Formation of combined solitons in two-component Bose–Einstein condensates*

Li Qiu-Yan(李秋艳)[a)b)], Li Zai-Dong(李再东)[a)b)†], Yao Shu-Fang(姚淑芳)[a)], Li Lu(李 禄)[c)], and Fu Guang-Sheng(傅广生)[b)‡]

[a)] *Department of Applied Physics, Hebei University of Technology, Tianjin* 300401, *China*
[b)] *School of Information Engineering, Hebei University of Technology, Tianjin* 300401, *China*
[c)] *College of Physics and Electronics Engineering, Shanxi University, Taiyuan* 030006, *China*

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We investigate the combined soliton solutions of two-component Bose–Einstein condensates with external potential. The "phase diagram" is obtained for the formation regions of different combined solitons. Our results show that the intraspecies (interspecies) interaction strengths and the external trapped potential clearly affect the formation of dark–dark, bright–bright, and dark–bright soliton solutions in different regions. Especially, we find that the bright–bright (dark–dark) soliton can exist in the case of both repulsive (attractive) intraspecies interaction strengths in the presence of external potential. This novel phenomenon is completely different from the formation of soliton solution of one-component Bose–Einstein condensates without external potential, and it will be useful for the study of two-component Bose–Einstein condensates.

Keywords: Bose–Einstein condensates, combined solitons

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The realization of Bose–Einstein condensations (BEC) of weakly interacting atomic gases[1−5] strongly stimulates a tremendous number of explorations for the nonlinear properties of matter waves. One specific topic of wide interest is the various types of excitations of multi-component BEC which possess complicated properties not seen in individual components.

For an individual atomic BEC the macroscopic wave function obeys the so-called Gross–Pitaevskii equation,[6−8] whose nonlinearity is a result of the interatomic interactions. Depending on the repulsive or attractive nature of the interatomic interactions, the Gross–Pitaevskii equation gives either dark or bright solitons. A dark soliton[9,10] in BECs is a macroscopic excitation characterized by a local density minimum.[11] Several theoretical studies were made for the dynamics and the stability of dark solitons.[12−24] A bright soliton was created in experiments with BECs of ^{7}Li atoms[25−27] with an attractive interparticle interaction. Recently, the controllable soliton solutions,[28−34] i.e. the term of nonautonomous solitons introduced first, are of interest in individual atomic BECs and the dynamics of nonautonomous soliton[35,36] can be affected by adjusting the external linear time-dependent potential.

The development of trapping techniques allowed the creation of multi-component condensates trapped in different internal (electronic) states.[37,38] They presented the novel and fundamentally different scenarios for its ground state[39−41] and excitations.[42−44] In particular, the separation of the species in different domains[38] was observed. Recently, two-species BECs with tunable interactions[45−47] were successfully produced. This important progress motivates further explorations of two-species BECs.[48−55]

In this paper, we study the family formation of combined soliton solutions of two-component BECs with an external potential in a closed form. As we shall set forth later, by choosing the sign (positive or negative) of A_j two component BECs allow for the existence of dark–dark, dark–bright and bright–bright soliton solutions.

As a theoretical model, we consider a trapped

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†Corresponding author. E-mail: zdli2003@yahoo.com
‡E-mail: fugs@hebut.edu.cn

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two-component BEC in which the dynamic process takes place only in one dimension due to the strong trap confinement in the transverse direction. This approximation is valid if the mean-field interaction is smaller than the typical energy separation in the other directions, and has been successfully employed in the analysis of dark solitons in single-component condensates.[56] For sufficiently low temperatures the dynamics is well described by[57−59]

$$\begin{aligned} \mathrm{i}\hbar\frac{\partial\Psi_1}{\partial t} &= \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2}+V+\lambda_1|\Psi_1|^2 \right. \\ &\quad \left. +\lambda_0|\Psi_2|^2\right)\Psi_1, \\ \mathrm{i}\hbar\frac{\partial\Psi_2}{\partial t} &= \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2}+V+\lambda_0|\Psi_1|^2 \right. \\ &\quad \left. +\lambda_2|\Psi_2|^2\right)\Psi_2, \end{aligned} \tag{1}$$

where $V(z)$ denotes the external confining potential, and the coupling constant, $\lambda_j = 4\pi\hbar^2 a_{jj}/m$, $j = 1, 2$, and $\lambda_0 = 4\pi\hbar^2 a_{12}/m = 4\pi\hbar^2 a_{21}/m$, represent intraspecies and interspecies interaction strengths, respectively, with a_{ij}, $i, j = 1, 2$, being the corresponding scattering lengths and m the reduced mass. In the experiment[60,61] of a very long cigar-shaped BEC, the validity of Eq. (1) requires the correlation length to be much larger than the mean interparticle separation along the axial direction.[62] Along the symmetric axis (z-axis), the confining potential is usually written as $V(z) = m\omega^2 z^2/2$ which can extend to infinity in principle. However, in a realistic experimental setup, the dimension of the trap is finite and the realistic trapping potential is truncated within the extension of the trap. That is to say, the realistic confining potential becomes flat near the edges of the trap and can be reasonably assumed to be the tanh-shaped potential $V(z) = V_0 \tanh^2(\beta z)$.

Generally, the stationary solution of Eq. (1) can be assumed to be $\Psi_j = R_j(z)\,\mathrm{e}^{\mathrm{i}(\theta_j(z)-\mu_j t/\hbar)}$ with $j = 1, 2$, where $R_j(z)$ and $u_j(z)$ are both real parameters. Substituting this expression into Eq. (1) and separating the imaginary part from the real part, we have

$$0 = 2\frac{\partial R_j}{\partial z}\frac{\partial\theta_j}{\partial z}+R_j\frac{\partial^2\theta_j}{\partial z^2}, \quad j=1,2, \tag{2}$$

$$\begin{aligned} \mu_1 &= \frac{\hbar^2}{2m}\left[\left(\frac{\partial\theta_1}{\partial z}\right)^2-\frac{1}{R_1}\frac{\partial^2 R_1}{\partial z^2}\right] \\ &\quad +V_1+\lambda_1 R_1^2+\lambda_0 R_2^2, \end{aligned} \tag{3}$$

$$\begin{aligned} \mu_2 &= \frac{\hbar^2}{2m}\left[\left(\frac{\partial\theta_2}{\partial z}\right)^2-\frac{1}{R_2}\frac{\partial^2 R_2}{\partial z^2}\right] \\ &\quad +V_2+\lambda_0 R_1^2+\lambda_2 R_2^2. \end{aligned} \tag{4}$$

Integrating Eq. (2), we can easily obtain

$$\theta_j(z) = \int_0^z \frac{C_j}{R_j^2(z')}\mathrm{d}z', \quad j=1,2. \tag{5}$$

The amplitude function can be assumed to be in the form, $R_j = (A_j \tanh^2(\beta z) + B_j)^{1/2}$, $j = 1, 2$. Substituting it into Eqs. (3) and (4) we obtain the parameters A_j, u_j, and C_j, separately, which reads

$$A_1 = \frac{\lambda_0-\lambda_2}{\lambda_0^2-\lambda_1\lambda_2}\left(\frac{\hbar^2\beta^2}{m}-V_0\right), \tag{6}$$

$$A_2 = \frac{\lambda_0-\lambda_1}{\lambda_0^2-\lambda_1\lambda_2}\left(\frac{\hbar^2\beta^2}{m}-V_0\right), \tag{7}$$

$$\mu_1 = \frac{\hbar^2\beta^2}{m}\left(1+\frac{B_1}{2A_1}\right)+\lambda_0 B_2+\lambda_1 B_1, \tag{8}$$

$$\mu_2 = \frac{\hbar^2\beta^2}{m}\left(1+\frac{B_2}{2A_2}\right)+\lambda_0 B_1+\lambda_2 B_2, \tag{9}$$

$$C_j = \beta\sqrt{\frac{B_j}{A_j}}\,(A_j+B_j), \tag{10}$$

where B_j, $j = 1, 2$, is the free parameter determined by the density of BEC.

With all the parameters obtained before we can obtain the exact soliton solutions of two-component BEC with an effective trapped potential in a closed form. From the expression of R_j it follows that the dark soliton exists under the condition of $A_j > 0$, while the bright soliton exists under the condition of $A_j < 0$. It implies that the combined soliton solutions, such as dark–dark, dark–bright, and bright–bright, can be obtained by adjusting the sign (positive or negative) of A_j determined by the combined effect of intraspecies, interspecies interaction strengths, and the terms of external trapped potential.

(i) **Dark–dark soliton solutions**

With the help of the expressions of R_j ($j = 1, 2$) we see that the dark–dark soliton solutions can exist in the case $A_j > 0$, $j = 1, 2$. And then the restriction of Eq. (10) demands the parameter $B_j \geq 0$, $j = 1, 2$. From Eqs. (6)–(10) we find that these dark–dark soliton solutions can be realized by adjusting the relation among scattering length (λ_0, λ_1, λ_2), width β and height V_0 of external trapped potential. For example, one can choose that $\hbar^2\beta^2/m > V_0$, $0 < \lambda_1 < \lambda_2 < \lambda_0$ (or $\lambda_1 < 0 < \lambda_2 < \lambda_0$), see Fig. 1 below. Each dark soliton has the form

$$\Psi_j = \sqrt{A_j\tanh^2(\beta z)+B_j}\;\mathrm{e}^{\mathrm{i}(\theta_j(z)-\mu_j t/\hbar)}, \tag{11}$$

where parameters A_j and μ_j have been given in Eqs. (6)–(9), $\theta_j(z)$ is obtained from Eqs. (5) and (10), $j = 1, 2$, and in the form

$$\theta_j(z) = \arctan\sqrt{\frac{A_j}{B_j}}\tanh(\beta z) + \beta\sqrt{\frac{B_j}{A_j}}z. \quad (12)$$

It is interesting to point out that the nonlinear term $\arctan\sqrt{A_j/B_j}\tanh(\beta z)$ enters into the phase.

(ii) **Bright–bright soliton solutions**

Similarly, the existence of bright–bright soliton solutions demands the conditions $A_j < 0$ and $B_j = -A_j$. It is also realized by adjusting the relation between scattering lengths λ_0, λ_1, λ_2 and the sign of the parameter $\left(\hbar^2\beta^2/m - V_0\right)$, such as $\hbar^2\beta^2/m > V_0$, $\lambda_0 < \lambda_1 < \lambda_2 < 0$ (or $\lambda_0 < -\sqrt{\lambda_1\lambda_2} < 0 < \lambda_1 < \lambda_2$), see Fig. 1 below. With the condition $B_j = -A_j$ and Eqs. (5) and (10) we have spacial phase $\theta_j(z) = 0$ and chemical potential $\mu_j = V_0 - \hbar^2\beta^2/2m$, $j = 1, 2$. With these consideration each bright soliton solution reduces into the form

$$\Psi_j = \sqrt{-A_j}\frac{1}{\cosh(\beta z)}\mathrm{e}^{\mathrm{i}\left(\hbar\beta^2/2m - V_0/\hbar\right)t},\ j = 1, 2. \quad (13)$$

(iii) **Dark–bright (bright–dark) soliton solutions**

In order to obtain the combined dark–bright soliton solutions we must demand that $A_1 > 0$, $A_2 < 0$ and $B_1 \geq 0, B_2 = -A_2$. This condition can be achieved by choosing an appropriate relation among scattering lengths λ_0, λ_1, λ_2, width β and height V_0 of potential, such as the scattering length $\hbar^2\beta^2/m > V_0$ and $\lambda_0^2 > \lambda_1\lambda_2, \lambda_1 > \lambda_0 > \lambda_2$, (or $\lambda_0^2 < \lambda_1\lambda_2$, $\lambda_1 < \lambda_0 < \lambda_2$), see Fig. 1 below. With these conditions and Eqs. (5) and (10) we have spacial phase $\theta_2(z) = 0$ and chemical potential $\mu_2 = V_0 - \hbar^2\beta^2/(2m)$. Then the combined dark–bright soliton solutions take the form

$$\Psi_1 = \sqrt{A_1\tanh^2(\beta z) + B_1}\ \mathrm{e}^{\mathrm{i}(\theta_1(z) - \mu_1 t/\hbar)},$$
$$\Psi_2 = \frac{\sqrt{-A_2}}{\cosh(\beta z)}\mathrm{e}^{\mathrm{i}\left(\hbar\beta^2/(2m) - V_0/\hbar\right)t}, \quad (14)$$

where all the parameters have been given in Eqs. (6)–(10) and Eq. (12). Repeating a similar procedure the combined bright–dark soliton solutions can also be constructed analytically.

The above discussion shows that the intraspecies and interspecies interaction strengths, width β and height V_0 of external trapped potential play the important roles in forming combined soliton solutions of two-component BEC. Next we will discuss this in detail. For the tightly trapped potential ($\hbar^2\beta^2/m > V_0 > 0$) or the expulsive potential ($V_0 < 0$), the family formation of combined soliton solutions is shown in Figs. 1(a)–1(c), in which we have assumed $\lambda_2 > \lambda_1$ without loss of generality. Without external potential, the dynamics of single-component BEC admits a dark (bright) soliton solution for the repulsive (attractive) interparticle interaction. However, in the presence of external potential and by adjusting interspecies interaction strength λ_0, two-component BEC supports the complex formation of combined soliton solutions. For both repulsive intraspecies interaction strengths ($\lambda_j > 0, j = 1, 2$) indicate in Fig. 1(a) that the system supports dark–dark soliton ($-\sqrt{\lambda_1\lambda_2} < \lambda_0 < \lambda_1$ or $\lambda_0 > \lambda_2$), bright–bright soliton ($\lambda_0 < -\sqrt{\lambda_1\lambda_2}$), dark–bright soliton ($\sqrt{\lambda_1\lambda_2} < \lambda_0 < \lambda_2$), and bright–dark soliton ($\lambda_1 < \lambda_0 < \sqrt{\lambda_1\lambda_2}$). For both attractive intraspecies interaction strengths ($\lambda_j < 0, j = 1, 2$) shown in Fig. 1(b), two-component BEC also admits combined soliton solutions in different regions, such as dark–dark soliton ($\lambda_0 < \sqrt{\lambda_1\lambda_2}$), bright–bright soliton ($\lambda_0 < \lambda_1$ or $\lambda_2 < \lambda_0 < \sqrt{\lambda_1\lambda_2}$), dark–bright soliton ($\lambda_1 < \lambda_0 < -\sqrt{\lambda_1\lambda_2}$), and bright–dark soliton ($-\sqrt{\lambda_1\lambda_2} < \lambda_0 < \lambda_2$). Finally, for the attractive and repulsive intraspecies interaction strengths ($\lambda_1 < 0$ and $\lambda_2 > 0$), there are dark–dark soliton ($\lambda_0 > \lambda_2$), bright–bright soliton ($\lambda_0 < \lambda_1$), and dark–bright soliton ($\lambda_1 < \lambda_0 < \lambda_2$).

The above results clearly indicate that the interspecies interaction strength λ_0 and the external potential determine mainly the complex formation of combined soliton solutions for two-component BECs. The surprising phenomena are the existence of bright–bright soliton for the case of both repulsive intraspecies interaction strengths and the existence of dark–dark soliton for the case of both attractive intraspecies interaction strengths. They are completely different from the soliton solutions of single-component BEC formed in the case of without an external potential. It implies that the external potential is also a key factor for the formation of bright (dark) soliton solutions in two-component BEC. If one tunes the trapped potential to the weak case, i.e. $0 < \hbar^2\beta^2/m < V_0$, the regions of combined soliton solutions are shown in Fig. 2, which are completely opposite to the results of the tightly trapped potential.

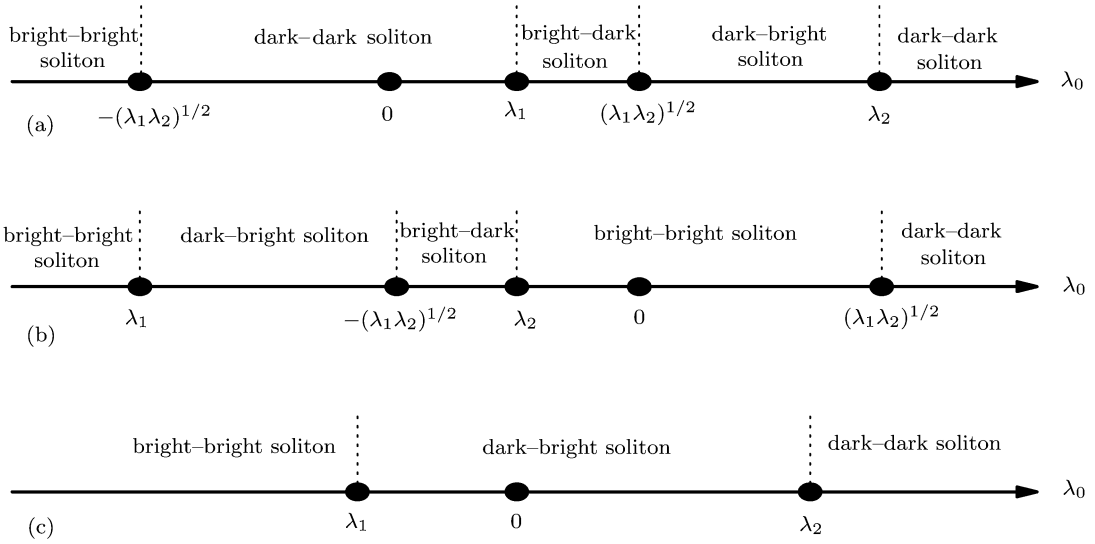


Fig. 1. Phase diagram of combined solitons versus interspecies interaction strength λ_0 for two-component BEC with the tightly trapped potential ($\hbar^2\beta^2/m > V_0 > 0$) or the expulsive potential ($V_0 < 0$). Panel (a) is for both repulsive intraspecies interaction strengths ($\lambda_j > 0, j = 1, 2$), panel (b) is for both attractive intraspecies interaction strengths ($\lambda_j < 0, j = 1, 2$), and panel (c) is for the attractive and repulsive intraspecies interaction strengths ($\lambda_1 < 0$ and $\lambda_2 > 0$).

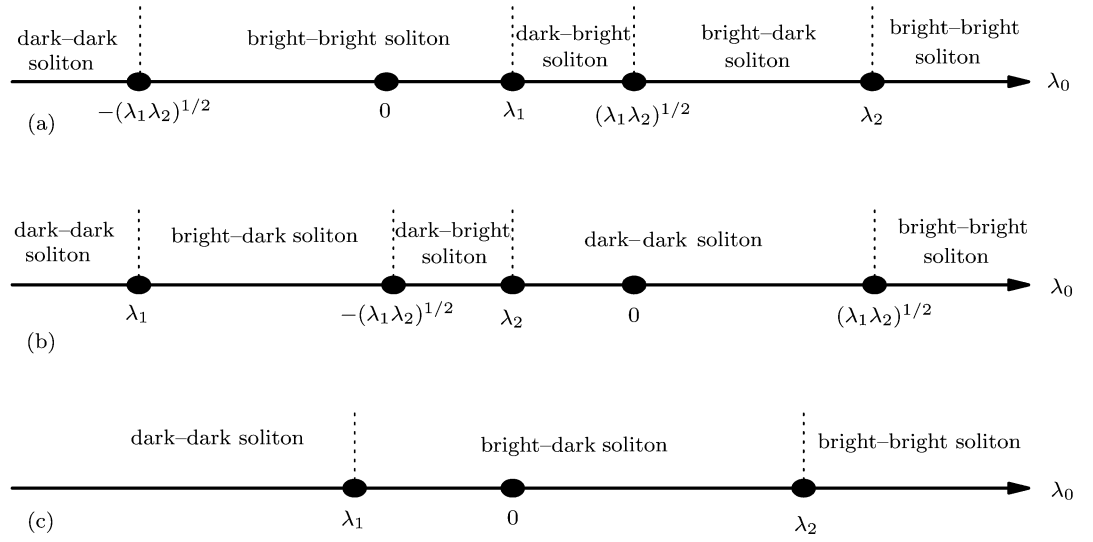


Fig. 2. Different regions for the formation of combined solitons versus interspecies interaction strength λ_0 with weakly trapped potential ($0 < \hbar^2\beta^2/m < V_0 > 0$). Panel (a) is for both repulsive intraspecies interaction strengths, panel (b) is for both attractive intraspecies interaction strengths, and panel (c) is for the attractive and repulsive intraspecies interaction strengths.

In summary, we analytically investigated the complex formation of the combined soliton solutions of two-component BEC in a closed form. The "phase diagram" distinctly indicates the formation regions of different combined solitons, where the intraspecies (interspecies) interaction strengths and external trapped potential play the complex roles for the existence of combined solitons. Especially, with the effect of external potential there exists the bright–bright (dark–dark) soliton for repulsive (attractive) intraspecies interaction strength. This is the novel and fundamental scenario for the study of two-component BEC.

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