Unconventional geometric logic gate in a strong-driving-assisted multi-mode cavity*

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We propose a scheme to implement an unconventional geometric logic gate separately in a two-mode cavity and a multi-mode cavity assisted by a strong classical driving field. The effect of the cavity decay is included in the investigation. The numerical calculation is carried out, and the result shows that our scheme is more tolerant to cavity decay than the previous one because the time consumed for finishing the logic gate is doubly reduced.

Keywords: unconventional geometric gate, multi-mode cavity, cavity decay

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1. Introduction

In recent years, there has been growing interest in quantum computers, which are based on the fundamental quantum mechanical principle of coherent superposition and quantum entanglement. In information processing,[1−8] this kind of machine can solve certain practical problems exponentially faster than classical computers. The quantum logic gates are the basic building blocks of a quantum computer. To realize these quantum gates, some theoretical schemes[9] based on dynamic evolution have been proposed, which are very sensitive to the parameter fluctuations and local inaccuracies in operation, and a relevant experiment has been reported.[10] On the other hand, it has been recognized that geometric quantum computation is a promising approach to the implementation of built-in fault-tolerant quantum gates.[11] It is well known that the geometric quantum computation is based on geometric phase shifts,[12−14] it depends only on some global geometric features, insensitive to the path shape, the driving Hamiltonian, the starting state distributions and the dynamics of the evolution. Therefore, the geometric phase gates possess more advantages than the dynamic ones. For example, it is robust against dephasing and has higher fidelity in the quantum computation.

Geometric phase gates are classified into two kinds by the difference of the canceling of dynamic phase involved in the dynamical evolution: the conventional geometric quantum gate (CGGs)[15,16] and the unconventional geometric quantum gate (UGGs).[17,18] Generally, the CGGs require additional operations to cancel the dynamical phase which may result in additional errors. However, the dynamic phase is proportional to the geometric phase in the process of the UGG, one can implement fault-tolerant quantum computation by using the total phase accumulated in the evolution, so it is unnecessary to eliminate it and simplify the experiment operations. The experiment has been done with trapped ions.[19]

For implementation of UGG, Chen *et al.*[20] proposed a potential scheme on two identical three-level atoms[21] in a cavity, strongly driven by a resonant classical field. But the influence of cavity decay is inevitable in practical operation, it brings us a serious challenge for the experimental realization. In order for the scheme to work efficiently, the increase of speed and toleration against decoherence of the UGG becomes another central problem. In this paper, we generalize the previous scheme[20] to carrying out the UGG in multi-mode cavity. In Chen's cavity-QED scheme, only one cavity mode is displaced along a closed path conditional on the atomic states. Accord-

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ing to the idea of creating the UGG,[17,18] in which a harmonic oscillator makes displacement along a closed path conditional on the state of the qubits, the phase accumulation is only the contribution of one cavity mode in Chen's cavity-QED scheme. In our scheme, by using two or more modes, the time consumed for realizing a certain phase gate is sharply reduced due to contributions of more modes. Through analytically investigation, we find that the scheme is more efficient and more robust against decoherence and more tolerant to the cavity decay. The fidelity and probability are higher than those of the previous one.[20]

The outline of this paper is as follows. In Section 2, the two-mode UGG is realized by means of the system of two atoms interacting with the quantized two-mode cavity field. In the cavity decay case, the specific calculations are done to show higher fidelity F and success probability P in our scheme. The multimode UGG is introduced in Section 3, then Section 4 is dedicated to the discussion of the feasibility of current experiment and our scheme.

2. Two-qubit logical gate in two-mode cavity field assisted by classical driving field

In the ideal case, we consider two identical cascade three-level atoms interacting with a two-mode field in the cavity, each of the atoms has two ground states $|g\rangle$ and $|e\rangle$ and one excited state $|i\rangle$. The transition $|e\rangle \to |i\rangle$ is driven by two quasiresonant normal modes in the cavity, assisted always by a strong external classical field. The associated Hamiltonian reads (assuming $\hbar = 1$)

$$\begin{aligned} H =& \omega_0 \sum_{j=1}^{2} S_{z,j} + \sum_{k=1}^{2} \omega_k a_k^{\dagger} a_k \\ &+ \sum_{j=1}^{2}\sum_{k=1}^{2} g_k (a_k^{\dagger} S_j^{-} + a_k S_j^{\dagger}) \\ &+ \Omega \sum_{j=1}^{2} (\mathrm{e}^{-\mathrm{i}\omega_L t} S_j^{\dagger} + \mathrm{e}^{\mathrm{i}\omega_L t} S_j^{-}), \end{aligned} \tag{1}$$

where ω_0, ω_k $(k = 1, 2)$ and ω_L are the frequencies of the resonant transition of atomic states $|e\rangle \to |i\rangle$, of the two cavity modes, and of the classical driving field; a_k and $a_k^{\dagger}$ $(k = 1, 2)$ are the annihilation operator and the creation operator, both associated with the intracavity fields, respectively;

$$\begin{aligned} &S_{z,j} = (|i\rangle\langle i| - |e\rangle\langle e|)/2\,, \\ &S_j^{\dagger} = |i\rangle\langle e|\,, \quad \text{and} \quad S_j^{-} = |e\rangle\langle i|\,, \quad (j = 1, 2) \end{aligned}$$

are the spin-flip operators between the lower state and the upper state; g_k $(k = 1, 2)$ and Ω are the coupling constants of the interaction of each atom with two cavity modes and with the driving field respectively. The Hamiltonian changed into a reference frame rotating with the driving field frequency ω_L is expressed as

$$\begin{aligned} H_{\mathrm{rot}} =& \sum_{k=1}^{2} \delta_k a_k^{\dagger} a_k + \Omega \sum_{j=1}^{2} (S_j^{\dagger} + S_j^{-}) \\ &+ \sum_{j=1}^{2}\sum_{k=1}^{2} g_k (a_k^{\dagger} S_j^{-} + a_k S_j^{\dagger}), \end{aligned} \tag{2}$$

where $\delta_k = \omega_k - \omega_L$ $(k = 1, 2)$, and $\omega_L = \omega_0$ is assumed in the above transformation. We now define

$$H_{\mathrm{rot}} = H_0 + H_{\mathrm{int}}$$

with

$$H_0 = \sum_{k=1}^{2} \delta_k a_k^{\dagger} a_k + \Omega \sum_{j=1}^{2} (S_j^{\dagger} + S_j^{-}), \tag{3}$$

$$H_{\mathrm{int}} = \sum_{j=1}^{2}\sum_{k=1}^{2} g_k (a_k^{\dagger} S_j^{-} + a_k S_j^{\dagger}). \tag{4}$$

Transforming H_{rot} into the interaction picture according to the above definition yields

$$H'_{\mathrm{int}} = \frac{1}{2} \sum_{j=1}^{2}\sum_{k=1}^{2} (|+\rangle_j\,{}_j\langle +| - |-\rangle_j\,{}_j\langle -| + \mathrm{e}^{2\mathrm{i}\Omega t} |+\rangle_j\,{}_j\langle -| - \mathrm{e}^{-2\mathrm{i}\Omega t} |-\rangle_j\,{}_j\langle +|) g_k a_k \mathrm{e}^{-\mathrm{i}\delta_k t} + \mathrm{H.c.}, \tag{5}$$

where $|\pm\rangle_j = (|e\rangle_j \pm |i\rangle_j)/\sqrt{2}$ are eigenstates of $S_j^x = S_j^{+} + S_j^{-}$ with eigenvalues ± 1, in the driving limit, $\Omega \gg \{g_1, g_2, \delta_1, \delta_2\}$, under the rotating-wave approximation, the terms with high frequencies can be eliminated, thus the effective Hamiltonian is given by

$$H_{\mathrm{int}}^{\mathrm{eff}} = \frac{1}{2} \sum_{j=1}^{2}\sum_{k=1}^{2} g_k (|+\rangle_j\,{}_j\langle +| - |-\rangle_j\,{}_j\langle -|)(a_k \mathrm{e}^{-\mathrm{i}\delta_k t} + a_k^{\dagger} \mathrm{e}^{\mathrm{i}\delta_k t}). \tag{6}$$

We encode the information in the qubit states $|g\rangle_j$ and $|-\rangle_j$, thus, the Hamiltonian that contributes the unconventional geometric phase gate is described by

$$\tilde{H}_{\rm int}^{\rm eff} = -\frac{1}{2}\sum_{j=1}^{2}\sum_{k=1}^{2} g_k(a_k\,{\rm e}^{-{\rm i}\delta_k t} + a_k^{\dagger}\,{\rm e}^{{\rm i}\delta_k t})\,|-\rangle_j\,{}_j\langle -|\,. \tag{7}$$

Here, because the states $|+\rangle_j$ are orthogonal to the states $|g\rangle_j$ and $|-\rangle_j$, they are not included in the computational subspace spanned by $|g\rangle_j$ and $|-\rangle_j$.

Using $\tilde{H}_{\rm int}^{\rm eff}$, the displacement operator is induced by the interaction of the atom states with two cavity modes field during the infinitesimal interval $[t, t+{\rm d}t]$ as shown in Ref. [20], and expressed as

$$U({\rm d}t) = D({\rm d}\alpha_1)D({\rm d}\alpha_2) = {\rm e}^{a_1^{\dagger}{\rm d}\alpha_1 - a_1{\rm d}\alpha_1^*}\,{\rm e}^{a_2^{\dagger}{\rm d}\alpha_2 - a_2{\rm d}\alpha_2^*} = {\rm e}^{-{\rm i}\tilde{H}_{\rm int}^{\rm eff}{\rm d}t}, \tag{8}$$

where

$${\rm d}\alpha_1 = {\rm i}\frac{g_1}{2}\,{\rm e}^{{\rm i}\delta_1 t}{\rm d}t,$$

and

$${\rm d}\alpha_2 = {\rm i}\frac{g_2}{2}\,{\rm e}^{{\rm i}\delta_2 t}{\rm d}t, \tag{9}$$

in which only the states $|-\rangle_j$ are evolved. In terms of the above equation, if l ($l = 1, 2, 3, \ldots, N$) states $|-\rangle_j$ are included in the qubit states, the displacement operator evolves into $D(l{\rm d}\alpha_1)D(l{\rm d}\alpha_2)$, but the states $|g\rangle_j$ will be unchanged. After an interval t', the displacement parameter satisfies the following relation:

$$\alpha_1(t') = \int_0^{t'} {\rm i}\frac{g_1}{2}\,{\rm e}^{{\rm i}\delta_1 t}{\rm d}t = \frac{g_1}{2\delta_1}({\rm e}^{{\rm i}\delta_1 t'} - 1),$$

and

$$\alpha_2(t') = \int_0^{t'} {\rm i}\frac{g_2}{2}\,{\rm e}^{{\rm i}\delta_2 t}{\rm d}t = \frac{g_2}{2\delta_2}({\rm e}^{{\rm i}\delta_2 t'} - 1). \tag{10}$$

According to the definition of the displacement operator[22,23] and based on the above formula, the total geometric phase shifts resulting from the interaction of the atomic state $|-\rangle_j$ or states $l\,|-\rangle_j$ ($l = 1, 2, 3, \ldots, N$) with two modes cavity fields are given, respectively, by

$$\Phi_{\rm tol} = \Phi_1 + \Phi_2, \tag{11}$$

and

$$\Phi_{\rm tol}^{l} = l^2\Phi_1 + l^2\Phi_2, \tag{12}$$

where

$$\Phi_1 = {\rm Im}\left(\int_c \alpha_1^*{\rm d}\alpha_1\right) = \frac{g_1}{4\delta_1^2}\left[\delta_1 t - \sin(\delta_1 t)\right],$$

and

$$\Phi_2 = {\rm Im}\left(\int_c \alpha_2^*{\rm d}\alpha_2\right) = \frac{g_2}{4\delta_2^2}\left[\delta_2 t - \sin(\delta_2 t)\right]. \tag{13}$$

Assuming that the cavity modes are initially in the vacuum states; then after a time interval $T = 2n_1\pi/\delta_1 = 2n_2\pi/\delta_2$ (n_1 and n_2 are positive integer), the cavity modes evolve to its initial state and complete a closed path in the phase space, thus the total phase is produced in the cyclic evolution process, $U(T) = {\rm e}^{{\rm i}\Phi_1}D(\alpha_1(T))\,{\rm e}^{{\rm i}\Phi_2}D(\alpha_2(T)) = {\rm e}^{{\rm i}\Phi_1+{\rm i}\Phi_2}D(0)D(0) = {\rm e}^{{\rm i}\Phi_1+{\rm i}\Phi_2}$. For the specific qubit states, we can obtain

$$\begin{aligned}
&|g\rangle_1\,|g\rangle_2\,|0\rangle_1\,|0\rangle_2\\
\rightarrow\ &|g\rangle_1\,|g\rangle_2\,|0\rangle_1\,|0\rangle_2\,,\\
&|g\rangle_1\,|-\rangle_2\,|0\rangle_1\,|0\rangle_2\\
\rightarrow\ &{\rm e}^{-{\rm i}\Omega T}\,{\rm e}^{{\rm i}\Phi_1+{\rm i}\Phi_2}\,|g\rangle_1\,|-\rangle_2\,|0\rangle_1\,|0\rangle_2\,,\\
&|-\rangle_1\,|g\rangle_2\,|0\rangle_1\,|0\rangle_2\\
\rightarrow\ &{\rm e}^{-{\rm i}\Omega T}\,{\rm e}^{{\rm i}\Phi_1+{\rm i}\Phi_2}\,|-\rangle_1\,|g\rangle_2\,|0\rangle_1\,|0\rangle_2\,,\\
&|-\rangle_1\,|-\rangle_2\,|0\rangle_1\,|0\rangle_2\\
\rightarrow\ &{\rm e}^{-{\rm i}2\Omega T}\,{\rm e}^{{\rm i}4\Phi_1+{\rm i}4\Phi_2}\,|-\rangle_1\,|-\rangle_2\,|0\rangle_1\,|0\rangle_2\,.
\end{aligned} \tag{14}$$

Here, the effect coming from the H_0 is considered, $|0\rangle_1\,|0\rangle_2$ is the initial cavity field vacuum state with the subscripts 1 and 2 representing cavity modes, after performing the one-qubit operation $|-\rangle_j \rightarrow {\rm e}^{{\rm i}\Omega T}\,{\rm e}^{-{\rm i}\Phi_1-{\rm i}\Phi_2}\,|-\rangle_j$ to the corresponding state, thereby leading to

$$\begin{aligned}
&|g\rangle_1\,|g\rangle_2\,|0\rangle_1\,|0\rangle_2 \rightarrow |g\rangle_1\,|g\rangle_2\,|0\rangle_1\,|0\rangle_2\,,\\
&|g\rangle_1\,|-\rangle_2\,|0\rangle_1\,|0\rangle_2 \rightarrow |g\rangle_1\,|-\rangle_2\,|0\rangle_1\,|0\rangle_2\,,\\
&|-\rangle_1\,|g\rangle_2\,|0\rangle_1\,|0\rangle_2 \rightarrow |-\rangle_1\,|g\rangle_2\,|0\rangle_1\,|0\rangle_2\,,
\end{aligned}$$

$$\begin{aligned}
&|-\rangle_1\,|-\rangle_2\,|0\rangle_1\,|0\rangle_2\\
\rightarrow\ &{\rm e}^{{\rm i}2(\Phi_1+\Phi_2)}\,|-\rangle_1\,|-\rangle_2\,|0\rangle_1\,|0\rangle_2\\
\rightarrow\ &{\rm e}^{{\rm i}2\Phi_{\rm tol}}\,|-\rangle_1\,|-\rangle_2\,|0\rangle_1\,|0\rangle_2\,,
\end{aligned} \tag{15}$$

where the condition of the closed-path evolution for the cavity mode, i.e., $\alpha_1(T) = \alpha_2(T) = 0$, has been

utilized. By substituting $T = 2n_1\pi/\delta_1 = 2n_2\pi/\delta_2$ into Eq. (11), we can express the total phase shift $\Phi_{\rm tol}$ as

$$\Phi_{\rm tol} = \frac{g_1^2 n_1 \pi}{2\delta_1^2} + \frac{g_2^2 n_2 \pi}{2\delta_2^2} \tag{16}$$

to realize a π-phase gate. Choose $2\Phi_{\rm tol} = \pi$, in practice, effective coupling constants g_1 and g_2 and detunings δ_1 and δ_2 that can be controlled by adjusting the driving light field, and appropriate integers n_1 and n_2, then the cyclic evolution condition and π-phase can be achieved. From the above discussion, it is easy to see that the π-phase is the contribution of two cavity modes simultaneously interacting with the atom, compared with the preceding scheme[20] in which there is only one cavity mode's contribution, so the time is reduced as much as that in the present case, thus the fidelity of the quantum gate operation with cavity decay can be largely increased, we will show this advantage in the next section.

A more realistic model of the cavity modes would include its interaction with a dissipative environment where the dissipation cannot be neglected; the evolution of the system is governed by the following non-Hermitian Hamiltonian due to its coupling to the reservoir:

$$\begin{aligned} H' =& \omega_0 \sum_{j=1}^{2} S_{z,j} + \sum_{k=1}^{2} \omega_k a_k^\dagger a_k \\ &+ \sum_{j=1}^{2}\sum_{k=1}^{2} g_k (a_k^\dagger S_j^- + a_k S_j^+) \\ &+ \Omega \sum_{j=1}^{2} (\,{\rm e}^{-{\rm i}\omega_L t} S_j^+ + {\rm e}^{{\rm i}\omega_L t} S_j^-) \\ &- {\rm i} \sum_{k=1}^{2} \frac{\kappa_k}{2} a_k^\dagger a_k, \end{aligned} \tag{17}$$

where κ_k $(k = 1, 2)$ is the cavity decay rate, by considering very weak decay, i.e., $\kappa_k t \ll \delta_k t$, using the above-mentioned method in the ideal case and assuming the cavity fields to be initially in the vacuum state, we can derive the effective Hamiltonian in the interaction picture,

$$H'^{\rm eff}_{\rm int} = \frac{1}{2}\sum_{j=1}^{2}\sum_{k=1}^{2} g_k (a_k\,{\rm e}^{-{\rm i}\delta_k t}\,{\rm e}^{-(\kappa_k/2)t} + a_k^\dagger\,{\rm e}^{{\rm i}\delta_k t}\,{\rm e}^{(\kappa_k/2)t})(|+\rangle_{j\;j}\langle +| - |-\rangle_{j\;j}\langle -|). \tag{18}$$

Similarly, the information is still encoded in states $|g\rangle_j$ and $|-\rangle_j$, so the Hamiltonian can be further simplified into

$$\tilde{H}'^{\rm eff}_{\rm int} = -\frac{1}{2}\sum_{j=1}^{2}\sum_{k=1}^{2} g_k (a_k\,{\rm e}^{-{\rm i}\delta_k t}\,{\rm e}^{-(\kappa_k/2)t} + a_k^\dagger\,{\rm e}^{{\rm i}\delta_k t}\,{\rm e}^{(\kappa_k/2)t})\,|-\rangle_{j\;j}\langle -|\,. \tag{19}$$

From the definition of the displacement-like operator in Ref. [20], during the infinitesimal interval $[t, t + {\rm d}t]$, the evolution operator in our model yields

$$U'({\rm d}t) = D'({\rm d}\alpha'_1)D'({\rm d}\alpha'_2) = {\rm e}^{a_1^\dagger {\rm d}\alpha'_1(t) - a_1 {\rm d}\alpha'_1(-t)}\,{\rm e}^{a_2^\dagger {\rm d}\alpha'_2(t) - a_2 {\rm d}\alpha'_2(-t)} = {\rm e}^{-{\rm i}\tilde{H}'^{\rm eff}_{\rm int}{\rm d}t}, \tag{20}$$

where

$${\rm d}\alpha'_1(t) = {\rm i}\frac{g_1}{2}\,{\rm e}^{{\rm i}\delta_1 t}\,{\rm e}^{\kappa_1 t/2}{\rm d}t, \quad \text{and} \quad {\rm d}\alpha'_2(t) = {\rm i}\frac{g_2}{2}\,{\rm e}^{{\rm i}\delta_2 t}\,{\rm e}^{\kappa_2 t/2}{\rm d}t. \tag{21}$$

Suppose that the cavity field is initially still in vacuum state $|0\rangle_j$, then after an interaction time t' the evolution operator of the cavity modes will be

$$\begin{aligned} &|g\rangle_1\,|g\rangle_2\,|0\rangle_1\,|0\rangle_2 \to |g\rangle_1\,|g\rangle_2\,|0\rangle_1\,|0\rangle_2\,, \\ &|g\rangle_1\,|-\rangle_2\,|0\rangle_1\,|0\rangle_2 \to {\rm e}^{-{\rm i}\Omega t'}\,{\rm e}^{-R_1-R_2}\,{\rm e}^{{\rm i}\Phi'_1+{\rm i}\Phi'_2} D(\alpha_1(t'))D(\alpha_2(t'))\,|g\rangle_1\,|-\rangle_2\,|0\rangle_1\,|0\rangle_2\,, \\ &|-\rangle_1\,|g\rangle_2\,|0\rangle_1\,|0\rangle_2 \to {\rm e}^{-{\rm i}\Omega t'}\,{\rm e}^{-R_1-R_2}\,{\rm e}^{{\rm i}\Phi'_1+{\rm i}\Phi'_2} D(\alpha_1(t'))D(\alpha_2(t'))\,|-\rangle_1\,|g\rangle_2\,|0\rangle_1\,|0\rangle_2\,, \\ &|-\rangle_1\,|-\rangle_2\,|0\rangle_1\,|0\rangle_2 \to {\rm e}^{-{\rm i}2\Omega t'}\,{\rm e}^{-4R_1-4R_2}\,{\rm e}^{{\rm i}4\Phi'_1+{\rm i}4\Phi'_2} D(2\alpha_1(t'))D(2\alpha_2(t'))\,|-\rangle_1\,|-\rangle_2\,|0\rangle_1\,|0\rangle_2\,, \end{aligned} \tag{22}$$

where

$$a'_k(t') = {\rm i}\int_0^{t'} \frac{g_k}{2}\,{\rm e}^{{\rm i}\delta_k t}\,{\rm e}^{\kappa_k t/2}{\rm d}t = \frac{2g_k\delta_k + {\rm i}g_k\kappa_k}{4\delta_k^2 + \kappa_k^2}\left[{\rm e}^{{\rm i}\delta_k t'}\,{\rm e}^{\kappa_k t'/2} - 1\right], \quad (k = 1, 2), \tag{23}$$

$$\begin{aligned}\Phi'_k &= \frac{1}{2}\mathrm{Im}\left(\int_c \alpha'_k(-t)\mathrm{d}\alpha'_k(t) - \int_c \alpha'_k(t)\mathrm{d}\alpha'_k(-t)\right)\\ &= \frac{g_k^2(4\delta_k-\kappa_k^2)}{(4\delta_k^2+\kappa_k^2)^2}\left[\delta_k t - \sin(\delta_k t)\mathrm{ch}\left(\frac{\kappa_k t}{2}\right)\right] + \frac{4g_k^2\delta_k\kappa_k}{(4\delta_k^2+\kappa_k^2)^2}\left[\frac{k_k t}{2} - \cos(\delta_k t)\mathrm{sh}\left(\frac{k_k t}{2}\right)\right], \qquad (24)\\ R_k &= -\frac{1}{2}\mathrm{Re}\left(\int_c \alpha'_k(-t)\mathrm{d}\alpha'_k(t) - \int_c \alpha'_k(t)\mathrm{d}\alpha'_k(-t)\right)\\ &= \frac{g_k^2(4\delta_k-\kappa_k^2)}{(4\delta_k^2+\kappa_k^2)^2}\left[\cos(\delta_k t)\mathrm{sh}\left(\frac{k_k t}{2}\right) - \frac{k_k t}{2}\right] + \frac{4g_k^2\delta_k\kappa_k}{(4\delta_k^2+\kappa_k^2)^2}\left[\delta_k t - \sin(\delta_k t)\mathrm{ch}\left(\frac{\kappa_k t}{2}\right)\right] \quad (k=1,2). \qquad (25)\end{aligned}$$

When the system evolves along the closed path, the following relations must be satisfied:

$$\delta_1 t = 2n_1\pi, \quad \text{and} \quad \delta_2 t = 2n_2\pi, \qquad (26)$$

where n_1 and n_2 are the positive integers, thus the evolution can be found as follows:

$$\begin{aligned}&|g\rangle_1|g\rangle_2|0\rangle_1|0\rangle_2 \to |g\rangle_1|g\rangle_2|0\rangle_1|0\rangle_2,\\ &|g\rangle_1|-\rangle_2|0\rangle_1|0\rangle_2 \to \mathrm{e}^{-\mathrm{i}\Omega\tau}e^{-R_1-R_2}\,\mathrm{e}^{\mathrm{i}\Phi'_1+\mathrm{i}\Phi'_2}|g\rangle_1|-\rangle_2|0\rangle_1|0\rangle_2,\\ &|-\rangle_1|g\rangle_2|0\rangle_1|0\rangle_2 \to \mathrm{e}^{-\mathrm{i}\Omega\tau}\,\mathrm{e}^{-R_1-R_2}\,\mathrm{e}^{\mathrm{i}\Phi'_1+\mathrm{i}\Phi'_2}|-\rangle_1|g\rangle_2|0\rangle_1|0\rangle_2,\\ &|-\rangle_1|-\rangle_2|0\rangle_1|0\rangle_2 \to \mathrm{e}^{-\mathrm{i}2\Omega T}\,\mathrm{e}^{-4R_1-4R_2}\,\mathrm{e}^{\mathrm{i}4\Phi'_1+\mathrm{i}4\Phi'_2}|-\rangle_1|-\rangle_2|0\rangle_1|0\rangle_2. \qquad (27)\end{aligned}$$

Then, a single-bit operation $|-\rangle_j \to \mathrm{e}^{-\mathrm{i}(\Phi'_1+\Phi'_2-\Omega t)}|-\rangle_j$ $(j=1,2\ )$ is performed, and the evolution is

$$\begin{aligned}&|g\rangle_1|g\rangle_2|0\rangle_1|0\rangle_2 \to |g\rangle_1|g\rangle_2|0\rangle_1|0\rangle_2,\\ &|g\rangle_1|-\rangle_2|0\rangle_1|0\rangle_2 \to \mathrm{e}^{-R_1-R_2}|g\rangle_1|-\rangle_2|0\rangle_1|0\rangle_2,\\ &|-\rangle_1|g\rangle_2|0\rangle_a|0\rangle_b \to \mathrm{e}^{-R_1-R_2}|-\rangle_1|g\rangle_2|0\rangle_1|0\rangle_2,\\ &|-\rangle_1|-\rangle_2|0\rangle_1|0\rangle_2 \to \mathrm{e}^{-4R_1-4R_2}\,\mathrm{e}^{\mathrm{i}2\Phi'_1+\mathrm{i}2\Phi'_2}|-\rangle_1|-\rangle_2|0\rangle_1|0\rangle_2 \qquad (28)\end{aligned}$$

with $2\Phi'_1+2\Phi'_2=\pi$. The $\pi-$phase gate is produced, and the influence of the cavity decay has been considered. In the following, we compare fidelity F and success probability P preparing an entangled state in our scheme with those in the preceding scheme in Ref. [20]. In a way similar to that in Ref. [20], by supposing the initial state $|\phi_0\rangle = (|g\rangle_1+|-\rangle_1)/\sqrt{2}\otimes(|g\rangle_2+|-\rangle_2)/\sqrt{2}\otimes|0\rangle_1|0\rangle_2$, we deduce the corresponding fidelity and the success probability in carrying out the above-mentioned phase gate, separately. For the sake of simplicity, assuming $\delta_1=\delta_2=\delta$, $g_1=g_2=\delta$, $n_1=n_2=n$, $\kappa_1=\kappa_2=\kappa$, then $R_1=R_2=R$. Thus we derive $F=|\langle\phi_0|\phi\rangle|^2=(1+2\,\mathrm{e}^{-2R}+\mathrm{e}^{-8R})^2/4(1+2\,\mathrm{e}^{-4R}+\mathrm{e}^{-16R})$ and $P=(1+\mathrm{e}^{-4R}+\mathrm{e}^{-16R})/4$.

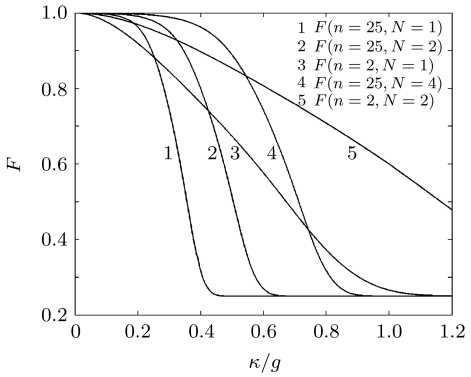


Fig. 1. Curves for fidelity F of the entangled state produced by using the phase gate versus decay κ/g for one mode ($N=1$), two modes ($N=2$) and four modes ($N=4$) cavity with $n=2$ and 25.

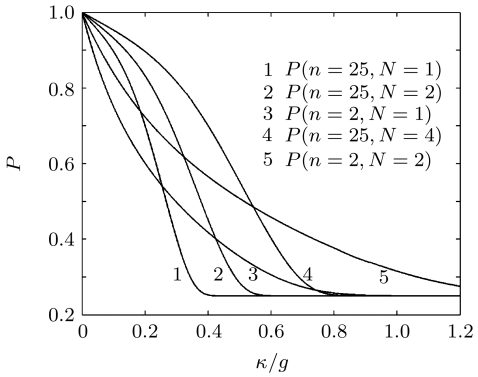


Fig. 2. Curves for success probability P of generating entangled state versus decay κ/g for one mode ($N=1$), two modes ($N=2$) and four modes ($N=4$) cavity with $n=2$ and 25.

In Figs. 1 and 2, we plot the fidelity and the success probability of generating entangled state each as a function of κ/g for different values of constant n ($n = 2$, 25) and different modes ($N = 1, 2$). The results show that when the value of κ/g is sufficiently small the fidelity and the probability each can keep a reasonable value, but when the value of κ/g increases, both of them sharply decrease. However, it is clear that fidelity F and success probability P of the phase gate proposed here diminish more slowly with the increase of decay than the previous one.[20] Thus it is more tolerant to decay in our logic gate than in the gate given in Ref. [20], which is very useful for realizing the quantum computation in practical physical modes which interact inevitably with the environment. Theoretically, we give a brief explanation about the more tolerance of decay. According to the definition of UGG, when the cavity mode is displaced along a closed path depending on the atomic states, a phase gate is produced. In our scheme, the phase is composed of two parts accumulated by two modes, so that the time consumed for realizing a certain phase gate is doubly reduced. Hence, the tolerance is improved.

3. Unconventional geometric logic gate in N-mode cavity

In the following, we consider the UGG in the multi-mode cavity assisted by a strong driving field. In the case of N modes, we can deduce the effective Hamiltonian of two atoms interacting with cavity fields as follows:

$$\tilde{H}'^{N\text{eff}}_{\text{int}} = -\frac{1}{2}\sum_{j=1}^{2}\sum_{k=1}^{N}\left[g_k\left(a_k\,\mathrm{e}^{-\mathrm{i}\delta_k t}\,\mathrm{e}^{-(\kappa_k/2)t} + a_k^{\dagger}\,\mathrm{e}^{\mathrm{i}\delta_k t}\,\mathrm{e}^{(\kappa_k/2)t}\right)\right]\Big| - \Big\rangle_j\,{}_j\langle -|\,, \tag{29}$$

where each cavity mode decay has been included with decay rate κ_k ($k = 1, 2\ldots, N$); g_k and δ_k are the corresponding coupling constant and the detuning; a_k and $a_k^{\dagger}$ are the annihilation operator and the creation operator of the k-th cavity, respectively. Following the derivation in Section 2, we can write the state evolution of the system governed by $\tilde{H}'^{N\text{eff}}_{\text{int}}$ as follows:

$$\begin{aligned}
&|g\rangle_1|g\rangle_2|0\rangle_1|0\rangle_2\cdots|0\rangle_N \to |g\rangle_1|g\rangle_2|0\rangle_1|0\rangle_2\cdots|0\rangle_N\,,\\
&|g\rangle_1|-\rangle_2|0\rangle_1|0\rangle_2\cdots|0\rangle_N \to \mathrm{e}^{-\sum_{k=1}^{N}R_k}|g\rangle_1|-\rangle_2|0\rangle_1|0\rangle_2\ldots|0\rangle_N\,,\\
&|-\rangle_1|g\rangle_2|0\rangle_1|0\rangle_2\cdots|0\rangle_N \to \mathrm{e}^{-\sum_{k=1}^{N}R_k}|-\rangle_1|g\rangle_2|0\rangle_1|0\rangle_2\cdots|0\rangle_N\,,\\
&|-\rangle_1|-\rangle_2|0\rangle_1|0\rangle_2\cdots|0\rangle_N \to \mathrm{e}^{-4\sum_{k=1}^{N}R_k}\,\mathrm{e}^{\mathrm{i}2\sum_{k=1}^{N}\Phi'_k}|-\rangle_1|-\rangle_2|0\rangle_1|0\rangle_2\cdots|0\rangle_N\,,
\end{aligned} \tag{30}$$

where Φ'_k and R_k are the same as those in Eqs. (24) and (25) respectively. To achieve a π-phase, condition $\sum_{k=1}^{N}\Phi'_k = \pi/2$ is satisfied. For comparison, we plot fidelity F (in Fig. 1) and success probability P (in Fig. 2) of the UGG in the cavity for $N = 4$, assuming $\delta_1 = \delta_2 = \delta_3 = \delta_4 = \delta$, $g_1 = g_2 = g_3 = g_4 = g$, $n_1 = n_2 = n_3 = n_4 = n$, and $\kappa_1 = \kappa_2 = \kappa_3 = \kappa_4 = \kappa$. It is found that both F and P are improved with the increase of the number of cavity modes. That is to say, the UGG possesses more decay-tolerances, owing to multi-mode contribution to the phase accumulation.

It is necessary to give a brief discussion on the experiment. For the Rydberg atom with principal quantum numbers 49, 50 and 51, the radiation time is $T_{\text{r}} = 3\times10^{-2}$ s. In the microwave cavity, the coupling constant of the cavity and the atom is $g = 2\pi\times 49$ kHz, the photon lifetime achieved is $T_c = 3.0\times10^{-3}$ s.[9] For the ideal two-mode cavity, the gating time is $t = 1.414\times10^{-5}$ s, much smaller than the lifetimes of the atom and the photon. It is clear that the time is shorter in multi-mode cavity, so that the UGG is more robust against the cavity decay in N-mode cavity.

4. Conclusions

We proposed a scheme to realize unconventional geometric two-qubit phase gates with atoms separately in a strong-driving-assisted two-mode cavity and a multi-mode cavity. Atoms interact simultaneously with multi-mode cavity fields and a classical field. The effect of the cavity decay is included in the investigation. Through the numerical calculation, it is found that the evolution time of the gate is less than that of the gate proposed in

Ref. [20], so our scheme is more tolerant to cavity decay than the previous one. With the developments of the technology of the multi-mode cavity and the application of strong-driving-assisted field, the scheme will have potential significance in the future.

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