Stochastic resonance in a monostable system driven by square-wave signal and dichotomous noise*

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This paper investigates the stochastic resonance in a monostable system driven by square-wave signal, asymmetric dichotomous noise as well as by multiplicative and additive white noise. By the use of the properties of the dichotomous noise, it obtains the expressions of the signal-to-noise ratio under the adiabatic approximation condition. It finds that the signal-to-noise ratio is a non-monotonic function of the asymmetry of the dichotomous noise, and which varies non-monotonously with the intensity of the multiplicative and additive noise as well as the system parameters. Moreover, the signal-to-noise ratio depends on the correlation rate and intensity of the dichotomous noise.

Keywords: stochastic resonance, monostable system, asymmetric dichotomous noise

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1. Introduction

The phenomenon of stochastic resonance (SR) observed in systems driven by random and periodic forces is the original work done by Benzi *et al.*[1,2] in the context of modeling the switch of the earth's climate between ice ages and periods of relative warmth. The SR phenomenon has been studied in a variety of nonlinear systems with additive and multiplicative noise.[1−21] It was thought that non-linearity, periodic and random force are the essential ingredients for the occurrence of SR. The SR phenomenon is not only presented in bistable systems,[3−7] but also occurred in mono-stable systems,[8−15,21] such as chemical, electronic, physical and biological systems. Stocks[8,9] investigated the zero-dispersion stochastic resonance (ZDSR) in a mono-stable system, for which the dependence of eigenfrequency upon energy can be maximized. Dykman[10] and Evstigneev[11] investigated the SR in a monostable overdamped system on the basis of linear response theory. Grigorenko[12] calculated the SR curves for several monostable systems. They found that a physical system of a general type manifests SR at higher harmonics. Vilar and Rubi[13] studied a class of monostable systems for which the signal-to-noise ratio (SNR) always increases during increasing the noise and diverges at infinite noise level. Damina[14] studied analytically the enhancement of the SNR in a monostable non-harmonic potential. Lindner[15] presented a simple monostable system which exhibits multiple distinct SR.

Dichotomous noise (DN) is widely used in physical, chemical and biological problems.[16−19] Fulinski [16] studied the SR of a linear model driven by non-Markovian DN, and found that the relaxation becomes nonmonotomous, the process exhibits SR when an external oscillation field is applied. Gitterman[17] investigated the SR phenomenon in a linear system subjected to multiplicative and additive DN. Their results showed that the SNR becomes a nonmomotonic function of the correlation time and the asymmetry of the DN. Robert[18] studied the linear responses of a stochastic bistable system driven by DN to a weak periodic signal. They showed that both SNR and the spectral power amplification reach much greater values than those in the standard SR setup. Ginzburg and Pustovoit[19] investigated voltage-gated ion channels in biological membranes, and found SR in two-state model of membrane channel with comparable opening and closing rates.

This paper investigates a monostable system driven simultaneously by a multiplicative noise, an additive DN, as well as by a periodic square-wave signal. Based on the adiabatic approximation theory of SR, we obtained the analytical expression of the system output SNR. It is shown that the SR takes place in

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the curves of SNR as a function of the intensities of the multiplicative and additive noise. Moreover, the influence of the DN on the SNR is also studied.

2. The monostable system and its output SNR

We consider a monostable system subjected to a multiplicative noise as well as an additive noise and an external force $f(t)$, which can be described by the following stochastic differential equation:[13]

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -bx^3 + x\xi(t) + \Gamma(t) + f(t) \tag{1}$$

with

$$f(t) = q(t) + Ah(t). \tag{2}$$

Here the noise terms $\xi(t)$ and $\Gamma(t)$ are Gaussian white noises, whose mean and correlation function are given by

$$\langle \xi(t) \rangle = 0, \quad \langle \xi(t_1)\xi(t_2) \rangle = 2D\delta(t_1 - t_2), \tag{3}$$

$$\langle \Gamma(t) \rangle = 0, \quad \langle \Gamma(t_1)\Gamma(t_2) \rangle = 2P\delta(t_1 - t_2), \tag{4}$$

$$\langle \xi(t_1)\Gamma(t_2) \rangle = \langle \xi(t_2)\Gamma(t_1) \rangle = 0. \tag{5}$$

The $q(t)$ is a zero mean DN, taking values $K_1, -K_2$, $K_i > 0, i = 1, 2$. Its correlation function has the form

$$\langle q(t_1)q(t_2) \rangle = \sigma \exp\left(-\lambda \left| t_1 - t_2 \right|\right), \tag{6}$$

where σ is the intensity of the DN, $\sigma = K_1K_2$, Δ denotes the asymmetry of the DN, $\Delta = K_1 - K_2$. The asymmetric DN $q(t)$ can be decomposed into two parts: a symmetric DN $\eta(t)$ and a constant, i.e.,

$$q(t) = \eta(t) + \Delta/2. \tag{7}$$

Obviously $\langle \eta(t) \rangle = -\Delta/2$, we assume $\eta(t) = \{c, -c\}$, then

$$c = (K_1 + K_2)/2. \tag{8}$$

The $h(t)$ is a periodic square wave with periods T,

$$h(t) = \begin{cases} -1, & 0 < t \leq T/2, \\ 1, & T/2 < t \leq T. \end{cases} \tag{9}$$

From Eqs. (2) and (7), $f(t)$ can be rewritten as

$$f(t) = m + c\zeta(t) + Ah(t), \quad m = \Delta/2, \tag{10}$$

where $\zeta(t)$ is a DN with correlation rate λ, $\zeta(t) = \pm 1$. From Eqs. (3)–(5), the corresponding Fokker–Planck equation of Eq. (1) can be given by

$$\begin{aligned} \frac{\partial \rho(x,t)}{\partial t} &= -\frac{\partial}{\partial x}[F(x,t)\rho(x,t)] \\ &\quad + \frac{\partial^2}{\partial x^2}[G(x)\rho(x,t)] \end{aligned} \tag{11}$$

with

$$\begin{aligned} F(x,t) &= Dx - bx^3 + f(t), \\ G(x) &= Dx^2 + P. \end{aligned} \tag{12}$$

Assuming that the frequency Ω of the driving square-wave signal is very small, i.e., frequency smaller than the rate of the intrawell relaxation, thus there is enough time for the system to reach the equilibrium during the period of $T = 2\pi/\Omega$, i.e., we make the assumption that the system satisfies the adiabatic approximation condition.[20] The solution of the corresponding Fokker–Planck equation may then be approximated as a steady-state distribution with a slow time variation, which can be obtained from Eqs. (11) and (12),

$$\rho_{st} = \frac{M}{\sqrt{G(x)}} \exp\left[-\frac{V(x)}{D}\right], \tag{13}$$

where M is the normalization constant, $V(x)$ is the rectified potential function

$$V(x) = \int_{-\infty}^{x} \frac{D[-U'(x) + f(t)]}{G(x)} \mathrm{d}x \tag{14}$$

with

$$U'(x) = \frac{\mathrm{d}U(x)}{\mathrm{d}x} = bx^3 - Dx. \tag{15}$$

From Eqs. (14) and (15), we can see that, for the presence of multiplicative noise $\xi(t)$, i.e. $D \neq 0$, the monostable system (1) can be considered as an equivalent bistable system, with $x_0 = 0$ and $x_\pm = \pm\sqrt{D/b}$ being its unstable and stable states. Under the adiabatic limit condition, the transition rates out of $x_\pm$ can by given by

$$\begin{aligned} N_\pm[f(t)] &= \frac{\sqrt{|U''(x_0)U''(x_\pm)|}}{2\pi} \\ &\quad \times \exp\left(\frac{V(x_\pm) - V(x_0)}{D}\right) \\ &= N_0 \exp[\mp\beta f(t)] \end{aligned} \tag{16}$$

with

$$\beta = \frac{1}{\sqrt{DP}} \arctan\left(\frac{D}{\sqrt{bP}}\right), \tag{17}$$

and N_0 is the characteristic switching frequency of the equivalent bistable system driven only by the multiplicative and additive noise, which is given by

$$N_0 = \frac{D}{\sqrt{2}\pi} \exp\left[-\frac{\left(D + \frac{bP}{D}\right) \ln\left(\frac{D^2}{bP} + 1\right) - D}{2D} \right]. \tag{18}$$

By making use of the method in Ref. [19], we obtain the expression of the correlation function as (see Appendix)

$$G(t) = [B_1(m,c,A) + B_3(m,c,A)] \exp(-\lambda |t|) + B_2(m,c,A)\varphi(t) + C(m,c,A)\delta(t), \tag{19}$$

where

$$B_1(m,c,A) = \frac{1}{16}\left[w(m+c+A) - w(m-c-A) + w(m+c-A) - w(m-c+A)\right]^2, \tag{20}$$

$$B_2(m,c,A) = \frac{1}{16}\left[w(m+c+A) - w(m-c-A) - w(m+c-A) + w(m-c+A)\right]^2, \tag{21}$$

$$B_3(m,c,A) = \frac{1}{16}\left[w(m+c+A) + w(m-c-A) - w(m+c-A) - w(m-c+A)\right]^2, \tag{22}$$

$$C(m,c,A) = \frac{1}{4}\left\{C_0(m+c+A) + C_0(m-c-A) + C_0(m+c-A) + C_0(m-c+A)\right\}, \tag{23}$$

$$C_0(\mu) = \frac{8N_0^2}{[N_-(\mu) + N_+(\mu)]^3}, \quad w(\mu) = \sqrt{\frac{D}{b}} \frac{\exp(2\beta\mu) - 1}{\exp(2\beta\mu) + 1}, \tag{24}$$

$$\varphi(t) = \frac{4}{\pi^2} \sum_{j=0}^{\infty} (2j+1)^{-2} \exp[-\mathrm{i}(2j+1)\Omega t]. \tag{25}$$

Applying Fourier transform on both sides of Eq. (19), we obtain

$$S(\omega) = S(0) + B_2(m,c,A)\, \Phi(\omega), \tag{26}$$

where

$$S(0) = \frac{2}{\lambda}\left[B_1(m,c,A) + B_3(m,c,A)\right] + C(m,c,A), \tag{27}$$

$$\Phi(\omega) = \frac{8}{\pi} \sum_{j=0}^{\infty} (2j+1)^{-2} \delta[\omega - (2j+1)\Omega], \quad \Omega = \frac{2\pi}{T}. \tag{28}$$

Here $S(0)$ denotes the noise background at zero frequency in the adiabatic limit case. The $B_2(m,c,A)$ is the amplitude of the output signal. The SNR is defined as the ratio of the power of fundamental harmonics of the signal to the noise background:

$$\mathrm{SNR} = \frac{8}{\pi} \frac{B_2(m,c,A)}{C(m,c,A) + 2[B_1(m,c,A) + B_3(m,c,A)]/\lambda}. \tag{29}$$

3. Discussion and conclusions

Multiplicative fluctuations emerge naturally in a variety of systems with ensuring applications in different areas ranging from physics to biology.[22−27] In fact, in a realistic model one must always deal with various sources for fluctuations acting upon collective variables. For example, the fluctuation of values of a resistor, a capacitor or an inductor of an electric circuit can lead to multiplicative noise.[24] Multiplicative processes always have a lot of different influences from additive ones. The most probable values in a multiplicative process depend strongly on the strength of the fluctuations, while in additive process the dependence is very weak. In this paper, we introduce a multiplicative noise term in a monostable system. By the analysis of the rectified potential function of the system, we find that the monostable system can be regarded as a bistable potential. From Eq. (29), we plot the curve of the SNR versus the multiplicative noise intensity D in Figs. 1 and 2. The SR phenomenon occurs in the SNR curve as a function of the strength of the multiplicative noise D. In addition, from Fig. 2, one can see that the height of the SNR decreases with

the increase of the intensity σ of the DN, which means that the addition of DN level weakens the output signal of the monostable system.

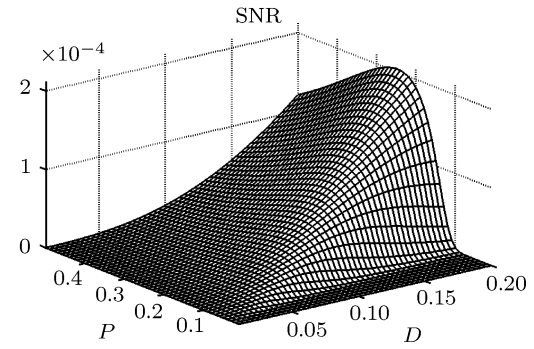


Fig. 1. The dependence of the SNR on the multiplicative noise D and additive noise P for $b = 1$, $\sigma = 1$, $\lambda = 10$, $\Delta = 0$, $A = 0.1$.

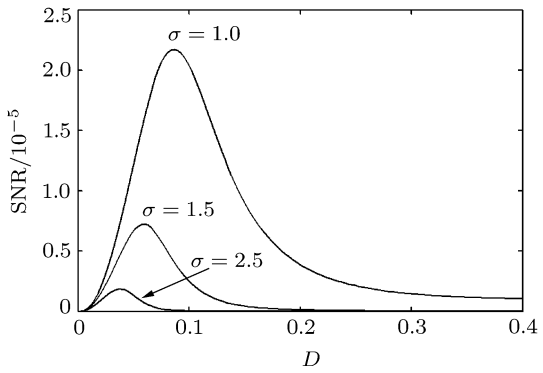


Fig. 2. The SNR versus the multiplicative noise intensity D for $b = 1$, $P = 0.1$, $A = 0.1$, $\Delta = 0.5$, $\lambda = 1$ for different values of DN strength σ.

The addition of additive noise to a bistable system can enhance the output signal of the system due to the SR effect. The result can be explained as the cooperation between the additive noise and the input signal on the system, at some appropriate level of the additive noise, the jumps of the particle moving in the bistable potential can be synchronized with the periods of the input signal, thus improves the output signal of the bistable system. Because the potential of the monostable system can be considered as a bistable potential, the influence of the additive noise of the monostable system has a similar effect on the monostable system. As shown in Fig. 3, the SNR is also a non-monotonous function of the additive noise strength P. Moreover, the SNR increases with the raise of the correlation rate λ of the DN. Therefore, the increase of λ can improve the output signal, which is an opposite effect to that done by the strength of the DN shown in Fig. 2.

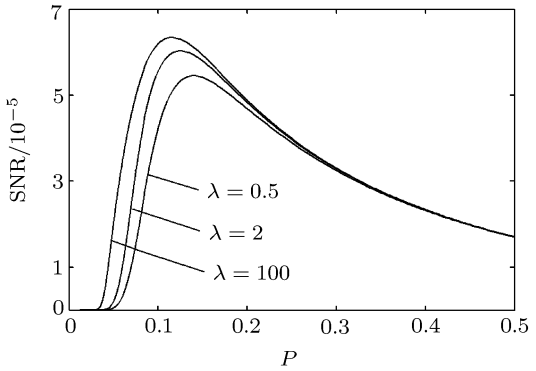


Fig. 3. The SNR versus the additive noise intensity P for $b = 1$, $D = 0.1$, $\sigma = 1$, $\Delta = 1$, $A = 0.1$ for different correlation rate λ of the dichotomous noise.

The effect of the asymmetry Δ of the DN on the SNR is analysed in Fig. 4. We see that the SR behaviour also takes place in the figures. For the given parameters, the SNR curve is almost laterally symmetric. The SNR raises with the increase of the amplitude A of the periodic square-wave signal.

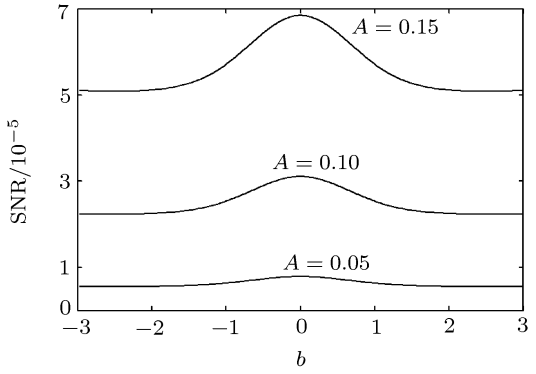


Fig. 4. The SNR versus the asymmetry of the dichotomous noise for $b = 0.5$, $D = 0.05$, $P = 0.25$, $\sigma = 0.5$, $\lambda = 10$ for different values of the amplitude A of the square-wave signal.

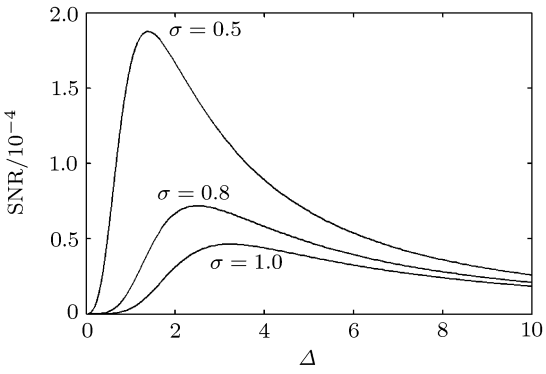


Fig. 5. The SNR versus the system parameter b of the monostable system for $D = 0.2$, $\lambda = 1$, $P = 0.1$, $\Delta = 0.5$, $A = 0.1$ for different values of the dichotomous noise intensity σ.

In Fig. 5, we give the curve of the SNR versus the system parameter b for different values of the strength σ of the DN. From Fig. 5, once more we observe the SR phenomenon. This can be explained by the analysis of the potential function $V(x)$ and the variation of its barrier height $\Delta V = V(x_0) - V(x_\pm)$ with the parameter b.[21]

To summarize, in the present paper, we investigated the SR phenomenon in a monostable system driven by a square wave periodic signal, an asymmetric DN, as well as by multiplicative and additive white noises. Based on the adiabatic approximation theory, the expression of the output SNR is obtained. We find that the SNR exhibits SR phenomenon as a function of the intensity of the multiplicative and additive noises, of the asymmetry of the DN as well as the system parameters. The strength of the DN weakens the output SNR, while the amplitude of the square-wave signal and the correlation rate of the DN improve the output signal of the system.

Appendix

The master equation of the equivalent bistable system for nonstationary probability density $P(x_\pm, t)$ is

$$\begin{aligned} \frac{\mathrm{d}P(x_+,t)}{\mathrm{d}t} &= N_-(t)P(x_-,t) - N_+(t)P(x_+,t), \\ \frac{\mathrm{d}P(x_-,t)}{\mathrm{d}t} &= N_+(t)P(x_+,t) - N_-P(x_-,t). \end{aligned} \tag{A1}$$

Since the local balance in the system under adiabatic condition is established much faster than change of the transition probabilities, the solution of Eq. (A1) is

$$P(x_+,t) = \frac{N_-(t)}{N_-(t)+N_+(t)}, \quad P(x_-,t) = \frac{N_+(t)}{N_-(t)+N_+(t)}. \tag{A2}$$

The probability of transition from state m at the moment t_2 to state n at the moment $t_1 > t_2$ can be written as

$$\begin{aligned} P(n,t_1|m,t_2) &= P(n,t_1) + \frac{\varphi_0(n,t_1)\varphi_0(m,t_1)}{P(m,t_1)} \exp[-(t_1-t_2)\lambda_0(t_1)], \\ \lambda_0(t) &= N_-(t) + N_+(t), \quad \varphi_0(x_+,t) = -\varphi_0(x_-,t) = \frac{\sqrt{N_-(t)N_+(t)}}{N_-(t)+N_+(t)}. \end{aligned} \tag{A3}$$

In Eq. (A3), $\lambda_0(t)$ and $\varphi_0(n,t)$ are the adiabatically slowly varying nonzero eigenvalue and the corresponding eigenfunction of Eq. (A1) respectively.

Taking into account Eqs. (A2) and (A3), we obtain the autocorrelation function

$$X(t_1,t_2) = \langle x(t_1)x(t_2)\rangle_x = w(t_1)w(t_2) + \frac{8N_-(t_1)N_+(t_1)}{[N_-(t_1)+N_+(t_1)]^3}\delta(t_1-t_2), \tag{A4}$$

where

$$w(t) = \langle x(t)\rangle_x = \sqrt{\frac{D}{b}}\frac{N_-(t)-N_+(t)}{N_-(t)+N_+(t)}.$$

Averaging Eq. (A4) by stochastic process $\Gamma(t)$ and by phase of periodic process $s(t)$, we can calculate the following autocorrelator:

$$G(t_1-t_2) = \langle X(t_1,t_2)\rangle_{\zeta,h} - \langle w(t_1)\rangle_{\zeta,h}\langle w(t_2)\rangle_{\zeta,h}. \tag{A5}$$

From Eqs. (A4) and (A5) we obtain

$$\begin{aligned} G(t) = G(t_1-t_2) &= \langle x(t_1)x(t_2)\rangle_{\zeta,h} - \langle x(t_1)\rangle_{x,\zeta,h}\langle x(t_2)\rangle_{x,\zeta,h} \\ &= \langle w(t_1)w(t_2)\rangle_{\zeta,h} - \langle w(t_1)\rangle_{\zeta,h}\langle w(t_2)\rangle_{\zeta,h} + 8\left\langle \frac{N_-(t_1)N_+(t_1)}{[N_-(t_1)+N_+(t_1)]^3}\right\rangle_{\zeta,h}\delta(t_1-t_2). \end{aligned} \tag{A6}$$

In the presence of both the DN and rectangular signal it is possible to obtain expressions for all quantities of our interest. We calculate the autocorrelation function $K(t)$. Since both signal and noise take only two values ± 1, taking into account Eq. (3) for arbitrary function $g[f(t)]$ we obtain

$$\begin{aligned} g[f(t)] &= \frac{1+\zeta(t)}{2}\frac{1+h(t)}{2}g(m+c+A) + \frac{1-\zeta(t)}{2}\frac{1-h(t)}{2}g(m-c-A) \\ &\quad + \frac{1+\zeta(t)}{2}\frac{1-h(t)}{2}g(m+c-A) + \frac{1-\zeta(t)}{2}\frac{1+h(t)}{2}g(m-c+A) \\ &= g_0(m,c,A) + g_1(m,c,A)\zeta(t) + g_2(m,c,A)h(t) + g_3(m,c,A)\zeta(t)h(t), \end{aligned} \tag{A7}$$

$$\begin{aligned} g_0(m,c,A) &= \frac{1}{4}\left\{g(m+c+A) + g(m-c-A) + g(m+c-A) + g(m-c+A)\right\}, \\ g_1(m,c,A) &= \frac{1}{4}\left\{g(m+c+A) - g(m-c-A) + g(m+c-A) - g(m-c+A)\right\}, \\ g_2(m,c,A) &= \frac{1}{4}\left\{g(m+c+A) - g(m-c-A) + g(m+c-A) - g(m-c+A)\right\}, \\ g_3(m,c,A) &= \frac{1}{4}\left\{g(m+c+A) - g(m-c-A) - g(m+c-A) + g(m-c+A)\right\}. \end{aligned} \tag{A8}$$

We can now easily obtain the following expression for arbitrary function $L^{(1)}$, $L^{(2)}$ of the random process $f(t)$:

$$\begin{aligned} &\left\langle L^{(1)}[f(t)]L^{(2)}[f(0)]\right\rangle_{\zeta,h} - \left\langle L^{(1)}[f(t)]\right\rangle_{\zeta,h}\left\langle L^{(2)}[x(0)]\right\rangle_{\zeta,h} \\ &= L_1^{(1)}(m,c,A)L_1^{(2)}(m,c,A)\mathrm{e}^{-\lambda|t|} + L_2^{(1)}(m,c,A)L_2^{(2)}(m,c,A)\varphi(t) \\ &\quad + L_3^{(1)}(m,c,A)L_3^{(2)}(m,c,A)\mathrm{e}^{-\lambda|t|}\varphi(t), \end{aligned} \tag{A9}$$

where

$$\varphi(t) = \langle h(t)h(0)\rangle_h = \frac{4}{\pi^2}\sum_{k=0}^{\infty}(2k+1)^{-2}\exp[-\mathrm{i}(2k+1)\Omega t], \quad \Omega = \frac{2\pi}{T}.$$

In our calculations of correlation functions we take $L^{(1)} = L^{(2)} = w(t)$ for $K(t)$. Thus we obtain,

$$\begin{aligned} G(t) &= B_1(m,c,A)\mathrm{e}^{-\lambda|t|} + B_2(m,c,A)\varphi(t) + B_3(m,c,A)\mathrm{e}^{-\lambda|t|}\varphi(t) + C(m,c,A)\delta(t), \\ B_1(m,c,A) &= \frac{1}{16}\left\{w(m+c+A) - w(m-c-A) + w(m+c-A) - w(m-c+A)\right\}^2, \\ B_2(m,c,A) &= \frac{1}{16}\left\{w(m+c+A) - w(m-c-A) - w(m+c-A) + w(m-c+A)\right\}^2, \\ B_3(m,c,A) &= \frac{1}{16}\left\{w(m+c+A) + w(m-c-A) - w(m+c-A) - w(m-c+A)\right\}^2, \\ C(m,c,A) &= \langle C_0(f)\rangle_{\zeta,h} = \frac{1}{4}\left\{C_0(m+c+A) + C_0(m-c-A) + C_0(m+c-A) + C_0(m-c+A)\right\}, \\ C_0(f) &= \frac{8N_-(f)N_+(f)}{[N_-(f)+N_+(f)]^3}. \end{aligned} \tag{A10}$$

With the adiabatic range, one assumes

$$\varphi(t)\mathrm{e}^{-\lambda|t|} \approx \varphi(0)\mathrm{e}^{-\lambda|t|} = \mathrm{e}^{-\lambda|t|}. \tag{A11}$$

Thus from Eqs. (A10) and (A11) we obtain

$$G(t) = [B_1(m,c,A) + B_3(m,c,A)]\exp(-\lambda|t|) + B_2(m,c,A)\varphi(t) + C(m,c,A)\delta(t). \tag{A12}$$

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