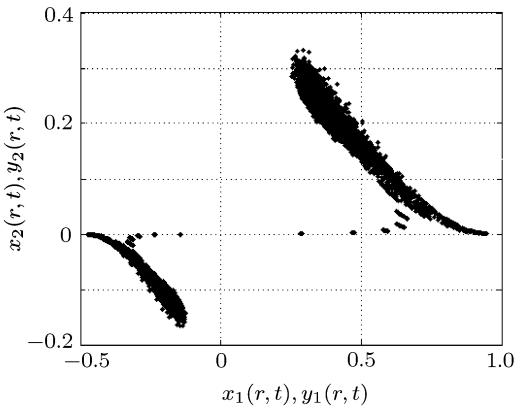


Fig. 11. Phase maps of nodes 1 and 2 when synchronization is realized.

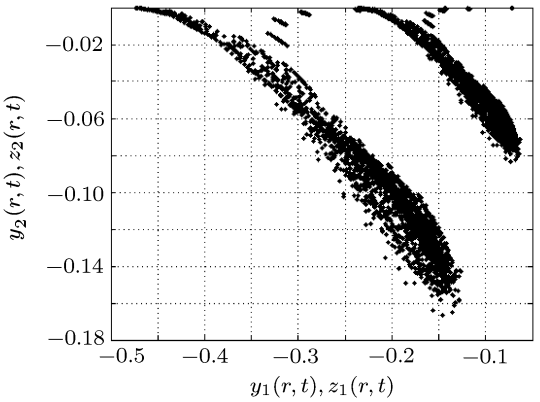


Fig. 12. Phase maps of nodes 2 and 3 when synchronization is realized.

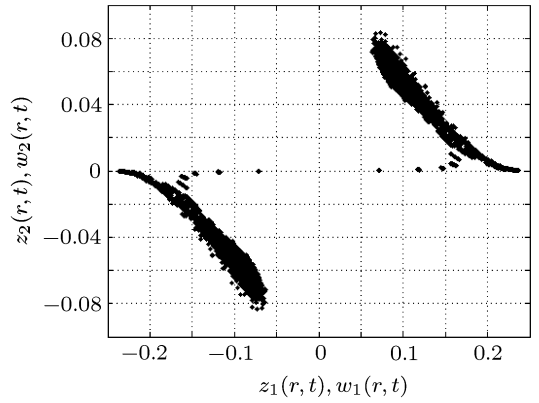


Fig. 13. Phase maps of nodes 3 and 4 when synchronization is realized.

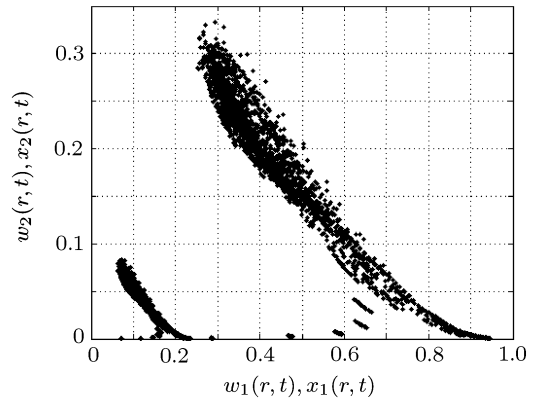


Fig. 14. Phase maps of nodes 4 and 1 when synchronization is realized.

4. Conclusion

A method of realizing projective synchronization of spatiotemporal chaos in a weighted complex net-

work is proposed in this paper. A spatiotemporal chaos system is taken as the node of the network, the nonlinear terms of the system are treated as coupling functions, and the relations between the nodes are built through a weighted connection. The range of the linear coefficient matrix $\boldsymbol{A}$ of separated configuration, when synchronization is implemented, is determined based on the Lyapunov stability theory. Furthermore, the Gray–Scott model exhibiting spatiotemporal behaviour is used as a node of the network. The simulation results show that in unidirectional star-connection, the projective synchronization of spatiotemporal chaos between Gray–Scott models at any different nodes can be realized even though the relative weight of the coupling strength between the nodes of the network is arbitrarily taken.

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