Growth of linear matter perturbations and current observational constraints*

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We discuss, using a recently proposed parametrization for the growth index of linear matter perturbations, the observational constraints on the wCDM model and the Dvali–Gabadadze–Porrati (DGP) model with all current growth factor data. We find that the wCDM model is allowed by the observations at the 1σ confidence level, while the DGP model is only consistent with observations at 2σ confidence level.

Keywords: dark energy, DGP model, matter perturbations
PACC: 9880, 9880B

1. Introduction

The growing observational evidences[1−9] show that the present expansion of our universe is accelerating. Within the framework of general relativity, if the universe is homogeneous and isotropic, the existence of an exotic energy with sufficient negative pressure, named dark energy, is required in our universe in order to explain this observed phenomenon. The simplest candidate of dark energy is the cosmological constant.[10−13] However, it suffers from two serious theoretical problems, i.e., why is the inferred value of the cosmological constant so tiny (120 orders of magnitude lower) compared to the typical vacuum energy values predicted by particle physics? and why is its energy density comparable to the matter density right now? So some other models are proposed, such as the quintessence,[14−18] phantom,[19−21] quintom,[22−24] holographic[25−28] and agegraphic[29−31] models, etc.

Modifying gravity is another way to explain the present comic accelerating expansion. Models based upon modified gravity, for example, the Dvali–Gabadadze–Porrati (DGP) brane-world model,[32−34] originate from the idea that our understanding of gravity is incorrect in the cosmic scale and the general relativity needs to be modified. If the effective equation of state of a modified gravity model is the same as the equation of state of a dark energy model, these two different models will share the same late time cosmological expansion according to the Friedmann equation, although they have a different theoretical basis. Therefore, an important task is to discriminate them to determine which one correctly describes the whole evolution of the universe. Recently, some attempts have been made[35−42] in this regard.

Since different models may produce different growths of matter perturbations[43] while they give the same late time expansion, the growth function $\delta(z) \equiv \delta\rho_{\rm m}/\rho_{\rm m}$ of matter perturbations[43−78] provides a particularly effective method for discriminating them. Here $\rho_{\rm m}$ is the energy density of matter. To the linear order of matter density perturbations and on subhorizon scales, $\delta(z)$ satisfies the following equation

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G_{\rm eff}\,\rho_{\rm m}\delta = 0, \tag{1}$$

where $G_{\rm eff}$ is an effective Newton gravity constant and the dot denotes the derivative with respect to the time t. Defining the growth factor $f \equiv {\rm d}\ln\delta/{\rm d}\ln a$, the above equation can be expressed as

$$\frac{{\rm d}f}{{\rm d}\ln a} + f^2 + \left(\frac{\dot{H}}{H^2} + 2\right) f = \frac{3}{2}\frac{G_{\rm eff}}{G_{\rm N}}\,\Omega_{\rm m}, \tag{2}$$

where $\Omega_{\rm m}$ is the fractional energy density of matter. In general, it is hard to obtain an analytical solution for Eq. (2), and we need to solve it numerically. It is interesting to note that this growth factor can be

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parameterized as[79−81]

$$f = \Omega_{\rm m}^{\gamma}, \tag{3}$$

where γ is called the growth index. Since $\Omega_{\rm m} \simeq 1$ at $z \gg 1$, one can easily see that $f = 1$ at the high redshift, which provides an initial condition for the numerical solution of Eq. (2). Substituting the above expression into Eq. (2), one can obtain the equation for γ

$$\ln \Omega_{\rm m} \frac{{\rm d}\gamma}{{\rm d}\ln a} + \gamma \frac{{\rm d}\ln \Omega_{\rm m}}{{\rm d}\ln a} + \Omega_{\rm m}^{\gamma} + \left(\frac{\dot{H}}{H^2} + 2\right) = \frac{3}{2}\frac{G_{\rm eff}}{G_{\rm N}}\Omega_{\rm m}^{1-\gamma}\,. \tag{4}$$

Usually γ is treated as a constant, and its theoretical value γ_∞ for some models can be obtained by expanding the above equation around $\Omega_{\rm m} = 1$. For the cosmological constant Lambda + cold dark matter (ΛCDM) model it was found that $\gamma_\infty = 6/11$[57,62] and for the DGP model $\gamma_\infty = 11/16$.[57,58] Therefore if one can determine the value of γ from the observations, different models can be discriminated.

However from Eq. (4), it is easy to see that γ should, in general, be a function of time. The error arising from treating it as a constant will not always be negligible, especially at the low redshifts. Recently the author in Ref. [60] presented a comparison between Ω^γ with γ as a constant and f, and found the error is nearly zero at the high redshift, but at low redshift $z < 2$, with $\Omega_{\rm m,0} = 0.27$, the error is larger than 1.2% for the ΛCDM model and 3% for the DGP model. Therefore, if one wants to discriminate different models by using the current observational data on the matter perturbations with γ being taken as a constant, the results may be biased, since there are about half of growth factor data points at the redshift region $z < 2$.

So, it seems necessary to consider an evolutionary growth index $\gamma(z)$. In this regard, the authors in Refs. [72]–[76] studied $\gamma(z)$ with a linear expansion, $\gamma \approx \gamma_0 + \gamma_0' z$, and found that γ_0' is different for different models. This may provide another signature to discriminate different models. Certainly this linear expansion gives a very good approximation at $z < 0.5$, but it is invalid at high redshift region. Recently, we used the three observed growth factor data at $z < 0.5$ to constrain the ΛCDM model and the DGP model and found that merely three data points cannot discriminate them.[76] To obtain stronger constraints which may clearly discriminate different models, we need a parametrized form of $\gamma(z)$, which is applicable for all the observational data. We proposed such a parameterized form on $\gamma(z)$[77]

$$\gamma(z) = \gamma_0 + \gamma_1 \frac{z}{1+z}, \tag{5}$$

and found that this parametrization can approximate f very well at both low and high redshifts for both the wCDM model and the DGP model. The error is below 0.03% for the ΛCDM model and 0.18% for the DGP model for all redshifts when $\Omega_{\rm m0} = 0.27$. As a result, form (5) is applicable to all the growth factor data points and can be used to discriminate these models. In this paper, we will discuss the observational constraints on the wCDM model and the DGP model, with the growth index in our proposed form. By comparing the theoretical values and observational results of parameters γ_0 and γ_1, we try to find out which model is consistent with the observations, and in fact, our results show that the DGP model is unfavoured.

2. The growth index

In this section, we will determine the regions for the theoretical values of γ_0 and γ_1 in the parameterized growth index given in Eq. (5), for both the wCDM model and the DGP model.

2.1. The wCDM model

For the wCDM model, from the Friedmann equation and the definition of $\Omega_{\rm m}$, one can easily obtain

$$\frac{\dot{H}}{H^2} = \frac{1}{2}\Omega_k - \frac{3}{2}[1 + w(1 - \Omega_{\rm m} - \Omega_k)], \tag{6}$$

and

$$\frac{{\rm d}\Omega_{\rm m}}{{\rm d}\ln a} = 3w\Omega_{\rm m}(1 - \Omega_{\rm m} - \Omega_k) - \Omega_{\rm m}\Omega_k\,. \tag{7}$$

Substituting Eqs. (5), (6), and (7) into Eq. (4), and using $G_{\rm eff}/G_{\rm N} = 1$ for the wCDM model yield an equation for γ_1

$$\gamma_1 = (\ln \Omega_{\rm m,0}^{-1})^{-1}\left[\frac{3}{2}\Omega_{\rm m,0}^{1-\gamma_0} - \Omega_{\rm m,0}^{\gamma_0} - \frac{3}{2}w(2\gamma_0 - 1)(1 - \Omega_{\rm m,0}\Omega_{k,0}) - \frac{1}{2}(1 + \Omega_{k,0} - 2\gamma_0\Omega_{k,0})\right]. \tag{8}$$

Here Ω_k denotes spatial curvature density. And here the subscript 0 denotes the present value. Using the numerical results of f under the initial condition $f = 1$ at the high redshift and $f_0 = \Omega_{\rm m,0}^{\gamma_0}$ with a prior determined value of $\Omega_{\rm m,0}$, one can obtain the value of

γ_0. Substituting this γ_0 and corresponding $\Omega_{\rm m,0}$ into Eq. (8), the value of γ_1 can be determined. For the current observational conservative range about $\Omega_{\rm m,0}$: $0.2 \leq \Omega_{\rm m,0} \leq 0.35$, our numerical calculation shows that, for the flat ΛCDM model, the value γ_0 ranges from 0.554 to 0.558, and γ_1 from −0.0197 to −0.016; for the ΛCDM model with $\Omega_{k,0} = -0.01$, γ_0 from 0.552 to 0.557 and γ_1 from −0.02 to −0.016; for wCDM model with $w = -0.8$ and $\Omega_{k,0} = -0.01$, γ_0 from 0.56 to 0.565 and γ_1 from −0.019 to −0.0152.

2.2. The DGP model

For the DGP model, the effective Newton constant takes the following form

$$G_{\rm eff} = G_{\rm N}\frac{4\Omega_{\rm m}^2 - 4(1-\Omega_k)^2 + 2\sqrt{1-\Omega_k}(3-4\Omega_k+2\Omega_{\rm m}\Omega_k+\Omega_k^2)}{3\Omega_{\rm m}^2 - 3(1-\Omega_k)^2 + 2\sqrt{1-\Omega_k}(3-4\Omega_k+2\Omega_{\rm m}\Omega_k+\Omega_k^2)}. \tag{9}$$

According to the calculations given in Refs. [58], [61], [82] and [83], by considering the form of $\gamma(z)$ given in Eq. (5), one can derive

$$\gamma_1 = (\ln\Omega_{\rm m,0}^{-1})^{-1}\bigg[-\Omega_{\rm m,0}^{\gamma_0} + \frac{3}{2}\bigg(1+\frac{1}{3\beta}\bigg)\Omega_{\rm m,0}^{1-\gamma_0} - \frac{1}{2}(1+\Omega_{k,0}-2\gamma_0\Omega_{k,0}) - 3\frac{-1+\Omega_{k,0}}{1+\Omega_{\rm m,0}-\Omega_{k,0}}(1-\Omega_{k,0}-\Omega_{\rm m,0})(\gamma_0-\frac{1}{2})\bigg]. \tag{10}$$

Our numerical calculation shows that, for $0.20 \leq \Omega_{\rm m,0} \leq 0.35$, γ_0 increases from 0.658 to 0.671 and γ_1 decreases from 0.042 to 0.035 for a spatially flat case; in a curved space with $\Omega_{k,0} = -0.01$, γ_0 increases from 0.659 to 0.668 and γ_1 decreases from 0.044 to 0.041. Apparently, γ_1 obtained in the DGP model is different from that in the wCDM model where γ_1 is negative. Therefore γ_1 provides another signature to discriminate these models.

3. Observational constraints

In order to obtain the observational constraints on γ_0 and γ_1, we first need to determine the value of $\Omega_{\rm m,0}$ from the observations. Here we use the results obtained in Refs. [60] and [61] where the author(s) used the 307 Union Sne Ia data, the BAO from the SDSS data, the shift parameter from the WMAP5 and the 11 Hubble parameter data. Combining these observational data sets, they found that, in a flat space, $\Omega_{\rm m,0} = 0.273 \pm 0.015$ for the ΛCDM model and $\Omega_{\rm m,0} = 0.278 \pm 0.015$ for the DGP model; in a curved space, $\Omega_{\rm m,0} = 0.278^{+0.015}_{-0.029}$, $\Omega_{k,0} = 0.000 \pm 0.016$ and $w = -0.93^{+0.14}_{-0.17}$ for the wCDM model, $\Omega_{\rm m,0} = 0.280^{+0.031}_{-0.028}$, $\Omega_{k,0} = -0.002 \pm 0.014$ for the ΛCDM model, and $\Omega_{\rm m,0} = 0.268^{+0.030}_{-0.025}$, $\Omega_{k,0} = 0.017^{+0.015}_{-0.014}$ for the DGP model. With these best fit values we can obtain the corresponding theoretical values of γ_0 and γ_1. The results are as follows: in the spatially flat case, $\gamma_0 = 0.555$ and $\gamma_1 = -0.018$ for the ΛCDM model, $\gamma_0 = 0.558$ and $\gamma_1 = -0.017$ for the wCDM model, and $\gamma_0 = 0.665$ and $\gamma_1 = 0.040$ for the DGP model; in a curved space, $\gamma_0 = 0.555$ and $\gamma_1 = -0.017$ for the ΛCDM model, and $\gamma_0 = 0.663$ and $\gamma_1 = 0.036$ for the DGP model.

In order to discriminate different models we need the observational constraints on γ_0 and γ_1, which can be obtained by the growth factor data. Here 12 data points given in Table 1 are used.

Table 1. The summary of the observational data on the growth factor f.

z	$f_{\rm obs}$	References
0.15	0.49 ± 0.1	[84], [85]
0.35	0.7 ± 0.18	[86]
0.55	0.75 ± 0.18	[87]
0.77	0.91 ± 0.36	[84], [85]
1.4	0.9 ± 0.24	[88]
3.0	1.46 ± 0.29	[89]
2.125 − 2.72	0.74 ± 0.24	[90]
2.2 − 3	0.99 ± 1.16	[91]
2.4 − 3.2	1.13 ± 1.07	[91]
2.6 − 3.4	1.66 ± 1.35	[91]
2.8 − 3.6	1.43 ± 1.34	[91]
3 − 3.8	1.3 ± 1.5	[91]

Let us note that although the data given in Refs. [90] and [91] are measured without 'any' bias, other data points are obtained by assuming a flat ΛCDM model with $\Omega_{\rm m,0}$ being a specific value, for example, $\Omega_{\rm m,0} = 0.25$ or 0.30. So, caution must be exercised when using these data. With this caveat in mind, it may still be worthwhile to apply the data to fit the models.[60,61,58,78] With the best fit values of $\Omega_{\rm m,0}$ we can find the constraints on γ_0 and γ_1 from the observations by using the following equation

$$\chi^2_f = \sum_{i=1}^{12} \frac{[f_{\rm obs}(z_i) - \Omega_{\rm m}^{\gamma_0+\gamma_1 z_i/(1+z_i)}]^2}{\sigma^2_{fi}}, \tag{11}$$

where σ_{fi} is the 1σ uncertainty of the $f(z_i)$ data. The results are shown in Figs. 1 and 2. Figure 1 gives the results of ΛCDM and wCDM models, from which one can see that at 1σ confidence level, these models are consistent with the observations. From Fig. 2 we find that at the 1σ confidence level the DGP model in a flat space is almost excluded, while in a spatially curved case it is completely ruled out, but in both cases it is consistent with the observations at 2σ confidence level.

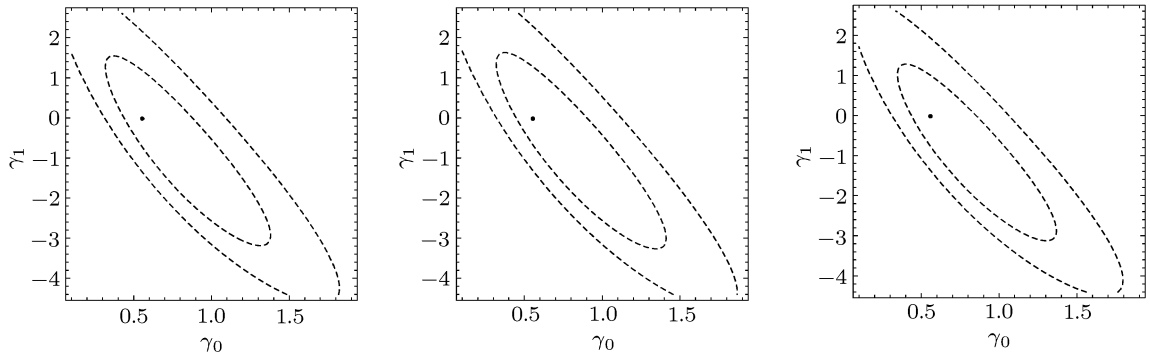


Fig. 1. The 1σ and 2σ contours of γ_0 and γ_1 by fitting the ΛCDM model and the wCDM model with the current growth factor data. The model parameters ($\Omega_{\rm m,0}$, $\Omega_{k,0}$ and w) take the best fit values obtained in Refs. [60] and [61]. The left, middle and right panels correspond to the results of the flat ΛCDM model, the ΛCDM model in the curved space and the flat wCDM model, respectively. The points denote the corresponding theoretical values of γ_0 and γ_1.

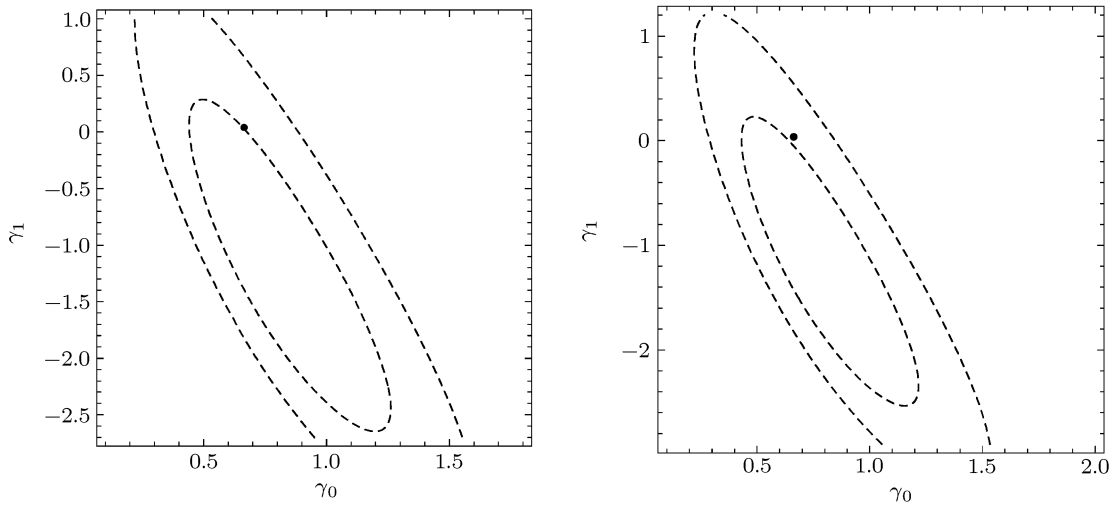


Fig. 2. The 1σ and 2σ contours of γ_0 and γ_1 by fitting the DGP model to the growth factor data with $\Omega_{\rm m,0}$ and $\Omega_{k,0}$ in the best fit values obtained in Refs. [60] and [61]. The left and right panels show the spatially flat and curved cases, respectively. The points denote the corresponding theoretical values of γ_0 and γ_1.

4. Conclusion

In a previous work,[77] we supposed a parameterized form of the growth index $\gamma(z)$: $\gamma(z) = \gamma_0 + z/1 + z\gamma_1$, which approximates f very well at both low and high redshifts for the wCDM model and the DGP model. In

this paper, we have discussed the observational constraints on the wCDM model and the DGP model from the growth factor data with the growth index taking the proposed form. By comparing the theoretical values and the observational results of parameters γ_0 and γ_1, we find that both the ΛCDM model and the wCDM model are consistent with the observations at the 1σ confidence level, however, the DGP model is consistent with observations only at the 2σ confidence level.

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