Stochastic resonance in a stochastic bistable system with additive noises and square-wave signal*

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This paper considers the stochastic resonance in a stochastic bistable system driven by a periodic square-wave signal and a static force as well as by additive white noise and dichotomous noise from the viewpoint of signal-to-noise ratio. It finds that the signal-to-noise ratio appears as stochastic resonance behaviour when it is plotted as a function of the noise strength of the white noise and dichotomous noise, as a function of the system parameters, or as a function of the static force. Moreover, the influence of the strength of the stochastic potential force and the correlation rate of the dichotomous noise on the signal-to-noise ratio is investigated.

Keywords: stochastic resonance, stochastic bistable system, dichotomous noise

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1. Introduction

The stochastic resonance (SR) phenomenon is a cooperative effect between a weak signal and noise in a nonlinear system, which leads an enhanced response to the periodic force. It first appeared in 1981, which was proposed by Benzi *et al.*[1] as a plausible mechanism for almost periodic occurrences of ice ages on earth. The SR was first demonstrated in laboratory in Schmitt triggers, and was experimentally observed in 1983.[2] The SR phenomenon has been studied in a variety of nonlinear systems with additive and multiplicative noise.[1−12] It was concluded that nonlinearity, periodic and random force are the essential ingredients for the onset of SR. On the other hand, behaviour similar to SR has also been found in linear systems subjected to multiplicative noise.[13−15] It was suggested that noise multiplicativity and time correlation are the necessary conditions for the SR to occur in a linear system. Lots of work about SR has been referred to the consideration of fluctuations depending on the time in the last two decades, yet some papers have come to study the fluctuations depending on the state variable.[3−9] Dunlap and co-workers[3−5] studied the nonlinear mobility of a classical particle moving in an infinite one-dimensional space spanned by the coordinate x, and subjected to a potential $U(x)$ with period L and external electric field, where the potential is a random stationary potential and depends on the state variable. Jia *et al.*[6] investigated the effects of random potential on the transport of two systems. Their results showed that the effects of the random potential on the transport process as the amplitude of increased random potential are much more remarkable than those as the correlation length of increased random potential. Zhao *et al.*[7] investigated the motion of an overdamped Brownian particle in a periodic potential subjected to a position-dependent perturbation and a sinusoidal external force. Ai and Liu[8] investigated the stochastic resonance in a bistable system in the presence of a stochastic potential. Li[9] investigated a single protein motor system with fluctuating potential barrier and subjected to sine electric field. They calculated the signal-to-noise ratio (SNR) in the adiabatic limit and found the SR phenomenon for the motor system.

In the present paper, with the introduction of a dichotomous noise into a stochastic bistable system driven by a square-wave periodic signal and additive white noise as well as a constant, we investigate the SR of the system SNR as a function of the parameter of the noise and the system and as a function of the input signal.

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2. The stochastic bistable system and its SNR

We consider the Brown motion of a particle in a bistable potential $U(x)$ driven by an external force $f(t)$ and a noise term $\eta(t)$, described by the following Langevin equation:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{\mathrm{d}U(x)}{\mathrm{d}x} + \eta(t) + f(t), \tag{1}$$

where $\eta(t)$ is a Gaussian white noise with

$$\langle \eta(t) \rangle = 0, \quad \langle \eta(t_1)\eta(t_2) \rangle = 2D\delta(t_1 - t_2), \tag{2}$$

$$f(t) = r + \varepsilon \Gamma(t) + As(t), \tag{3}$$

and r is a constant denoting a static force, $\Gamma(t)$ is a dichotomous noise with correlation rate λ, $\Gamma(t) = \pm 1$. The $\Gamma(t)$ is uncorrelated with $\eta(t)$, $s(t)$ is a periodic square-wave signal with period T,

$$s(t) = \begin{cases} -1, & 0 < t \leq T/2, \\ 1, & T/2 < t \leq T. \end{cases} \tag{4}$$

The potential function of the system has the form

$$U(x) = U_0(x) + \xi(x), \tag{5}$$

where $U_0(x) = -(a/2)x^2 + (b/4)x^4$ is a deterministic bistable potential with stable states $x_\pm = \pm\sqrt{a/b}$ and unstable state $x_0 = 0$. The $\xi(x)$ is a single dichotomous potential

$$\xi(x) = \Delta(-1)^{n(x,0)}, \tag{6}$$

where the random function $n(x, 0)$ counts the number of jumps between $+\Delta$ and $-\Delta$ in the interval of $(0, x)$, i.e.,

$$\langle n(x_2, x_1) \rangle = \frac{|x_2 - x_1|}{l}, \tag{7}$$

where l is the correlation length denoting the mean distance between jumps. Applying the properties of the dichotomous potential $\xi(x)$, one can get the following equations:[9]

$$\langle \xi(x) \rangle = 0, \tag{8}$$

$$\langle \xi(x_1)\xi(x_2) \rangle = \Delta^2 \exp\left(-\frac{2\,|x_1 - x_2|}{l}\right), \tag{9}$$

$$[\xi(x)]^{2n} = \Delta^{2n}, \quad [\xi(x)]^{2n+1} = \Delta^{2n}\xi(x), \tag{10}$$

$$\left\langle \exp\left[\frac{\xi(x_1) - \xi(x_2)}{D}\right] \right\rangle_\xi = \exp\left\{\left(\frac{\Delta}{D}\right)^2 \left[1 - \exp\left(-\frac{2\,|x_1 - x_2|}{l}\right)\right]\right\}. \tag{11}$$

Assume that the frequency Ω of the driving square-wave signal is very small, i.e., smaller than the rate of the intrawell relaxation, thus there is enough time for the system to reach the equilibrium during the period of $T = 2\pi/\Omega$, i.e., we make the assumption that the system satisfies the adiabatic approximation condition.[16] The solution of the corresponding Fokker–Planck equation may then be approximated as a steady-state distribution with a slow time variation, which can be written as

$$\rho_{\mathrm{st}} = N \exp\left[-\frac{\Phi(x,t)}{D}\right], \tag{12}$$

where N is the normalization constant, $\Phi(x, t)$ is the rectified potential function

$$\Phi(x,t) = U_0(x) + \xi(x) - xf(t). \tag{13}$$

Under the adiabatic limit condition, the transition rates out of $x_\pm$ can be obtained by

$$\begin{aligned} & W_\pm\left(f(t)\right) \\ &= \frac{a}{\sqrt{2}\pi} \exp\left[-\frac{\Phi(x_0) - \Phi(x_\pm)}{D}\right] \\ &= \frac{a}{\sqrt{2}\pi} \exp\left\{\left(\frac{\Delta}{D}\right)^2 \left[1 - \exp\left(-\frac{2\sqrt{a}}{l\sqrt{b}}\right)\right]\right\} \\ &\quad \times \exp\left[-\frac{a^2}{4Db} \mp \sqrt{\frac{a}{b}}\frac{f(t)}{D}\right] \\ &= W_0 \exp\left[\mp\sqrt{\frac{a}{b}}\frac{f(t)}{D}\right], \end{aligned} \tag{14}$$

where W_0 denotes the characteristic switching frequency of the bistable system when it is only driven by the additive noise $\eta(t)$,

$$W_0 = \frac{a}{\sqrt{2}\pi} \exp\left\{\left(\frac{\Delta}{D}\right)^2 \left[1 - \exp\left(-\frac{2\sqrt{a}}{l\sqrt{b}}\right)\right] - \frac{a^2}{4Db}\right\}. \tag{15}$$

Applying the method in Ref. [17], we can get the expression of the correlation function (see Appendix A)

$$K(\tau) = [B_1(r, \varepsilon, A) + B_3(r, \varepsilon, A)] \exp(-\lambda\,|t|) + B_2(r, \varepsilon, A)\varphi(t) + C(r, \varepsilon, A)\delta(t), \tag{16}$$

where

$$\varphi(t) = \frac{4}{\pi^2}\sum_{j=0}^{\infty}(2j+1)^{-2}\exp[-\mathrm{i}(2j+1)\Omega t], \tag{17}$$

$$B_1(r,\varepsilon,A) = \frac{1}{16}\left[w(r+\varepsilon+A) - w(r-\varepsilon-A) + w(r+\varepsilon-A) - w(r-\varepsilon+A)\right]^2, \tag{18}$$

$$B_2(r,\varepsilon,A) = \frac{1}{16}\left[w(r+\varepsilon+A) - w(r-\varepsilon-A) - w(r+\varepsilon-A) + w(r-\varepsilon+A)\right]^2, \tag{19}$$

$$B_3(r,\varepsilon,A) = \frac{1}{16}\left[w(r+\varepsilon+A) + w(r-\varepsilon-A) - w(r+\varepsilon-A) - w(r-\varepsilon+A)\right]^2 , \tag{20}$$

$$C(r,\varepsilon,A) = \frac{1}{4}\left\{C_0(r+\varepsilon+A) + C_0(r-\varepsilon-A) + C_0(r+\varepsilon-A) + C_0(r-\varepsilon+A)\right\}, \tag{21}$$

$$w(\mu) = \sqrt{\frac{a}{b}}\frac{W_- - W_+}{W_- + W_+} = \sqrt{\frac{a}{b}}\frac{\exp[2\sqrt{a}\mu/(D\sqrt{b})] - 1}{\exp[2\sqrt{a}\mu/(D\sqrt{b})] + 1}\ , \tag{22}$$

$$C_0(\mu) = \frac{8W_0^2}{[W_-(\mu) + W_+(\mu)]^3}. \tag{23}$$

The power spectrum, defined as the Fourier transform of the correlation function, can be given as

$$S(\omega) = S_1(0) + S_2(\omega), \tag{24}$$

where

$$S_1(0) = \frac{2}{\lambda}\left[B_1(r,\varepsilon,A) + B_3(r,\varepsilon,A)\right] + C(r,\varepsilon,A), \tag{25}$$

$$S_2(\omega) = B_2(r,\varepsilon,A)\varphi(\omega), \tag{26}$$

$$\varphi(\omega) = \frac{8}{\pi}\sum_{j=0}^{\infty}(2j+1)^{-2}\delta[\omega - (2j+1)\Omega],$$

$$\Omega = \frac{2\pi}{T}. \tag{27}$$

Here $S_1(0)$ denotes the power density at the zero frequency associated with the noise background, $S_2(\omega)$ is the power spectrum of the output signal. The output SNR is the ratio between the power density of the signal and the noise at the signal frequency, which has the form

$$\mathrm{SNR} = \frac{8}{\pi}\frac{B_2(r,\varepsilon,A)}{C(r,\varepsilon,A) + 2[B_1(r,\varepsilon,A) + B_3(r,\varepsilon,A)]/\lambda}. \tag{28}$$

3. Discussion and conclusions

Dichotomous noise (DN) becomes more and more popular since its relatively simple calculation and well-defined procedures lead from DN to both Gaussian white noise and white shot noise, especially when the noise is introduced in a nonlinear equation. The DN can be generated by a two-state Poisson process and be formed by quasiparticles (defects, impurities, spins, etc.) jumping between two-level systems.[18−21] For example, a digital circuit may be perturbed by Gaussian white noise, which results from the background noise around the circuit; meanwhile, it can also be disturbed by random telephone noise (one form of DN), which is induced by other digital circuits close to it. Therefore, the study of a system with white noise and DN is of practical significance. The influence of the DN strength on the system output SNR is analysed in Fig. 1. As seen from Fig. 1, the SNR varies non-monotonically with the increase of the strength ε of the DN, for fixed other parameters, the SNR reaches its maximum at some mediate value ε, so the SR phenomenon takes place in this case. Meanwhile, the SNR increases monotonically with the variety of the increase of the intensity Δ of the dichotomous potential $\xi(x)$, which means that the dichotomous potential can improve the output SNR.

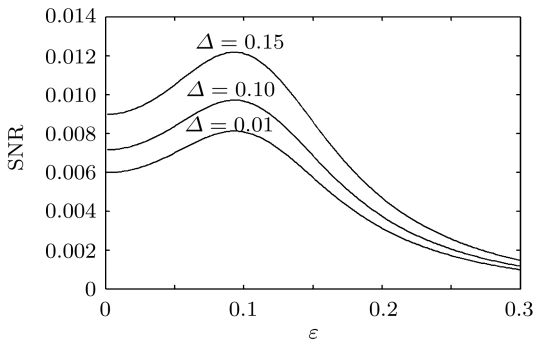


Fig. 1. The SNR versus the DN strength ε for $a = 1, b = 1, D = 0.1, l = 10, \lambda = 100, A = 0.05, r = 0.1$ for varied intensity Δ of the stochastic potential.

In practice, physical systems are usually asymmetric. Therefore, some new phenomena may oc-

cur, for example, the nonzero currents induced in stochastic ratches and the occurrence of even and odd spectral harmonics in the power spectral density of a bistable system. In this study, we consider the case of the stochastic bistable potential $U(x)$ subjected to a symmetry-breaking direct-current signal, i.e., the constant r, which can be also regarded as a static force. Since r can break the asymmetry of the system potential function, it will consequently influence the potential barrier, and then affects the system output. From Fig. 2, we see that the SNR also varies nonmonotonically with the variety of the constant r. The SNR curve is almost symmetric for positive and negative values r. When the absolute value $|r|$ increases, the SNR curve achieves a peak and then decreases for larger values $|r|$. In addition, the SNR increases with the increment of the correlation rate λ of the DN.

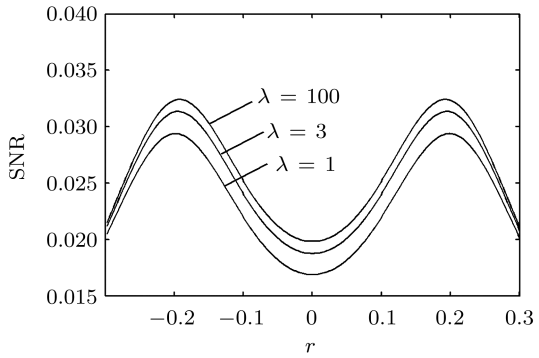


Fig. 2. The SNR versus the system bias r for $a = 1, b = 1, D = 0.18, l = 10, \Delta = 0.1, A = 0.1, \varepsilon = 0.2$ for varied correlation rate λ of the dichotomous noise.

For a bistable system subjected to additive noise, when a small periodic modulation is applied to the potential at a modulation frequency much smaller than the intrawell relaxation rate, the thermal activation rates are modulated periodically in time. At some optimal noise level, the transitions between the two wells also occur almost periodically in time, the timescale associated with the Kramers rate equals approximately half the signal period. In this case the SR takes place. In this paper, we analyse the SR phenomenon in Figs. 3–5. From these figures, one immediately concludes that the SR phenomenon occurs in the SNR curves versus the additive noise intensity D. From Figs. 3 and 4, one can see two peaks on the SNR curve. With the increase of the correlation rate λ of the DN and the intensity Δ of the dichotomous potential $\xi(x)$, the height of first peak increases.

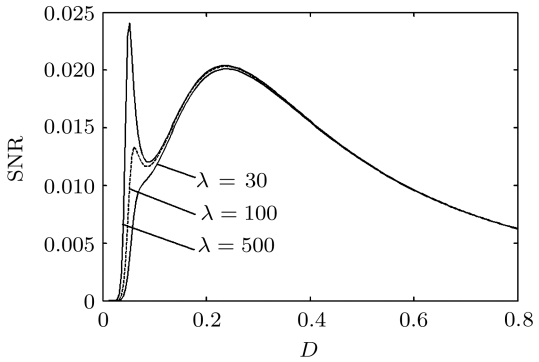


Fig. 3. The SNR versus the additive noise strength D for $a = 1, b = 1, \varepsilon = 0.3, l = 10, \Delta = 0.3, r = 0.1$ for varied correlation rate λ of the DN.

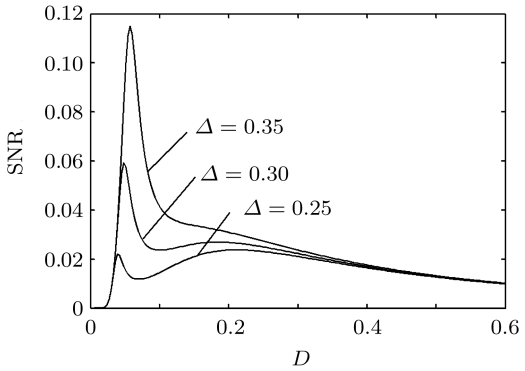


Fig. 4. The SNR versus the additive noise strength D for $a = 1, b = 1, \varepsilon = 0.25, l = 10, \lambda = 20, r = 0.1, A = 0.1$ for varied intensity Δ of the stochastic potential.

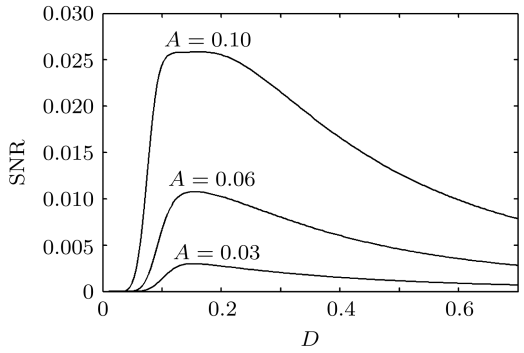


Fig. 5. The SNR versus the additive noise strength D for $a = 1, b = 1, \varepsilon = 0.3, l = 5, \lambda = 10, \Delta = 0.3, r = 0.1$ for varied amplitude of the square-wave signal.

When we plot the SNR as a function of the system parameters a and b in Figs. 6 and 7, we find that the SNR exhibits SR phenomenon both as a function of the system parameters, i.e., the SR in a broad sense occurs. This can be explained by using the potential function $\Phi(x,t)$.[22] From Eq. (13), one can see that the potential function $\Phi(x,t)$ is a function of the system parameters a and b, so the height $\Delta\Phi = \Phi(x_0,t) - \Phi(x_\pm,t)$ of the potential barrier and the distance Δx between the two minima of the potential function are also functions of the parameters.

Therefore, there may be some mediate values of the parameters a and b, at which the system SNR can be maximized.

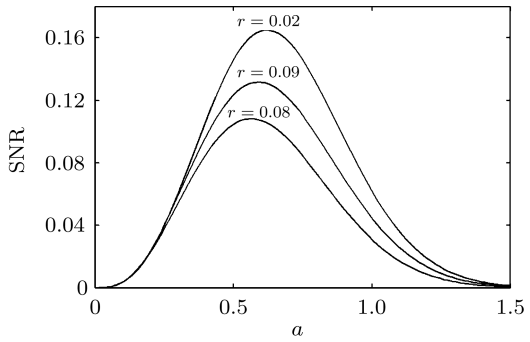


Fig. 6. The SNR versus the system parameter a for $A = 0.15, b = 2, \Delta = 0.17, \lambda = 10, D = 0.03, l = 50, \varepsilon = 0.1$ for varied system bias r.

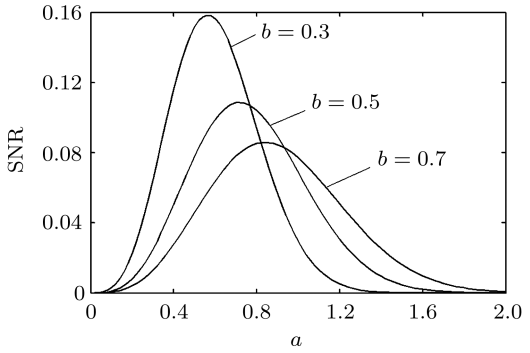


Fig. 7. The SNR versus the system parameter a for $A = 0.1, \Delta = 0.01, \lambda = 10, D = 0.15, l = 1, r = 0.01, \varepsilon = 0.01$ for varied system parameter b.

In summary we have studied the SR in a stochastic bistable system driven simultaneously by a static force, an additive white noise, a square-wave signal and by a DN. Under the adiabatic approximation condition, the analytical expression of the SNR is obtained. Results show that the SNR presents SR behaviour when it is plotted versus the intensity of the additive noise and DN as well as versus the values of the static force. In addition, the intensity of the dichotomous potential and the correlation rate of the DN can improve the output SNR.

Appendix

The master equation for nonstationary probability density $P(x_\pm, t)$ is

$$\begin{aligned}\frac{\mathrm{d}P(x_+,t)}{\mathrm{d}t} &= W_-(t)P(x_-,t) - W_+(t)P(x_+,t),\\ \frac{\mathrm{d}P(x_-,t)}{\mathrm{d}t} &= W_+(t)P(x_+,t) - W_-(t)P(x_-,t). \end{aligned} \quad \text{(A1)}$$

Since the local balance in the system in adiabatic condition is established much faster than the transition probabilities change, the solution of Eq. (A1) reads as

$$\begin{aligned}P(x_+,t) &= \frac{W_-(t)}{W_-(t)+W_+(t)},\\ P(x_-,t) &= \frac{W_+(t)}{W_-(t)+W_+(t)}. \end{aligned} \quad \text{(A2)}$$

The probability of transition from state m at the moment t_2 to state n at the moment $t_1 > t_2$ can be written as:

$$\begin{aligned}&P(n,t_1|m,t_2)\\ &= P(n,t_1) + \frac{\varphi_0(n,t_1)\varphi_0(m,t_1)}{P(m,t_1)}\\ &\quad\times \exp[-(t_1-t_2)\lambda_0(t_1)],\\ &\lambda_0(t) = W_-(t) + W_+(t),\\ &\varphi_0(x_+,t) = -\varphi_0(x_-,t) = \frac{\sqrt{W_-(t)W_+(t)}}{W_-(t)+W_+(t)}. \end{aligned} \quad \text{(A3)}$$

In Eq. (A3), $\lambda_0(t)$ and $\varphi_0(n,t)$ are the adiabatically slowly varying nonzero eigenvalue and the corresponding eigenfunction of Eq. (A1).

Taking into account Eqs. (A2) and (A3), we obtain the autocorrelation function

$$\begin{aligned}&X(t_1,t_2)\\ &= \langle x(t_1)x(t_2)\rangle_x\\ &= w(t_1)w(t_2) + \frac{8W_-(t_1)W_+(t_1)}{[W_-(t_1)+W_+(t_1)]^3}\delta(t_1-t_2),\\ &w(t) = \langle x(t)\rangle_x = \sqrt{\frac{a}{b}}\frac{W_-(t)-W_+(t)}{W_-(t)+W_+(t)}. \end{aligned} \quad \text{(A4)}$$

Averaging Eq. (A4) by stochastic process $\Gamma(t)$ and by phase of periodic process $s(t)$, once can calculate the following autocorrelator:

$$\begin{aligned}&K(t_1-t_2)\\ &= \langle X(t_1,t_2)\rangle_{\Gamma,s} - \langle w(t_1)\rangle_{\Gamma,s}\langle w(t_2)\rangle_{\Gamma,s}. \end{aligned} \quad \text{(A5)}$$

From Eqs. (A4) and (A5) we obtain

$$\begin{aligned}&K(t)\\ &= K(t_1-t_2)\\ &= \langle x(t_1)x(t_2)\rangle_{\Gamma,s} - \langle x(t_1)\rangle_{x,\Gamma,s}\langle x(t_2)\rangle_{x,\Gamma,s}\\ &= \langle w(t_1)w(t_2)\rangle_{\Gamma,s} - \langle w(t_1)\rangle_{\Gamma,s}\langle w(t_2)\rangle_{\Gamma,s}\\ &\quad + 8\left\langle \frac{W_-(t_1)W_+(t_1)}{[W_-(t_1)+W_+(t_1)]^3}\right\rangle_{\Gamma,s}\delta(t_1-t_2). \end{aligned} \quad \text{(A6)}$$

In the presence of both the dichotomous noise and rectangular signal it is possible to obtain expressions for all quantities of our interest. We now calculate the autocorrelation function $K(t)$. Since both signal and noise take only two values ± 1, we get, taking into account Eq. (3), for arbitrary function $g[f(t)]$:

$$\begin{aligned} g[f(t)] &= \frac{1+\Gamma(t)}{2}\frac{1+s(t)}{2}g(r+\varepsilon+A) + \frac{1-\Gamma(t)}{2}\frac{1-s(t)}{2}g(r-\varepsilon-A) \\ &\quad + \frac{1+\Gamma(t)}{2}\frac{1-s(t)}{2}g(r+\varepsilon-A) + \frac{1-\Gamma(t)}{2}\frac{1+s(t)}{2}g(r-\varepsilon+A) \\ &= g_0(r,\varepsilon,A) + g_1(r,\varepsilon,A)\Gamma(t) + g_2(r,\varepsilon,A)s(t) + g_3(r,\varepsilon,A)\Gamma(t)s(t), \end{aligned} \tag{A7}$$

$$\begin{aligned} g_0(r,\varepsilon,A) &= \frac{1}{4}\left\{g(r+\varepsilon+A) + g(r-\varepsilon-A) + g(r+\varepsilon-A) + g(r-\varepsilon+A)\right\}, \\ g_1(r,\varepsilon,A) &= \frac{1}{4}\left\{g(r+\varepsilon+A) - g(r-\varepsilon-A) + g(r+\varepsilon-A) - g(r-\varepsilon+A)\right\}, \\ g_2(r,\varepsilon,A) &= \frac{1}{4}\left\{g(r+\varepsilon+A) - g(r-\varepsilon-A) + g(r+\varepsilon-A) - g(r-\varepsilon+A)\right\}, \\ g_3(r,\varepsilon,A) &= \frac{1}{4}\left\{g(r+\varepsilon+A) - g(r-\varepsilon-A) - g(r+\varepsilon-A) + g(r-\varepsilon+A)\right\}. \end{aligned} \tag{A8}$$

We can now easily get the following expression for arbitrary function $L^{(1)}$, $L^{(2)}$ of the random process $f(t)$:

$$\begin{aligned} &\left\langle L^{(1)}[f(t)]L^{(2)}[f(0)]\right\rangle_{\Gamma,s} - \left\langle L^{(1)}[f(t)]\right\rangle_{\Gamma,s}\left\langle L^{(2)}[x(0)]\right\rangle_{\Gamma,s} \\ &= L_1^{(1)}(r,\varepsilon,A)L_1^{(2)}(r,\varepsilon,A)\,\mathrm{e}^{-\lambda|t|} + L_2^{(1)}(r,\varepsilon,A)L_2^{(2)}(r,\varepsilon,A)\varphi(t) + L_3^{(1)}(r,\varepsilon,A)L_3^{(2)}(r,\varepsilon,A)\,\mathrm{e}^{-\lambda|t|}\varphi(t), \\ &\varphi(t) = \langle s(t)s(0)\rangle_s = \frac{4}{\pi^2}\sum_{k=0}^{\infty}(2k+1)^{-2}\exp[-\mathrm{i}(2k+1)\Omega t], \quad \Omega = \frac{2\pi}{T}. \end{aligned} \tag{A9}$$

In our calculations of correlation functions we take $L^{(1)} = L^{(2)} = w(t)$ for $K(t)$. Thus we obtain,

$$\begin{aligned} K(t) &= B_1(r,\varepsilon,A)\,\mathrm{e}^{-\lambda|t|} + B_2(r,\varepsilon,A)\varphi(t) + B_3(r,\varepsilon,A)\,\mathrm{e}^{-\lambda|t|}\varphi(t) + C(r,\varepsilon,A)\delta(t), \\ B_1(r,\varepsilon,A) &= \frac{1}{16}\left\{w(r+\varepsilon+A) - w(r-\varepsilon-A) + w(r+\varepsilon-A) - w(r-\varepsilon+A)\right\}^2, \\ B_2(r,\varepsilon,A) &= \frac{1}{16}\left\{w(r+\varepsilon+A) - w\,(r-\varepsilon-A) - w(r+\varepsilon-A) + w(r-\varepsilon+A)\right\}^2, \\ B_3(r,\varepsilon,A) &= \frac{1}{16}\left\{w(r+\varepsilon+A) + w(r-\varepsilon-A) - w(r+\varepsilon-A) - w(r-\varepsilon+A)\right\}^2, \\ C(r,\varepsilon,A) &= \langle C_0(f)\rangle_{s,R} \\ &= \frac{1}{4}\left\{C_0(r+\varepsilon+A) + C_0(r-\varepsilon-A) + C_0(r+\varepsilon-A) + C_0(r-\varepsilon+A)\right\}, \\ C_0(f) &= \frac{8W_-(f)W_+(f)}{[W_-(f)+W_+(f)]^3}. \end{aligned} \tag{A10}$$

With the adiabatic range, one assumes

$$\varphi(t)\,\mathrm{e}^{-\lambda|t|} \approx \varphi(0)\,\mathrm{e}^{-\lambda|t|} = \mathrm{e}^{-\lambda|t|}. \tag{A11}$$

Thus we obtain from Eqs. (A10) and (A11)

$$K(t) = [B_1(r,\varepsilon,A) + B_3(r,\varepsilon,A)]\exp(-\lambda|t|) + B_2(r,\varepsilon,A)\varphi(t) + C(r,\varepsilon,A)\delta(t). \tag{A12}$$

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