Probabilistic joint remote preparation of a high-dimensional equatorial quantum state

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This paper proposes a scheme for probabilistic joint remote preparation of an arbitrary high-dimensional equatorial quantum state by using high-dimensional single-particle orthogonal projective measurement and appropriate unitary operation. As a special case, a scheme of joint remote preparation of a single-qutrit equatorial state is presented in detail. The scheme is also generalized to the multi-party high-dimensional case. It shows that, only if when all the senders collaborate with each other, the receiver can reconstruct the original state with a certain probability.

Keywords: probabilistic joint remote preparation, high-dimensional equatorial quantum state, single-qudit projective measurement

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1. Introduction

Quantum communication plays a significant role in the ongoing field of information theory. Quantum teleportation, proposed by Bennett *et al.*,[1] is the process that transmits an unknown quantum state from a sender to a spatially distant receiver via a quantum channel with the help of some classical information. In the last decade, Lo,[2] Pati,[3] and Bennett *et al.*[4] presented a new quantum communication scheme that uses classical communication and a previously shared entangled resource to remotely prepare a known quantum state. This communication scheme is called remote state preparation (RSP). Compared with teleportation, RSP requires less classical communication cost than teleportation. Since then, various protocols[5−20] for the generalization of RSP have been presented by using various kinds of methods, including low-entanglement RSP,[5] higher-dimensional RSP,[6] optimal RSP,[7] generalized RSP,[8] oblivious RSP,[9] continuous variable RSP,[10] faithful RSP,[11] etc. Meanwhile, some RSP schemes have been implemented experimentally.[21−25]

All the above schemes assume the case that only one sender knows the original state to be remotely prepared and wants to help the receiver recover the original state. However, if two-party or multiparty shares a original quantum state, and they want to remotely prepare it in the laboratory of receiver, how can they do it? To answer this problem, recently, a novel aspect of RSP, called the joint RSP (JRSP), has been proposed.[26−29] In these schemes of the JRSP,[26−29] two senders (or N senders) know partly the original state they want to remotely prepare, respectively. If and only if all the senders agree to collaboration, the receiver can reconstruct the original quantum state. Nevertheless, in Refs. [26]–[29] only the single- or multi-qubit state was considered. Though various schemes of RSP of high-dimensional quantum state have been proposed in recent years,[6,11,30−32] no scheme has yet been reported for the JRSP of higher-dimensional quantum state. In this paper, we propose a scheme for JRSP of a higher-dimensional equatorial quantum state. In our scheme, a bipartite higher-dimensional partial entangled state is used as the quantum channel, and the single-particle higher-dimensional projective measurement and appropriate unitary operation are needed. It is shown that, if and only if all the senders collaborate with each other, the receiver can recover the original state with a certain probability.

2. Probabilistic JRSP of a three-dimensional equatorial quantum state

To present our scheme more clearly, let us first begin with JRSP in three dimensions. Suppose that Al-

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ice and Bob wish to help the receiver Charlie remotely prepare a three-dimensional equatorial quantum state

$$|\phi\rangle = \frac{1}{\sqrt{3}}(|0\rangle + \mathrm{e}^{\mathrm{i}\delta_1}|1\rangle + \mathrm{e}^{\mathrm{i}\delta_2}|2\rangle), \quad (1)$$

where the parameters $\delta_1 = a_1 + b_1$ and $\delta_2 = a_2 + b_2$ and a_1, a_2, b_1 and b_2 are real. Suppose that Alice and Bob know the original state $|\phi\rangle$ partly, i.e., Alice knows a_1 and a_2, Bob knows b_1 and b_2, but unknown to Charlie. We also suppose that the tripartite three-dimensional non-maximal entangled state $|\varphi\rangle_{123}$ shared by Alice, Bob and Charlie can be written as

$$|\psi\rangle_{123} = \alpha_0|000\rangle_{123} + \alpha_1|111\rangle_{123} + \alpha_2|222\rangle)_{123} , \quad (2)$$

where $|\alpha_0|^2 + |\alpha_1|^2 + |\alpha_2|^2 = 1$ and $|\alpha_0|^2 \geq |\alpha_2|^2$, $|\alpha_1|^2 \geq |\alpha_2|^2$. Assume that qutrit 1 belongs to Alice, qutrit 2 belongs to Bob and qutrit 3 belongs to Charlie, respectively. In order to help Charlie remotely prepare the original state, what Alice and Bob need to do is to perform single-qutrit projective measurements on their own qutrits 1 and 2 respectively. The measurement bases chosen by Alice are mutually orthogonal basis vectors $\{|x_0\rangle, |x_1\rangle, |x_2\rangle\}$, which are related to computation basis vectors $\{|0\rangle, |1\rangle, |2\rangle\}$ and are given by

$$|x_k\rangle = \frac{1}{\sqrt{3}} \sum_j \mathrm{e}^{2\pi \mathrm{i} jk/3} \mathrm{e}^{-\mathrm{i}a_j}|j\rangle , \quad (3)$$

where $a_0 = 0$ and $j, k = 0, 1, 2$. Specifically,

$$\begin{aligned}
|x_0\rangle &= \frac{1}{\sqrt{3}}(|0\rangle + \mathrm{e}^{-\mathrm{i}a_1}|1\rangle + \mathrm{e}^{-\mathrm{i}a_2}|2\rangle) , \\
|x_1\rangle &= \frac{1}{\sqrt{3}}(|0\rangle + \mathrm{e}^{2\pi\mathrm{i}/3}\mathrm{e}^{-\mathrm{i}a_1}|1\rangle + \mathrm{e}^{4\pi\mathrm{i}/3}\mathrm{e}^{-\mathrm{i}a_2}|2\rangle) , \\
|x_2\rangle &= \frac{1}{\sqrt{3}}(|0\rangle + \mathrm{e}^{4\pi\mathrm{i}/3}\mathrm{e}^{-\mathrm{i}a_1}|1\rangle + \mathrm{e}^{2\pi\mathrm{i}/3}\mathrm{e}^{-\mathrm{i}a_2}|2\rangle) . \quad (4)
\end{aligned}$$

And the measurement bases chosen by Bob are mutually orthogonal basis vectors $\{|y_0\rangle, |y_1\rangle, |y_2\rangle\}$, which are related to computation basis vectors $\{|0\rangle, |1\rangle, |2\rangle\}$ and are given by

$$\begin{aligned}
|y_0\rangle &= \frac{1}{\sqrt{3}}(|0\rangle + \mathrm{e}^{-\mathrm{i}b_1}|1\rangle + \mathrm{e}^{-\mathrm{i}b_2}|2\rangle) , \\
|y_1\rangle &= \frac{1}{\sqrt{3}}(|0\rangle + \mathrm{e}^{2\pi\mathrm{i}/3}\mathrm{e}^{-\mathrm{i}b_1}|1\rangle + \mathrm{e}^{4\pi\mathrm{i}/3}\mathrm{e}^{-\mathrm{i}b_2}|2\rangle) , \\
|y_2\rangle &= \frac{1}{\sqrt{3}}(|0\rangle + \mathrm{e}^{4\pi\mathrm{i}/3}\mathrm{e}^{-\mathrm{i}b_1}|1\rangle + \mathrm{e}^{2\pi\mathrm{i}/3}\mathrm{e}^{-\mathrm{i}b_2}|2\rangle) . \quad (5)
\end{aligned}$$

Using Eqs. (4) and (5), we can describe the state (2)

$$\begin{aligned}
|\psi\rangle_{123} = \frac{1}{\sqrt{3}}[&|x_0\rangle_1|y_0\rangle_2(\alpha_0|0\rangle + \alpha_1\mathrm{e}^{\mathrm{i}\delta_1}|1\rangle + \alpha_2\mathrm{e}^{\mathrm{i}\delta_2}|2\rangle)_3 \\
&+ |x_0\rangle_1|y_1\rangle_2(\alpha_0|0\rangle + \alpha_1\mathrm{e}^{4\pi\mathrm{i}/3}\mathrm{e}^{\mathrm{i}\delta_1}|1\rangle + \alpha_2\mathrm{e}^{2\pi\mathrm{i}/3}\mathrm{e}^{\mathrm{i}\delta_2}|2\rangle)_3 \\
&+ |x_0\rangle_1|y_2\rangle_2(\alpha_0|0\rangle + \alpha_1\mathrm{e}^{2\pi\mathrm{i}/3}\mathrm{e}^{\mathrm{i}\delta_1}|1\rangle + \alpha_2\mathrm{e}^{4\pi\mathrm{i}/3}\mathrm{e}^{\mathrm{i}\delta_2}|2\rangle)_3 \\
&+ |x_1\rangle_1|y_0\rangle_2(\alpha_0|0\rangle + \alpha_1\mathrm{e}^{4\pi\mathrm{i}/3}\mathrm{e}^{\mathrm{i}\delta_1}|1\rangle + \alpha_2\mathrm{e}^{2\pi\mathrm{i}/3}\mathrm{e}^{\mathrm{i}\delta_2}|2\rangle)_3 \\
&+ |x_1\rangle_1|y_1\rangle_2(\alpha_0|0\rangle + \alpha_1\mathrm{e}^{2\pi\mathrm{i}/3}\mathrm{e}^{\mathrm{i}\delta_1}|1\rangle + \alpha_2\mathrm{e}^{4\pi\mathrm{i}/3}\mathrm{e}^{\mathrm{i}\delta_2}|2\rangle)_3 \\
&+ |x_1\rangle_1|y_2\rangle_2(\alpha_0|0\rangle + \alpha_1\mathrm{e}^{\mathrm{i}\delta_1}|1\rangle + \alpha_2\mathrm{e}^{\mathrm{i}\delta_2}|2\rangle)_3 \\
&+ |x_2\rangle_1|y_0\rangle_2(\alpha_0|0\rangle + \alpha_1\mathrm{e}^{2\pi\mathrm{i}/3}\mathrm{e}^{\mathrm{i}\delta_1}|1\rangle + \alpha_2\mathrm{e}^{4\pi\mathrm{i}/3}\mathrm{e}^{\mathrm{i}\delta_2}|2\rangle)_3 \\
&+ |x_2\rangle_1|y_1\rangle_2(\alpha_0|0\rangle + \alpha_1\mathrm{e}^{\mathrm{i}\delta_1}|1\rangle + \alpha_2\mathrm{e}^{\mathrm{i}\delta_2}|2\rangle)_3 \\
&+ |x_2\rangle_1|y_2\rangle_2(\alpha_0|0\rangle + \alpha_1\mathrm{e}^{4\pi\mathrm{i}/3}\mathrm{e}^{\mathrm{i}\delta_1}|1\rangle + \alpha_2\mathrm{e}^{2\pi\mathrm{i}/3}\mathrm{e}^{\mathrm{i}\delta_2}|2\rangle)_3] . \quad (6)
\end{aligned}$$

Since a_1 and a_2 (b_1 and b_2) being necessary for determining the bases $\{|x_k\rangle_1\}$ ($\{|y_k\rangle_2\}$) are known only to Alice (Bob), Alice and Bob are always able to make the measurements independently of each other.[29] Now let Alice and Bob measure their own qutrits 1 and 2 respectively, and then they inform Charlie

of their outcomes by the classical channels. According to Alice's and Bob's measurement results, Charlie can reconstruct the original state at his side. Without loss of generality, assume Alice's measurement result is $|x_0\rangle_1$ and Bob's result is $|y_2\rangle_2$, the qutrit 3 will collapse into the state $(\alpha_0|0\rangle+\alpha_1\,\mathrm{e}^{2\pi\mathrm{i}/3}\,\mathrm{e}^{\mathrm{i}\delta_1}|1\rangle+\alpha_2\,\mathrm{e}^{4\pi\mathrm{i}/3}\,\mathrm{e}^{\mathrm{i}\delta_2}|2\rangle)_3/3$. Charlie needs to perform a local unitary operation U_1 on qutrit 3, this means that

$$\begin{aligned}&U_1\otimes(\alpha_0|0\rangle_3+\alpha_1\,\mathrm{e}^{2\pi\mathrm{i}/3}\,\mathrm{e}^{\mathrm{i}\delta_1}|1\rangle_3\\&+\alpha_2\,\mathrm{e}^{4\pi\mathrm{i}/3}\,\mathrm{e}^{\mathrm{i}\delta_2}|2\rangle_3)/3\\=&\frac{1}{3}(\alpha_0|0\rangle+\alpha_1\,\mathrm{e}^{\mathrm{i}\delta_1}|1\rangle+\alpha_2\,\mathrm{e}^{\mathrm{i}\delta_2}|2\rangle)_3\ ,\end{aligned}\tag{7}$$

where $U_1=|0\rangle\langle0|+\mathrm{e}^{2\pi\mathrm{i}/3}|1\rangle\langle1|+\mathrm{e}^{4\pi\mathrm{i}/3}|2\rangle\langle2|$. Next, Charlie should introduce an auxiliary qutrit C in the state $|0\rangle_C$, and performs another unitary operation U' on both qutrits 3 and C. The U' is a 9×9 matrix in the two-qutrit bases $\{|00\rangle_{3C},|10\rangle_{3C},|20\rangle_{3C},|01\rangle_{3C},|11\rangle_{3C},|21\rangle_{3C},|02\rangle_{3C},|12\rangle_{3C},|22\rangle_{3C}\}$

$$U'=\begin{pmatrix}\omega_0&\omega_1&0\\\omega_1&-\omega_0&0\\0&0&I\end{pmatrix}\ ,\tag{8}$$

where I is a 3×3 identity matrix, ω_0 and ω_1 are 3×3 diagonal matrices. The ω_0 and ω_1 take the forms as follows:

$$\omega_0=\mathrm{diag}(\lambda_0,\lambda_1,1)\ ,\tag{9}$$

$$\omega_1=\mathrm{diag}(\sqrt{1-{\lambda_0}^2},\sqrt{1-{\lambda_1}^2},0)\ ,\tag{10}$$

where $\lambda_0=\alpha_2/\alpha_0$ and $\lambda_1=\alpha_2/\alpha_1$. The unitary operation U' will transform the state $\frac{1}{3}(\alpha_0|0\rangle+\alpha_1\,\mathrm{e}^{\mathrm{i}\delta_1}|1\rangle+\alpha_2\,\mathrm{e}^{\mathrm{i}\delta_2}|2\rangle)_3\otimes|0\rangle_C$ into

$$\begin{aligned}&\frac{\alpha_2}{\sqrt{3}}\left[\frac{1}{\sqrt{3}}(|0\rangle+\mathrm{e}^{\mathrm{i}\delta_1}|1\rangle+\mathrm{e}^{\mathrm{i}\delta_2}|2\rangle)_3\right]\otimes|0\rangle_C\\&+\frac{1}{3}\left(\sqrt{\alpha_0^2-\alpha_2^2}|0\rangle+\sqrt{\alpha_1^2-\alpha_2^2}\,\mathrm{e}^{\mathrm{i}\delta_1}|1\rangle\right)_3\otimes|1\rangle_C\ .\end{aligned}\tag{11}$$

Then Charlie performs a computation basis measurement on the auxiliary qutrit C. If the measurement result is $|1\rangle_C$, the remote preparation of the original state fails. If the measurement outcome is $|0\rangle_C$, the state of the qutrit 3 will collapse to $(|0\rangle+\mathrm{e}^{\mathrm{i}\delta_1}|1\rangle+\mathrm{e}^{\mathrm{i}\delta_2}|2\rangle)/\sqrt{3}$, which is exactly the original state $|\phi\rangle$. Thus the JRSP of single qutrit state is successfully realized and the success probability is $|\alpha_2|^2/3$. If the measurement results of Alice and Bob are the other 8 cases in Eq. (8), Charlie should perform an appropriate unitary operation $U_k=\sum_j\mathrm{e}^{2\pi\mathrm{i}jk/3}|j\rangle\langle j|$ $(j,k=0,1,2)$ on his qutrit 3, respectively, the state of qutrit 3 can evolves to $(\alpha_0|0\rangle+\alpha_1\,\mathrm{e}^{\mathrm{i}\delta_1}|1\rangle+\alpha_2\,\mathrm{e}^{\mathrm{i}\delta_2}|2\rangle)_3/3$. Then, similar to the above deal, the JRSP can be perfectly realized, the total probability of successful JRSP is $3|\alpha_2|^2$. Specially, if quantum channel is a maximally tripartite-qutrit entangled state, namely, $|\alpha_0|=|\alpha_1|=|\alpha_2|=1/\sqrt{3}$, the total successful probability equals 1.

Classical communication cost plays an important role in RSP, more RSP schemes were discussed in detail.[15,18,32–34] In the above scheme of JRSP of single qutrit state, without loss of generality, if Alice's and Bob's measurement outcomes are $|x_0\rangle_1$ and $|y_2\rangle_2$, respectively, and Charlie's measurement result is $|0\rangle_C$, the classical communication cost required is $-(|\alpha_2|^2/3)\log_2(|\alpha_2|^2/3)$ bits. As the above analyses, there are 9 kinds of measurement results by Alice and Bob, so the amount of the classical communication is $-3|\alpha_2|^2\log_2(|\alpha_2|^2/3)$ bits. For the maximally entangled channel, the total classical communication cost will equal $2\log_2 3$ bits.

3. Probabilistic JRSP of a *d*-dimensional equatorial quantum state

Now we will generalize the above scheme to arbitrarily higher dimensions. Suppose that Alice and Bob wish to help Charlie remotely prepare d-dimensional equatorial quantum state

$$|\Phi\rangle=\sum_{j=0}^{d-1}\mathrm{e}^{\mathrm{i}\delta_j}|j\rangle/\sqrt{d}\ ,\tag{12}$$

where parameters δ_j is real and let $\delta_0=0$. We assume that Alice and Bob know the state $|\Phi\rangle$ partly, that is, Alice knows a_j and Bob knows b_j, where $a_j+b_j=\delta_j$ $(j=0,1,\ldots,d-1)$, but Charlie does not know them at all. We also suppose that a tripartite d-dimensional nonmaximally entangled state shared by Alice, Bob and Charlie can be written as

$$|\Psi\rangle=\sum_{j=0}^{d-1}\alpha_j|jjj\rangle_{123}/\sqrt{d}\ ,\tag{13}$$

where $\sum_{j=0}^{d-1}|\alpha_j|^2=1$, $|\alpha_j|>|\alpha_{d-1}|$ $(j=0,1,\ldots,d-2)$, and the qudit 1 belongs to Alice, the qudit 2 to Bob and the qudit 3 to Charlie, respectively.

To achieve the JRSP scheme, Alice should make a single-qudit projective measurement on her qudit 1. The measurement bases of Alice are a set of mutually orthogonal vectors $\{|x_0\rangle, |x_1\rangle, \ldots, |x_2\rangle\}$, which are related to computation basis vectors $\{|0\rangle, |1\rangle, \ldots, |d-1\rangle\}$ and are given by

$$|x_k\rangle = \sum_{j=0}^{d-1} \mathrm{e}^{2\pi \mathrm{i} jk/d}\, \mathrm{e}^{-\mathrm{i}a_j} |j\rangle/\sqrt{d}\,, \tag{14}$$

where $k, j = 0, 1, \ldots, d-1$. Meantime Bob performs a single-qudit projective measurement on his qudit 2 in the bases $\{|y_0\rangle, |y_1\rangle, \ldots, |y_2\rangle\}$, which are related to the computation basis vectors $\{|0\rangle, |1\rangle, \ldots, |d-1\rangle\}$ and are given by

$$|y_{k'}\rangle = \sum_{j=0}^{d-1} \mathrm{e}^{2\pi \mathrm{i} jk'/d}\, \mathrm{e}^{-\mathrm{i}b_j} |j\rangle/\sqrt{d}\,, \tag{15}$$

where $k', j = 0, 1, \ldots, d-1$. In terms of Eqs. (14) and (15) the state $|\Psi\rangle_{123}$ can be rewritten as

$$|\Psi\rangle_{123} = \frac{1}{d} \sum_{k=0}^{d-1} \left\{ |x_k\rangle_1 \sum_{k'=0}^{d-1} [|y_{k'}\rangle_2 \sum_{j=0}^{d-1} \alpha_j \times \mathrm{e}^{-2\pi \mathrm{i} j(k+k')/d}\, \mathrm{e}^{\mathrm{i}\delta_j} |j\rangle_3] \right\}. \tag{16}$$

After the measurements Alice and Bob inform Charlie of their results by using the classical channels. According to the measurement results of Alice and Bob, Charlie can reconstruct the original state $|\Phi\rangle$ on his qudit 3. If the measurement results of Alice and Bob are $|x_k\rangle_1$ and $|y_{k'}\rangle_2$, $(k, k' = 0, 1, \ldots, d-1)$, respectively, Charlie should perform a unitary operation

$$U_d = \sum_{j=0}^{d-1} \mathrm{e}^{2\pi \mathrm{i} j(k+k')/d} |j\rangle\langle j| \tag{17}$$

on his qudit 3, respectively. In this case, the state of qudit 3 will evolve to $\sum_{j=0}^{d-1} \alpha_j\, \mathrm{e}^{\mathrm{i}\delta_j} |j\rangle_3$. Next, Charlie introduces an auxiliary qudit A in the state $|0\rangle_A$, and then performs another unitary operation U_d' on both qudits 3 and A. The U_d' is a $d^2 \times d^2$ matrix in the bases $\{|jl\rangle_{3A}\}$ $(j, l = 0, 1, \ldots, d-1)$

$$U_d' = \begin{pmatrix} \nu_0 & \nu_1 & 0 \\ \nu_1 & -\nu_0 & 0 \\ 0 & 0 & I \end{pmatrix}, \tag{18}$$

where I is a $d \times d$ identity matrix, ν_0 and ν_1 are $d \times d$ diagonal matrices. The ν_0 and ν_1 take the forms as follows:

$$\nu_0 = \mathrm{diag}(\lambda_0, \lambda_1, \ldots, \lambda_{d-2}, 1)\,, \tag{19}$$

$$\nu_1 = \mathrm{diag}\left(\sqrt{1-\lambda_0{}^2}, \sqrt{1-\lambda_1{}^2}, \ldots, \sqrt{1-\lambda_{d-2}^2}, 0\right), \tag{20}$$

where $\lambda_j = \alpha_{d-1}/\alpha_j$ $(j = 0, 1, \ldots, d-2)$. The unitary operation U_d' will transform the state $(\sum_{j=0}^{d-1} \alpha_j\, \mathrm{e}^{\mathrm{i}\delta_j} |j\rangle_3/\sqrt{d}) \otimes |0\rangle_A$ into

$$\left(\frac{\alpha_{d-1}}{\sqrt{d}} \sum_{j=0}^{d-1} \mathrm{e}^{\mathrm{i}\delta_j} |j\rangle_3/\sqrt{d}\right) \otimes |0\rangle_A + \frac{1}{d} \left(\sum_{j=0}^{d-2} \sqrt{\alpha_j^2 - \alpha_{d-1}^2}\, \mathrm{e}^{\mathrm{i}\delta_j} |j\rangle_3\right) \otimes |1\rangle_A\,. \tag{21}$$

Then Charlie performs a single-qudit measurement on the auxiliary qudit A. If the measurement result is $|1\rangle_A$, the JRSP of the original state fails. If the measurement outcome is $|0\rangle_A$, the state of the qudit 3 will collapse to the state $\sum_{j=0}^{d-1} \mathrm{e}^{\mathrm{i}\delta_j} |j\rangle_3/\sqrt{d}$. This means that the JRSP of the original state $|\Phi\rangle$ is successfully realized with the successful probability being $|\alpha_{d-1}|^2/d$. The required classical communication cost is $-(|\alpha_{d-1}|^2/d)\log_2(|\alpha_{d-1}|^2/d)$ bits. Similarly, from Eq. (16), if the measurement results of Alice and Bob are the other d^2-1 cases, Charlie will also obtain the same successful probability $|\alpha_{d-1}|^2/d$ by the above steps. Therefore the total probability of successful joint RSP is $d|\alpha_{d-1}|^2$ and the total amount of classical communication required is $-d|\alpha_{d-1}|^2 \log_2(|\alpha_{d-1}|^2/d)$ bits. For the special case, that is, if the quantum channel is a maximally tripartite-qudit entangled state, i.e., $|\alpha_j\rangle = 1/\sqrt{d}$ $(j = 0, 1, \ldots, d-1)$, the total successful probability equals 1, and the total classical communication cost is $2\log_2 d$ bits.

The above scheme can be generalized to the multi-party case. Suppose N senders Alice_1, Alice_2, …, Alice_N share a d-dimensional equatorial quantum state

$$|\Xi\rangle = \sum_{j=0}^{d-1} \mathrm{e}^{\mathrm{i}\eta_j} |j\rangle/\sqrt{d}\,, \tag{22}$$

where the parameters η_j is real and let $\eta_0 = 0$. Suppose Alice_1, Alice_2, …, Alice_N know partly the state which they wish to help the receiver Charlie remotely prepare, i.e., Alice_1 knows a_j^1, Alice_2 knows a_j^2, …, Alice_N knows a_j^N, where $\sum_{j=0}^{d-1} a_j^{(m)} = \eta_j$ $(m = 0, 1, \ldots, N)$, and all $a_j^{(m)}$ are real. We also

suppose that the $(N+1)$-partite d-dimensional nonmaximally entangled state shared by N senders and Charlie can be described as

$$|\Gamma\rangle = \frac{1}{d}\sum_{j=0}^{d-1}\alpha_j|\underbrace{jj\cdots j}_{N+1}\rangle_{12\ldots(N+1)}\ , \qquad (23)$$

where $\sum_{j=0}^{d-1}|\alpha_j|^2=1$, $|\alpha_j|>|\alpha_{d-1}|$ $(j=0,1,\ldots,d-2)$, and the qudit $1,2,\ldots,N$ belongs to Alice$_1$, Alice$_2$,..., Alice$_N$, and qudit $(N+1)$ belongs to Charlie, respectively.

To help the receiver Charlie remotely prepare state $|\Xi\rangle$, all senders need to perform a single-qudit projective measurements on their own qudit in the orthogonal basis vectors $\{|x_{k_m}\rangle\}$, respectively. Here

$$|x_{k_m}\rangle = \sum_j \mathrm{e}^{2\pi \mathrm{i} j(k_m)/d}\,\mathrm{e}^{-\mathrm{i}a_j^{(m)}}|j\rangle/\sqrt{d}\ , \qquad (24)$$

where $m=0,1,\ldots,N$; $k_m,j=0,1,\ldots,d-1$. Then, they all inform Charlie their measurement results via classical channels. In accord with the measurement results of the senders, Charlie can recover the original state $|\Xi\rangle$ on his qudit by appropriate operations $U_d^{(N)}$ and U_d', where

$$U_d^{(N)} = \sum_{j=0}^{d-1}\mathrm{e}^{2\pi \mathrm{i} j(\sum_{m=1}^{N}k_m)/d}|j\rangle\langle j|\ , \qquad (25)$$

and U_d' is in the form as Eq. (18). That is, the JRSP of the state $|\Xi\rangle$ is successfully realized. The total probability of successful JRSP is $d|\alpha_{d-1}|^2$. When the quantum channel is a maximally $(N+1)$ partite-qudit entangled state, the total successful probability equals 1, and the total amount of classical communication required is $N\log_2 d$ bits.

4. Conclusion

In conclusion, we have proposed a scheme for joint remote preparation of a high-dimensional equatorial quantum state. In this scheme, two senders share an original state which they wish to help the receiver to remotely prepare, but the senders only partly know the state. It shows that only if all the senders collaborate with each other can the receiver remotely reconstruct the original state with certain probability. To present this scheme more clearly, we first investigated a scheme for JRSP of a three-dimensional equatorial quantum state in detail. Then we have presented a scheme for joint remote preparation of a d-dimensional equatorial quantum state. We also generalize the scheme to the multi-party d-dimensional case. These schemes require resources such as bipartite three- or d-dimensional nonmaximally entangled state as the quantum channel, single-qutrit or single-qudit projective measurement, classical communication and appropriate unitary operation. The total successful probability and the total classical communication cost required in the JRSP process have been calculated, respectively.

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