Scheme to implement optimal asymmetric economical 1→3 phase-covariant telecloning via cavity QED*

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(Received 3 December 2009; revised manuscript received 7 January 2010)

We propose an experimentally feasible scheme to implement the optimal asymmetric economical 1→3 phase-covariant telecloning protocol, which works without ancilla, based on cavity quantum electrodynamics (QED). The scheme is insensitive to the cavity field states and the cavity decay. In the telecloning process, the cavity is only virtually excited, it greatly prolongs the efficient decoherent time. Therefore, the scheme may be experimentally realized in the field of current cavity QED techniques.

Keywords: telecloning, cavity QED

PACC: 0367

1. Introduction

It is impossible to create a machine that produces an exact copy of an unknown state because of the no-cloning theorem.[1] Because the exact copy cannot be obtained, much more attention has been paid to approximate copies in the deterministic cloning process. Bužek and Hillery first discussed this probability, which is very important to quantum communication and cryptography.[2] They accounted for the potentiality to realize copies (approximate clones) of unknown quantum states with a quality independence of initial states. Many investigators began to make efforts to clone quantum states under all these limitations, that is, the inexact cloning of quantum input states without knowing their information completely. For discrete variable quantum systems, quantum clonings can be divided into the optimal universal quantum cloning (UQC),[2−5] the optimal phase-covariant quantum cloning (PCC),[6,7] and the optimal real state cloning (RS).[8,9] According to the difference in fidelity among clones, three cloners can be divided into symmetric[2−7] and asymmetric[8−10] cloners. For the optimal symmetric cloning, the fidelities of copies are maximal and equal. The optimal asymmetric cloning was first proposed by N. J. Cerf,[10] which locally produces two approximate copies with the best match of the fidelities of two clones. Or in other words, given the fidelity of one clone the fidelity of the other can reach a maximum. Recently, some schemes were proposed to implement the optimal symmetric cloning.[11,12] In experiment, the optimal asymmetric economical 1→2 phase-covariant quantum cloning has been realized.[13] The so-called optimal economical phase-covariant cloning machine without ancilla, was first proposed by Niu and Griffiths.[14] Afterwards, Fiurášek generalized this kind of cloner.[15] The transformation proposed in Ref. [15] has been realized in experiment[16] by using nuclear-magnetic-resonance techniques. More recently, several protocols of the quantum cloning have been presented in the circumstance of cavity QED.[17−20] The economical phase-covariant quantum cloning in d-dimension[21,22] is further researched, and some schemes[23−25] to implement the symmetric phase-covariant quantum cloning have appeared.

Telecloning[26] combines quantum teleportation and cloning. Telecloning protocol depends on the establishment of particular multiparticle entangled states, which function as multiuser quantum information channels. It can be more economical in terms of the entanglement cost than the local clone. Later on, the optimal symmetric $1\to M$ telecloning in d-dimension, the optimal asymmetric 1→2 telecloning,[27] and the optimal $N \to M$ $(M > N)$ quantum telecloning[28] have been studied.

In this paper, we propose an experimentally feasible scheme to implement an optimal asymmetric economical 1→3 phase-covariant telecloning machine based on cavity QED. First, four atoms are used to

*Project supported by the National Natural Science Foundation of China (Grant No. 10674001), and the Personal Development Foundation of Anhui Province, China (Grant No. 2008Z018).

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prepare a maximally entangled state of the port and the two-output basis as an information distribution channel, and the sender, Alice, delivers three atoms to her three partners, Bob, Claire and David, respectively. When Alice wants to clone, she will perform the Bell states measurement on the initial and the port atoms and then informs her partners of the measurement results. The three partners can use the local operations to obtain their clones. In our scheme the Bell measurement is needed. Recently, Ye and Guo[29] have proposed a scheme to distinguish the Bell states via detecting the atom state accurately. In the telecloning process, the cavity is only virtually excited, thus it greatly prolongs the efficient decoherent time. Recently, the scheme proposed by Zheng and Guo[30] through using two Rydberg atoms crossing an off-resonant cavity has been demonstrated by Osnaghi *et al.*[31] Therefore our scheme may be achieved in experiment.

The rest of the present paper is organized as follows. In Section 2, we briefly review the optimal asymmetric economical 1→3 PCC cloning and telecloning in two dimensions. In Section 3, we present an experimentally feasible scheme to implement the PCC telecloning protocol in two dimensions, based on the cavity QED technique. The paper ends with a conclusion in Section 4.

2. Optimal asymmetric economical 1→3 PCC cloning and telecloning

To clone the input states on the equator $|\psi\rangle_0^{(\mathrm{in})} = \big(|0\rangle + \mathrm{e}^{\mathrm{i}\varphi}|1\rangle\big)/\sqrt{2}$ with $\varphi \in [0, 2\pi)$ unknown, the optimal asymmetric economical 1→3 phase-covariant quantum cloning is defined as[32]

$$\begin{aligned} |000\rangle_{1,2,3} &\to (a|001\rangle + b|010\rangle + c|100\rangle)_{1,2,3} \\ &= |\psi\rangle_{1,2,3}^{(0)}, \\ |100\rangle_{1,2,3} &\to (a|110\rangle + b|101\rangle + c|011\rangle)_{1,2,3} \\ &= |\psi\rangle_{1,2,3}^{(1)}, \end{aligned} \tag{1}$$

where a, b and c are the parameters satisfying $a^2 + b^2 + c^2 = 1$ $(a, b, c \geq 0)$. The fidelities of three copies 1, 2 and 3 are given by

$$F_1 = ab + 1/2, \quad F_2 = ac + 1/2$$

and

$$F_3 = bc + 1/2. \tag{2}$$

For an input state to be cloned, the output state is

$$|\psi\rangle_{1,2,3} = \Big[|\psi\rangle_{1,2,3}^{(0)} + \mathrm{e}^{\mathrm{i}\varphi}|\psi\rangle_{1,2,3}^{(1)}\Big]\Big/\sqrt{2}. \tag{3}$$

Next we briefly review the telecloning protocol.[33] The information channel is a maximally entangled state of port qubit p and two-output basis $|\psi\rangle_{1,2,3}^{(0)}$ and $|\psi\rangle_{1,2,3}^{(1)}$. It is given by

$$|\Psi\rangle_{p,1,2,3} = \Big(|0\rangle_p|\psi\rangle_{1,2,3}^{(0)} + |1\rangle_p|\psi\rangle_{1,2,3}^{(1)}\Big)\Big/\sqrt{2}. \tag{4}$$

The particle p is held by Alice, and she sends particles 1, 2 and 3 to Bob, Claire, and David, respectively. When she wants to clone the initial state, Alice makes the Bell state on particles 0 and p. The whole state $|\Psi\rangle_{0,p,1,2,3}$ can be rewritten as

$$\begin{aligned} &|\Psi\rangle_{0,p,1,2,3} \\ &= |\psi\rangle_0^{(\mathrm{in})} \otimes |\Psi\rangle_{p,1,2,3} \\ &= \frac{1}{2\sqrt{2}}\Big[\,|\phi^+\rangle_{0,p}\Big(|\psi\rangle_{1,2,3}^{(0)} + \mathrm{e}^{\mathrm{i}\varphi}|\psi\rangle_{1,2,3}^{(1)}\Big) \\ &\quad + |\phi^-\rangle_{0,p}\Big(|\psi\rangle_{1,2,3}^{(0)} - \mathrm{e}^{\mathrm{i}\varphi}|\psi\rangle_{1,2,3}^{(1)}\Big) \\ &\quad + |\psi^+\rangle_{0,p}\Big(\mathrm{e}^{\mathrm{i}\varphi}|\psi\rangle_{1,2,3}^{(0)} + |\psi\rangle_{1,2,3}^{(1)}\Big) \\ &\quad + |\psi^-\rangle_{0,p}\Big(\mathrm{e}^{\mathrm{i}\varphi}|\psi\rangle_{1,2,3}^{(0)} - |\psi\rangle_{1,2,3}^{(1)}\Big)\Big], \end{aligned} \tag{5}$$

where $|\phi^\pm\rangle = (|00\rangle \pm |11\rangle)/\sqrt{2}$ and $|\psi^\pm\rangle = (|01\rangle \pm |10\rangle)/\sqrt{2}$ are the Bell states. After the Bell measurement on qubits 0 and p, Alice informs her partners of the measurement results. Then the receivers can obtain their clones by using the local operators, defined as

$$\begin{aligned} &\sigma_z^1 \otimes \sigma_z^2 \otimes \sigma_z^3 \otimes \Big(|\psi\rangle_{1,2,3}^{(0)} - \mathrm{e}^{\mathrm{i}\varphi}|\psi\rangle_{1,2,3}^{(1)}\Big) \\ &\to \Big(|\psi\rangle_{1,2,3}^{(0)} + \mathrm{e}^{\mathrm{i}\varphi}|\psi\rangle_{1,2,3}^{(1)}\Big), \end{aligned} \tag{6}$$

$$\begin{aligned} &\sigma_x^1 \otimes \sigma_x^2 \otimes \sigma_x^3 \otimes \Big(\mathrm{e}^{\mathrm{i}\varphi}|\psi\rangle_{1,2,3}^{(0)} + |\psi\rangle_{1,2,3}^{(1)}\Big) \\ &\to \Big(|\psi\rangle_{1,2,3}^{(0)} + \mathrm{e}^{\mathrm{i}\varphi}|\psi\rangle_{1,2,3}^{(1)}\Big), \end{aligned} \tag{7}$$

$$\begin{aligned} &\sigma_x^1 \otimes \sigma_x^2 \otimes \sigma_x^3 \otimes \sigma_z^1 \otimes \sigma_z^2 \otimes \sigma_z^3 \\ &\otimes \Big(\mathrm{e}^{\mathrm{i}\varphi}|\psi\rangle_{1,2,3}^{(0)} - |\psi\rangle_{1,2,3}^{(1)}\Big) \\ &\to \Big(|\psi\rangle_{1,2,3}^{(0)} + \mathrm{e}^{\mathrm{i}\varphi}|\psi\rangle_{1,2,3}^{(1)}\Big), \end{aligned} \tag{8}$$

where σ_z^i and σ_x^i $(i = 1, 2, 3)$ are the Pauli operators. If Alice's measurement result is $|\phi^+\rangle_{0,p}$, then the subsystem containing particles 1, 2 and 3 is projected accurately into the optimal copy. She informs Bob, Claire and David, of the result, and her three partners can obtain their copies without doing anything. If the result is one of $|\phi^-\rangle_{0,p}$, $|\psi^+\rangle_{0,p}$ and $|\psi^-\rangle_{0,p}$,

her partners can perform local operations given by Eqs. (6)–(8) on their own particles, and acquire their clones.

3. Scheme to implement optimal asymmetric economical 1→3 phase-covariant quantum telecloning

For the implementation of the scheme, we first consider the interaction of two-level atoms with a single-mode cavity field. In the interaction picture the Hamiltonian is[34]

$$H_e = \lambda \left[\sum_{j=1}^{N} \left(|e_j\rangle \langle e_j| aa^\dagger - |g_j\rangle \langle g_j| a^\dagger a \right) + \sum_{j,\, k=1, j\neq k}^{N} S_j^+ S_k^- \right], \quad (9)$$

where $S_j^+ = |e_j\rangle \langle g_j|$ and $S_k^- = |g_k\rangle \langle e_k|$, with $|e_j\rangle$ ($|e_k\rangle$) and $|g_j\rangle$ ($|g_k\rangle$) being the excited and ground states of the j-th (k-th) atom, $a^\dagger$ and a are the creation operator and the annihilation operator for the cavity mode, and $\lambda \approx \Omega^2/\delta$, Ω is the single-photon Rabi frequency, $\delta \gg \Omega\sqrt{\bar{n}+1}$, with $\bar{n}$ being the mean photon number of the cavity field.

If the cavity field is initially in a vacuum state, the effective Hamiltonian is reduced into[34]

$$H_e = \lambda \left[\sum_{j=1}^{N} |e_j\rangle \langle e_j| + \sum_{j,\, k=1, j\neq k}^{N} S_j^+ S_k^- \right]. \quad (10)$$

In the following, we can use qubits $|1\rangle$ and $|0\rangle$ to denote atomic states $|e\rangle$ and $|g\rangle$, respectively. And we assume that all cavities are prepared in the vacuum state.

The telecloning scheme can be divided into three processes. First, we prepare the information channel (The process is depicted in Fig. 1).

The initial state of the four atoms $p, 1, 2, 3$ is $|\Psi\rangle_{p,1,2,3} = |0100\rangle_{p,1,2,3}$ and our task is to complete the evolution

$$|0100\rangle_{p,1,2,3} \to \left(|0\rangle_p |\psi\rangle_{1,2,3}^{(0)} + |1\rangle_p |\psi\rangle_{1,2,3}^{(1)} \right)/\sqrt{2}. \quad (11)$$

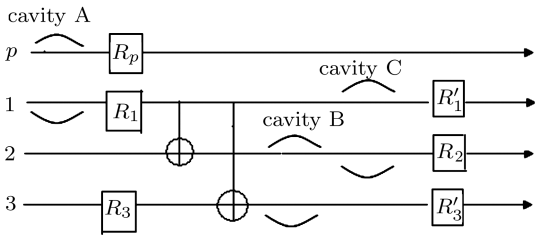


Fig. 1. Schematic diagram for the preparation of information channel, containing four atoms p, 1, 2 and 3, three cavities A, B and C and two C-Not gates. $R_{\rm p}$, R_1', R_2, and R_3' represent the rotation operators $R_z^{(p)}(\theta) = {\rm e}^{-{\rm i}\theta/2} |g\rangle_j \langle g| + {\rm e}^{{\rm i}\theta/2} |e\rangle_j \langle e|$ exerted on atoms p, 1, 2, 3 with angles $(\pi/2 + 2\theta_1)$, $2\theta_2$, $2\theta_3$, $2\theta_4$, respectively. R_1 denotes operation σ_x on particle 1. R_3 is transformation $|0\rangle_3 \to |1\rangle_3$.

Step 1 We sent atoms p and 1 into a vacuum cavity A simultaneously, after interaction time τ, which is chosen to be $\lambda\tau = \pi/4$, the system becomes

$$\begin{aligned} &|\Psi\rangle_{p,1,2,3}^{(1)} \\ &= {\rm e}^{-{\rm i}\lambda\tau} \left[\cos(\lambda\tau) |01\rangle_{p,1} - {\rm i}\sin(\lambda\tau) |10\rangle_{p,1} \right] |00\rangle_{2,3} \\ &= {\rm e}^{-{\rm i}\pi/4} \frac{1}{\sqrt{2}} \left[|01\rangle_{p,1} + {\rm e}^{-{\rm i}\pi/2} |10\rangle_{p,1} \right] |00\rangle_{2,3}. \end{aligned} \quad (12)$$

Step 2 Then a rotation operator, defined as $R_z^{(p)}(\theta) = {\rm e}^{-{\rm i}\theta/2} |g\rangle_j \langle g| + {\rm e}^{{\rm i}\theta/2} |e\rangle_j \langle e|$, is exerted on atom p with an angle $(\pi/2 + 2\theta_1)$, one can obtain

$$\begin{aligned} &|\Psi\rangle_{p,1,2,3}^{(2)} \\ &= {\rm e}^{-{\rm i}\pi/4} \frac{1}{\sqrt{2}} \left[{\rm e}^{-{\rm i}(\pi/4+\theta_1)} |01\rangle_{p,1} \right. \\ &\quad \left. + {\rm e}^{{\rm i}(\pi/4+\theta_1)} {\rm e}^{-{\rm i}\pi/2} |10\rangle_{p,1} \right] |00\rangle_{2,3} \\ &= \frac{1}{\sqrt{2}} \left[{\rm e}^{-{\rm i}\theta_1} |01\rangle_{p,1} + {\rm e}^{{\rm i}\theta_1} |10\rangle_{p,1} \right] |00\rangle_{2,3}, \end{aligned} \quad (13)$$

where we discard the whole phase factor ${\rm e}^{-{\rm i}\pi/2}$, and θ_1 will be determined later.

Step 3 A classical field is exerted on the atom 1 to obtain $|0\rangle_1 \Leftrightarrow |1\rangle_1$, equivalent to the operator $\sigma_x^{(1)}$, the state $|\Psi\rangle_{p,1,2,3}^{(2)}$ becomes

$$\begin{aligned} &|\Psi\rangle_{p,1,2,3}^{(3)} \\ &= \frac{1}{\sqrt{2}} \left[{\rm e}^{-{\rm i}\theta_1} |00\rangle_{p,1} + {\rm e}^{{\rm i}\theta_1} |11\rangle_{p,1} \right] |00\rangle_{2,3}. \end{aligned} \quad (14)$$

Step 4 A classical field is applied to atom 3 to obtain $|0\rangle_3 \to |1\rangle_3$, the system reads

$$\begin{aligned} &|\Psi\rangle_{p,1,2,3}^{(4)} \\ &= \frac{1}{\sqrt{2}} \left[{\rm e}^{-{\rm i}\theta_1} |0\rangle_p |001\rangle_{1,2,3} + {\rm e}^{{\rm i}\theta_1} |1\rangle_p |101\rangle_{1,2,3} \right]. \end{aligned} \quad (15)$$

Step 5 Atoms 1 and 2, 1 and 3 undergo two C-not gate operators[28] with atom 1 used as a control qubit, defined as

$$U_{\mathrm{C-Not}}\,|00\rangle \to |00\rangle\,,\quad U_{\mathrm{C-Not}}\,|01\rangle \to |01\rangle\,,$$
$$U_{\mathrm{C-Not}}\,|10\rangle \to |11\rangle\,,\quad U_{\mathrm{C-Not}}\,|11\rangle \to |10\rangle\,. \quad (16)$$

Equation (15) can evolve into

$$|\Psi\rangle^{(5)}_{p,1,2,3} = \frac{1}{\sqrt{2}}\left[\mathrm{e}^{-\mathrm{i}\theta_1}\,|0\rangle_p\,|001\rangle_{1,2,3} + \mathrm{e}^{\mathrm{i}\theta_1}\,|1\rangle_p\,|110\rangle_{1,2,3}\right]. \quad (17)$$

Step 6 Let atoms 2 and 3 cross a vacuum cavity B simultaneously, after interaction time τ_1, state $|\Psi\rangle^{(5)}_{p,1,2,3}$ becomes

$$\begin{aligned}|\Psi\rangle^{(6)}_{p,1,2,3} = {}& \frac{1}{\sqrt{2}}\big\{\mathrm{e}^{-\mathrm{i}\theta_1}\,|0\rangle_p\,\big[\cos(\lambda\tau_1)|001\rangle_{1,2,3} \\ &+ \mathrm{e}^{-\mathrm{i}\pi/2}\sin(\lambda\tau_1)|010\rangle_{1,2,3}] \\ &+ \mathrm{e}^{\mathrm{i}\theta_1}|1\rangle_p[\cos(\lambda\tau_1)|110\rangle_{1,2,3} \\ &+ \mathrm{e}^{-\mathrm{i}\pi/2}\sin(\lambda\tau_1)|101\rangle_{1,2,3}]\big\}. \end{aligned} \quad (18)$$

Step 7 The two atoms 1 and 2 cross cavity C, and the state of the system becomes

$$\begin{aligned}|\Psi\rangle^{(7)}_{p,1,2,3} = {}& \frac{1}{\sqrt{2}}\big\{\mathrm{e}^{-\mathrm{i}\theta_1}|0\rangle_p[\cos(\lambda\tau_1)|001\rangle_{1,2,3} \\ &+ \mathrm{e}^{-\mathrm{i}\pi/2}\sin(\lambda\tau_1)\cos(\lambda\tau_2)|010\rangle_{1,2,3} \\ &+ \mathrm{e}^{-\mathrm{i}\pi}\sin(\lambda\tau_1)\sin(\lambda\tau_2)|100\rangle_{1,2,3}] \\ &+ \mathrm{e}^{\mathrm{i}\theta_1}|1\rangle_p[\mathrm{e}^{-\mathrm{i}2\lambda\tau_2}\cos(\lambda\tau_1)|110\rangle_{1,2,3} \\ &+ \mathrm{e}^{-\mathrm{i}\pi/2}\sin(\lambda\tau_1)\cos(\lambda\tau_2)|101\rangle_{1,2,3} \\ &+ \mathrm{e}^{-\mathrm{i}\pi}\sin(\lambda\tau_1)\sin(\lambda\tau_2)|011\rangle_{1,2,3}]\big\}. \end{aligned} \quad (19)$$

Taking $a = \cos(\lambda\tau_1)$, $b = \sin(\lambda\tau_1)\cos(\lambda\tau_2)$, and $c = \sin(\lambda\tau_1)\sin(\lambda\tau_2)$, equation (19) can be expressed as

$$\begin{aligned}|\Psi\rangle^{(7)}_{p,1,2,3} = {}& \frac{1}{\sqrt{2}}\big\{\mathrm{e}^{-\mathrm{i}\theta_1}|0\rangle_p[a|001\rangle_{1,2,3} \\ &+ \mathrm{e}^{-\mathrm{i}\pi/2}b|010\rangle_{1,2,3} + \mathrm{e}^{-\mathrm{i}\pi}c|100\rangle_{1,2,3}] \\ &+ \mathrm{e}^{\mathrm{i}\theta_1}|1\rangle_p[\mathrm{e}^{-\mathrm{i}2\lambda\tau_2}a|110\rangle_{1,2,3} \\ &+ \mathrm{e}^{-\mathrm{i}\pi/2}b|101\rangle_{1,2,3} + \mathrm{e}^{-\mathrm{i}\pi}c|011\rangle_{1,2,3}]\big\}. \end{aligned} \quad (20)$$

Step 8 We apply three classical fields to atoms 1, 2 and 3 by exploring rotation operator which has been used in Step 2, and the rotated angles of atoms 1, 2 and 3 are $2\theta_2$, $2\theta_3$, $2\theta_4$, respectively. Equation (20) becomes

$$\begin{aligned}|\Psi\rangle^{(8)}_{p,1,2,3} = {}& \frac{1}{\sqrt{2}}\big\{\mathrm{e}^{-\mathrm{i}\theta_1}|0\rangle_p[\mathrm{e}^{-\mathrm{i}\theta_2}\,\mathrm{e}^{-\mathrm{i}\theta_3}\,\mathrm{e}^{-\mathrm{i}\theta_4}a|001\rangle_{1,2,3} \\ &+ \mathrm{e}^{-\mathrm{i}\theta_2}\,\mathrm{e}^{\mathrm{i}\theta_3}\,\mathrm{e}^{-\mathrm{i}\theta_4}\,\mathrm{e}^{-\mathrm{i}\pi/2}b|010\rangle_{1,2,3} \\ &+ \mathrm{e}^{\mathrm{i}\theta_2}\,\mathrm{e}^{-\mathrm{i}\theta_3}\,\mathrm{e}^{-\mathrm{i}\theta_4}\,\mathrm{e}^{-\mathrm{i}\pi}c|100\rangle_{1,2,3}] \\ &+ \mathrm{e}^{\mathrm{i}\theta_1}|1\rangle_p[\mathrm{e}^{\mathrm{i}\theta_2}\,\mathrm{e}^{\mathrm{i}\theta_3}\,\mathrm{e}^{-\mathrm{i}\theta_4}\,\mathrm{e}^{-\mathrm{i}2\lambda\tau_2}a|110\rangle_{1,2,3} \\ &+ \mathrm{e}^{\mathrm{i}\theta_2}\,\mathrm{e}^{-\mathrm{i}\theta_3}\,\mathrm{e}^{\mathrm{i}\theta_4}\,\mathrm{e}^{-\mathrm{i}\pi/2}b|101\rangle_{1,2,3} \\ &+ \mathrm{e}^{-\mathrm{i}\theta_2}\,\mathrm{e}^{\mathrm{i}\theta_3}\,\mathrm{e}^{\mathrm{i}\theta_4}\,\mathrm{e}^{-\mathrm{i}\pi}c|011\rangle_{1,2,3}]\big\}. \end{aligned} \quad (21)$$

In order to obtain the final state, the angles θ_1, θ_2, θ_3, and θ_4 should satisfy the following equations:

$$\begin{aligned}&\varphi_1 = -\theta_1 - \theta_2 - \theta_3 + \theta_4,\\ &\varphi_2 = -\theta_1 - \theta_2 + \theta_3 - \theta_4 - \frac{\pi}{2},\\ &\varphi_3 = -\theta_1 + \theta_2 - \theta_3 - \theta_4 - \pi,\\ &\varphi_4 = \theta_1 + \theta_2 + \theta_3 - \theta_4 - 2\lambda\tau_2,\\ &\varphi_5 = \theta_1 + \theta_2 - \theta_3 + \theta_4 - \frac{\pi}{2},\\ &\varphi_6 = \theta_1 - \theta_2 + \theta_3 + \theta_4 - \pi,\\ &\varphi_1 = \varphi_2 = \varphi_3 = \varphi_4 = \varphi_5 = \varphi_6. \end{aligned} \quad (22)$$

The solutions are

$$\begin{aligned}&\theta_2 = \frac{3\pi}{4} + \lambda\tau_2 - \theta_1,\quad \theta_3 = \frac{\pi}{2} + \lambda\tau_2 - \theta_1,\\ &\theta_4 = \frac{\pi}{4} + \lambda\tau_2 - \theta_1,\quad \varphi_1 = \pi + \lambda\tau_2. \end{aligned} \quad (23)$$

By inserting the solutions given by Eq. (22) into Eq. (21), and then discarding the whole phase factor $\mathrm{e}^{\pi+\lambda\tau_2}$, we can obtain the required state as follows:

$$\begin{aligned}|\Psi\rangle^{(\mathrm{final})}_{p,1,2,3} = {}& \frac{1}{\sqrt{2}}\Big\{|0\rangle_p\Big[a|001\rangle_{1,2,3} + b|010\rangle_{1,2,3} + c|100\rangle_{1,2,3}\Big] \\ &+|1\rangle_p\Big[a|110\rangle_{1,2,3} + |101\rangle_{1,2,3} + c|011\rangle_{1,2,3}\Big]\Big\}. \end{aligned} \quad (24)$$

Here, we complete our task as given by Eq. (11).

Next, Alice holds particle p, the other particles 1, 2, and 3 are distributed to her parties. When she wants to copy state $|\psi\rangle_0 = \big(|0\rangle + \mathrm{e}^{\mathrm{i}\varphi}|1\rangle\big)/\sqrt{2}$ with $\varphi \in [0, 2\pi)$ unknown, she will make the Bell state measurement on atoms 0 and p. The whole system can be decomposed as Eq. (5). Then she informs her partners of the measurement results. If the result is $|\phi^+\rangle_{0,p}$, then the subsystem is accurately projected into the optimal copies $(|\psi\rangle^{(0)}_{1,2,3} + \mathrm{e}^{\mathrm{i}\varphi}\,|\psi\rangle^{(1)}_{1,2,3})/\sqrt{2}$. Her partners can acquire their clones without doing anything. If the measurement result is one of $|\phi^-\rangle_{0,p}$, $|\psi^+\rangle_{0,p}$

and $|\psi^-\rangle_{0,p}$, the receivers can perform local operations defined as Eqs. (6)–(8) to obtain their clones, respectively.

Finally, we briefly discuss the feasibility of our scheme in experiment. To implement the scheme, we need to preserve the coherence of the cavity field before the atoms fly out of the cavity. In a recent cavity QED experiment with principal quantum numbers 49, 50 and 51, the coupling constant is $g = 2\pi \times 34$ MHz.[35] In order to control the entanglement in the cavity-assisted collision process, detuning δ should be much greater than g. If we choose $\delta = 10\ g$, the atom–cavity–field interaction time will be $\pi\delta/2g^2 = 7.4 \times 10^{-2}\ \mu$s. In the experiment,[36] a practical superconducting cavity has a typical Q value of about 10^9 if the cavity temperature is low enough, i.e. the cavity lifetime for a high-Q superconducting cavity can be as long as 0.01 s. We can find the atom–cavity–field interaction time to be shorter than efficient decay time $T_{\rm r} = 3 \times 10^{-2}$ s and photon lifetime 0.01 s in the present cavity. Bell states can be exactly distinguished via detecting the atomic state.[29] On the other hand, taking the atomic spontaneous emission into consideration, the Hamiltonian of the system of Eq. (10) can be rewritten as[37]

$$H_e = \lambda \left[\sum_{j=1}^{N} |e_j\rangle \langle e_j| + \sum_{j,k=1,j\neq k}^{N} S_j^+ S_k^- \right] - {\rm i}\frac{\varepsilon}{2} \sum_{j=1}^{N} |e_j\rangle \langle e_j|, \tag{25}$$

where ε is the atomic spontaneous emission rate. If we send the atoms 1 and 2 through the vacuum cavity, choose the interaction time $\lambda\tau = \pi/4$, and set $\varepsilon = 0.1\lambda$, the state of $|\Psi\rangle_{p,1,2,3}^{(1)}$ becomes

$$|\Psi\rangle_{p,1,2,3}^{(1')} = {\rm e}^{-\pi/20}\,{\rm e}^{-{\rm i}\pi/4}\frac{1}{\sqrt{2}}\left[|01\rangle_{p,1} + {\rm e}^{-{\rm i}\pi/2}\,|10\rangle_{p,1}\right]|00\rangle_{2,3}\,. \tag{26}$$

The fidelity of the state in Eq. (26) relative to the state in Eq. (12) is $[({\rm e}^{-\pi/20}\,{\rm e}^{-{\rm i}\pi/4} + {\rm e}^{-\pi/20}\,{\rm e}^{-{\rm i}3\pi/4})/({\rm e}^{-{\rm i}\pi/4} + {\rm e}^{-{\rm i}3\pi/4})]^2 \approx 0.9755$. In addition, our scheme requires two atoms to be simultaneously sent through a cavity; otherwise, there will be an error. Assume that during the generation of a two-atom cluster state one atom enters the cavity 0.01τ earlier than another atom, with τ being the time for each atom to stay in the cavity. We can obtain fidelity $f = 0.999$ for generating a two-atom entangled state. Obviously the effect of the errors on the fidelity of the resulting state is slight. Thus our scheme may be realized with the present cavity techniques.

4. Conclusion

We proposed an experimentally feasible scheme to implement the optimal asymmetric economical 1→3 phase-covariant telecloning, which works without ancilla, based on cavity QED. In the case of the input states on the equator, the telecloning can be completed with probability 100%. In the telecloning process, the cavity is only virtually excited, thus it greatly prolongs the efficient decoherent time. Our scheme is insensitive to cavity field states and cavity decay. Therefore, the scheme may be realized in experiment.

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