Simulation study of the effects of time delay on the correlation function of a bistable system with correlated noises*

Du Lu-Chun(杜鲁春)[a], Dai Zu-Cheng(戴祖诚)[b], and Mei Dong-Cheng(梅冬成)[a]†

[a] *Department of Physics, Yunnan University, Kunming* 650091, *China*
[b] *Department of Physics, Kunming Univertity, Kunming* 650031, *China*

(Received 1 February 2010; revised manuscript received 2 March 2010)

The effects of time delay on the fluctuation properties of a bistable system are investigated by simulating its normalised correlation function $C(s)$. Three cases including linear delay, cubic delay and global delay in the system are considered respectively. The simulation results indicate that the linear delay enhances the fluctuation of the system (reduces the stability of the system) while the cubic delay and global delay weaken it (enforce the stability of the system), and the effect of cubic delay is more pronounced than the linear delay and global delay.

Keywords: bistable system, time delay, correlation function

PACC: 0540, 0250

1. Introduction

For a system in a stable state, the variable is not at rest but fluctuating. The fluctuation of the state variable is described by its correlation function in two-time. The correlation function in a non-equilibrium steady state is an important quantity on characterizing system's fluctuation[1−8] and ordering.[9] The state variable correlation function in two-time is also an important intermediate quantity for further detection on the relaxation time,[8] characteristic correlation time[9] and signal-to-noise ratio.[10] As a prototype model of stochastic dynamics, bistable system's state variable correlation function $C(s)$ has been widely studied without considering time delay feedback.[1−3] The projection-operator method,[7,8] in which the memory kernel effects of the correlation function are taken into account, was employed in Refs. [2]–[5]. Explicit expression of the correlation function of a bistable system with correlated noises was first obtained in Ref. [2] by using this method. The first-order approximation of this method was proved to be an effective tool by the earlier experience with the Duffing oscillator and laser fluctuations,[7] and by the numerical simulations.[1,3,4,8]

The delay-induced dynamics of the stochastic system has received more and more attention.[10−38] The effects of time delay on the coherence resonance,[10−12] stochastic resonance,[13−21] stationary properties,[22−32] laser population dynamics[33−35] and rate process[36−38] have been studied extensively. However, how the time delay can affect the correlation function of a stochastic system has not been fully researched yet. Time delay arises in the linear term of the state variable in most of the previous works. This is mainly due to the analytical difficulty of dealing with nonlinear time delay feedback, particularly for large delay or delay arising in a nonlinear term. In Ref. [19], the cubic delay has been introduced to a bistable system and its effect on the resonance properties is studied. For a bistable system where the time delay exists in linear and cubic terms simultaneously, its stationary properties and rate process are given by a small delay expansion.[38] It is indicated that the transition rate and the mean first passage time both follow Arrhenius' law when the additive noise intensity is small, even if the delay is large. In order to fully understand the dynamical properties of the stochastic-delayed system, the correlation function of state variable should be further considered. Since there is no analytical method available, the numerical simulation should be employed. Numerical simulations have now become a powerful tool for checking the scope of validity in the theoretical work, and since the domain of

*Project supported by the National Natural Science Foundation of China (Grant No. 10865006), the Science Foundation of Yunnan University (Grant No. 2009A01z), and the Graduate Science Foundation of Yunnan University (Grant No. ynuy200926).
†Corresponding author. E-mail: meidch@ynu.edu.cn
© 2010 Chinese Physical Society and IOP Publishing Ltd

the analytical solution cannot be obtained, numerical simulations are vital for understanding the statistical properties of the stochastic systems.

In this paper, the effects of time delay on the fluctuation properties of a bistable system are presented. In Section 2, the simulation results of normalized state variable correlation function are given . In Section 3, discussion and conclusion end the paper.

2. The effects of time delay on the fluctuation properties of the bistable system subjected to cross-correlated noises

To fully investigate the effects of time delay on normalized correlation function of a bistable system, we consider that the time delay occurs in linear term, cubic term, and all terms of state variables of the bistable model.

2.1. The case for the bistable system with linear delay

Consider a bistable system with delayed linear term and subjected to correlated multiplicative and additive noises:[24]

$$\frac{\mathrm{d}x}{\mathrm{d}t} = x(t-\tau) - x(t)^3 + x(t-\tau)\xi(t) + \eta(t), \quad (1)$$

where τ is the delay time, $\xi(t)$ and $\eta(t)$ are the cross-correlated Gaussian white noises, and have the following statistical properties:

$$\langle \xi(t) \rangle = \langle \eta(t) \rangle = 0, \quad (2)$$

$$\langle \xi(t)\xi(t') \rangle = 2D\delta(t-t'), \quad (3)$$

$$\langle \eta(t)\eta(t') \rangle = 2\alpha\delta(t-t'), \quad (4)$$

and

$$\langle \xi(t)\eta(t') \rangle = \langle \eta(t)\xi(t') \rangle = 2\lambda\sqrt{D\alpha}\delta(t-t'), \quad (5)$$

where D and α denote the multiplicative and additive noise intensities, respectively. The parameter λ measures the strength of correlations between $\xi(t)$ and $\eta(t)$. The stationary normalized correlation function of the state variable x is defined by[1−9]

$$C(s) = \frac{\langle \delta x(t+s)\delta x(t) \rangle_{\mathrm{st}}}{\langle (\delta x)^2 \rangle_{\mathrm{st}}}. \quad (6)$$

It describes the fluctuation decay of a dynamics variable $\delta x = x - \langle x \rangle_{\mathrm{st}}$ in the stationary state, and s represents the decay time.

The simulation can be performed by integrating the stochastic time-delayed Langevin equation (1) with standard Euler forward procedure.[1] The Box–Mueller algorithm is used to generate Gaussian noises, and the decomposition method is employed to deal with the noises correlation.[39,40] Using the Euler forward procedure, the time-discrete numerical data are calculated with the integration step of $\Delta t = 0.001$. An ensemble of $N_0 = 2 \times 10^4$ realizations of $x(t)$ is obtained. For each realization of $x(t)$ the cycle is repeated for 1.8×10^4 times before $C(s)$ is calculated which can ensure the system to be in stationary state. The initial condition of $x(t)$ is set to $x(t=0) = 0$ and $x(t-\tau) = x(0)$ for $t < \tau$. Control runs under different initial conditions revealed no significant difference in the dynamics.

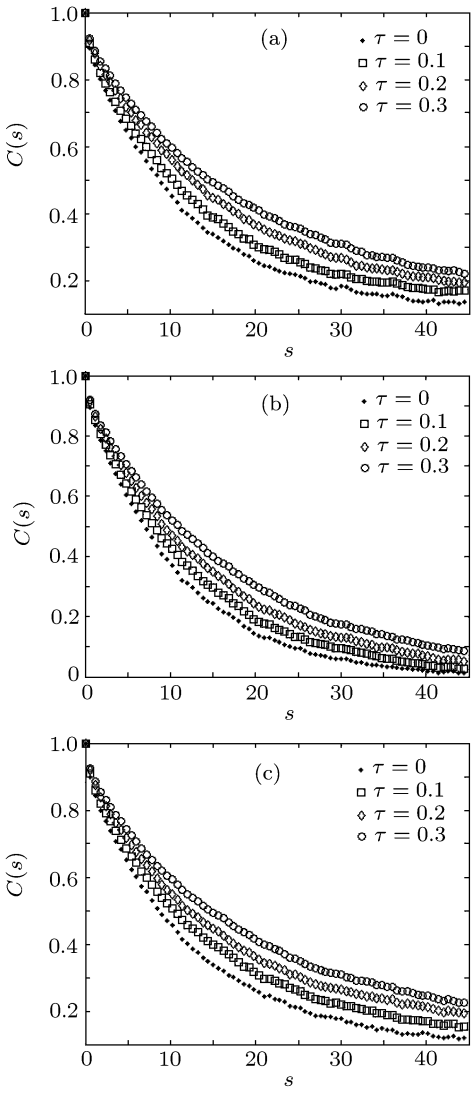


Fig. 1. $C(s)$ versus s for the case of linear delay, in which τ takes 0, 0.1, 0.2, 0.3 and $D = 0.2$, $\alpha = 0.1$. (a) $\lambda = 0.5$; (b) $\lambda = 0$; (c) $\lambda = -0.5$.

Our simulation aims at studying how the $C(s)$ depends on s and τ, and how this dependence varies with the value of λ. During simulation we fix $a = 1$, $b = 1$, the additive noise strength $\alpha = 0.1$, and multiplicative noise strength $D = 0.2$ for convenience. Some results are plotted in Figs. 1–3.

In Fig. 1, the normalized intensity correlation function $C(s)$ versus the decay time s is depicted for different values of τ and λ. $C(s)$ decays monotonically as the decay time s increases. The time for the curves' decay to zero characterizes the process when the system reaches steady state after a disturbing. The increase of τ slows down the fluctuation decay of the state variable x for three cases ($\lambda > 0$, $\lambda = 0$ and $\lambda < 0$). This means τ enhances the system fluctuation in this case. Equivalently, time delay reduces the stability of the system.

2.2. The case for the bistable system with cubic delay

Consider a bistable system with delayed cubic term when subjected to correlated multiplicative and additive noises:[19]

$$\frac{\mathrm{d}x}{\mathrm{d}t} = x(t) - x(t-\tau)^3 + x(t)\xi(t) + \eta(t), \qquad (7)$$

All the terms in Eq. (7) have the same meaning as those in Eq. (1), and the simulation technique is the same as that given in Section 1.

In Fig. 2, $C(s)$ versus s is depicted for different values of τ and λ. $C(s)$ decays also monotonically as the decay time s increases. The increase of τ speeds up the fluctuation decay of the state variable x for the three cases ($\lambda > 0$, $\lambda = 0$ and $\lambda < 0$). This means τ weakens the system fluctuation in the case of cubic delay. In this case, the system's stability is enforced due to the emergence of cubic delay. It is seen also that the cubic delay is much more pronounced than linear delay.

2.3. The case for the bistable system with global delay

Consider a bistable system with global delay and subjected to correlated multiplicative and additive noises:[38]

$$\frac{\mathrm{d}x}{\mathrm{d}t} = x(t-\tau) - x(t-\tau)^3 + x(t-\tau)\xi(t) + \eta(t). \quad (8)$$

All the terms in Eq. (8) have the same meaning as those in Eq. (1), and the simulation technique is the same as that given in Section 1.

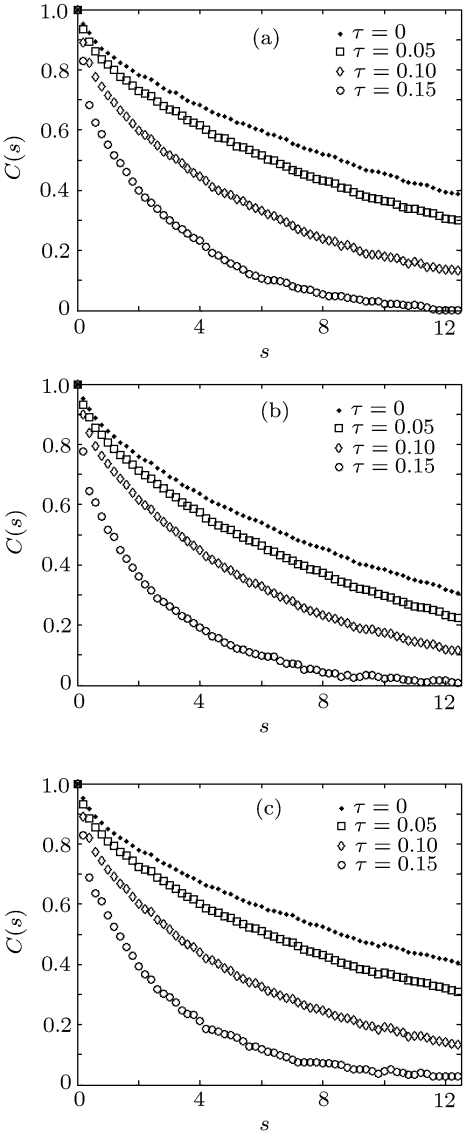


Fig. 2. $C(s)$ versus s for the case of cubic delay, in which τ takes 0, 0.05, 0.1, 0.15 and the other parameters are the same as those in Fig. 1.

In Fig. 3, $C(s)$ versus s is depicted for different values of τ and λ. $C(s)$ decays monotonically as the decay time s increases. The increase of τ speeds up the fluctuation decay of the state variable x for the three cases ($\lambda > 0$, $\lambda = 0$ and $\lambda < 0$). This means τ weakens the system fluctuation for the case of global delay, in other words, global delay enhances the stability of the system. The effect of the global delay is more pronounced than the linear delay, but less pronounced than the cubic delay.

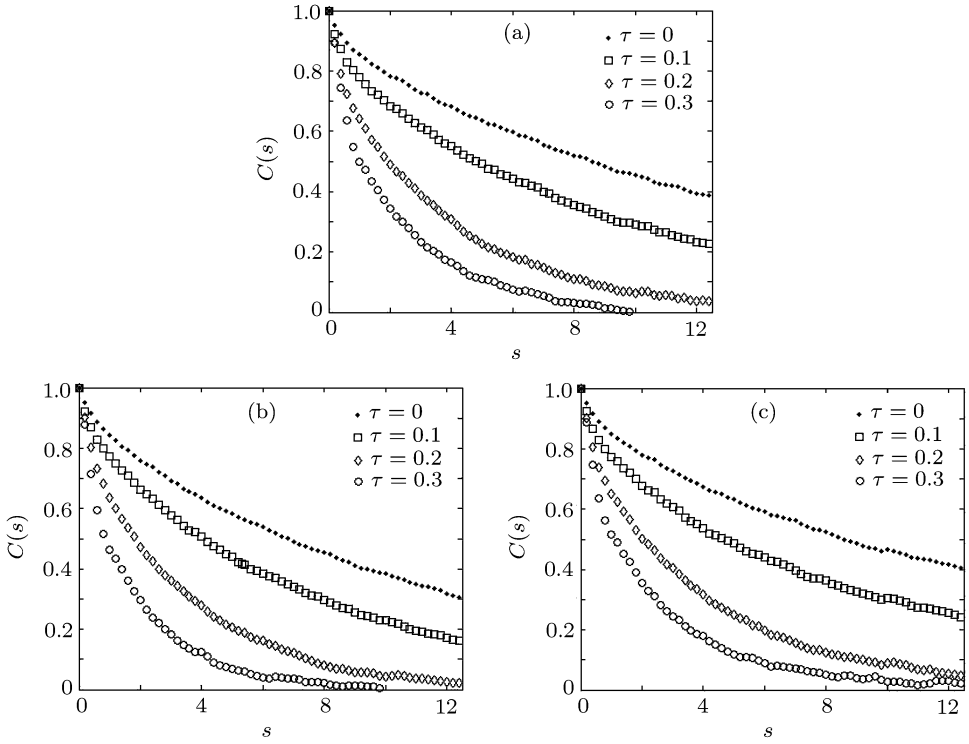


Fig. 3. $C(s)$ versus s for the case of global delay, in which τ takes 0, 0.1, 0.2, 0.3 and the other parameters are the same as those in Fig. 1.

3. Discussion and conclusion

We have studied the effects of time delay on the normalized autocorrelation function $C(s)$ of a bistable system by numerical simulation. In our results, linear delay and cubic delay have opposite effects on $C(s)$, i.e., the former enhances the fluctuation of the system while the latter weakens it. In other words, linear delay reduces while cubic delay enhances the stability of the system. There exists a competition between the two kinds of delay for the fluctuation properties of the bistable system. However the cubic delay has more pronounced effects than the linear delay. So in the case of global delay (the linear and cubic delay arise simultaneously), $C(s)$ is weakened by increasing delay time. Our results indicate that the fluctuation of the system is not only dependent on the noise intensity, but also the time delay. Whether the time delay restrains or enforces the fluctuation of the system depends on its style in the Langevin equation.

References

[1] Sancho J M, San Miguel M, Katz S L and Gunton J D 1982 *Phys. Rev.* A **26** 1589

[2] Mei D C, Xie C W and Xiang Y L 2004 *Physica* A **343** 167

[3] Mei D C, Xiang Y L and Xie C W 2006 *Phys. Scr.* **74** 123

[4] Du L C and Mei D C 2008 *Phys. Lett.* A **372** 5529

[5] Long F, Du L C and Mei D C 2009 *Phys. Scr.* **79** 045007

[6] Despósito M A and Viñales A D 2009 *Phys. Rev.* E **80** 021111

[7] Fujisaka H and Grossmann S 1981 *Z. Phys.* B: *Cond. Matter* **43** 69

[8] Hernández-Machado A, San Migeul M and Sancho J M 1984 *Phys. Rev.* A **29** 3388

[9] Pikovsky A S and Kurths J 1997 *Phys. Rev. Lett.* **78** 775

[10] Tsimring L S and Pikovsky A S 2001 *Phys. Rev. Lett.* **87** 250602

[11] Huber D and Tsimring L S 2003 *Phys. Rev. Lett.* **91** 260601

[12] Piwonski T, Houlihan J, Busch T and Huyet G 2005 *Phys. Rev. Lett.* **95** 040601

[13] Kim S, Park S H and Pyo H B 1999 *Phys. Rev. Lett.* **82** 1620

[14] Ohira T and Sato Y 1999 *Phys. Rev. Lett.* **82** 2811

[15] Du L C and Mei D C 2009 *Chin. Phys.* B **18** 946

[16] Stephen Yeung M K and Strogatz S H 1999 *Phys. Rev. Lett.* **82** 648

[17] Wu D and Zhu S Q 2007 *Phys. Lett.* A **363** 202

[18] Du L C and Mei D C 2008 *J. Stat. Mech.* **11** P11020

[19] Wu D, Zhu S Q and Luo X Q 2009 *Europhys. Lett.* **86** 50002

[20] Mei D C, Du L C and Wang C J 2009 *J. Stat. Phys.* **137** 625
[21] Jia Z L 2010 *Chin. Phys.* B **19** 020504
[22] Nie L R and Mei D C 2007 *Europhys. Lett.* **79** 20005
[23] Nie L R and Mei D C 2008 *Phys. Rev.* E **77** 031107
[24] Gu X, Zhu S Q and Wu D 2007 *Eur. Phys. J.* D **42** 461
[25] Du L C and Mei D C 2010 *Physica* A **389** 1189
[26] Han L B 2008 *Acta Phys. Sin.* **57** 2699 (in Chinese)
[27] Long F, Du L C and Mei D C 2009 *Chin. Phys.* B **18** 4738
[28] Frank T D 2002 *Phys. Rev.* E **66** 011914
[29] Hasegawa H 2004 *Phys. Rev.* E **70** 021911
[30] Frank T D 2005 *Phys. Rev.* E **71** 031106
[31] Frank T D 2005 *Phys. Rev.* E **72** 011112
[32] Pomplum J, Balanov A G and Schöll E 2007 *Phys. Rev.* E **75** 040101(R)
[33] Kim M Y, Roy R, Aron J L, Carr T W and Schwartz I B 2005 *Phys. Rev. Lett.* **94** 088101
[34] Gu X, Zhu S Q and Wu D 2007 *Commun. Theor. Phys.* **48** 1055
[35] Gu X and Zhu S Q 2010 *Eur. Phys. J.* D **56** 215
[36] Guillouzic S, LHeureux I and Longtin A 2000 *Phys. Rev.* E **61** 4906
[37] Li J C and Mei D C 2008 *Acta Phys. Sin.* **57** 6792 (in Chinese)
[38] Lin L, Yan Y and Mei D C 2010 *Acta Phys. Sin.* **59** 2240 (in Chinese)
[39] Zhu S Q 1993 *Phys. Rev.* A **47** 2405
[40] Xiang Y L and Mei D C 2010 *Chin. Phys.* B **19** 010501