Frustration and intermediate dimerized phase in spin ladder system*

Shi Dong(石 洞)[a)†], Wen Rui(文 瑞)[a)],
Liu Guang-Hua(刘光华)[b)], and Tian Guang-Shan(田光善)[a)]

[a)] *School of Physics, Peking University, Beijing* 100871, *China*
[b)] *Department of Physics, Tianjin Polytechnic University, Tianjin* 300160, *China*

(Received 17 March 2010; revised manuscript received 29 March 2010)

This paper studies a spin ladder model which possesses frustrating interactions. By using both the bosonization and the density matrix renormalization group techniques, it shows that the intermediate columnar dimerized phase, which exists in a narrow parameter region of the so-called $J_1 - J_2$ model, vanishes if the interchain frustration is weak and anisotropic. Therefore, it concludes that the frustrating interaction indeed plays an important role in producing such a phase. As a complementary to our previous investigation, it reaches a more complete picture of the quantum phase transition in the frustrated spin ladder systems.

Keywords: spin ladder, dimerized phases, frustration
PACC: 7510J, 7540C, 7540F

1. Introduction

The frustration effect in quasi-one-dimensional magnetic materials has been intensively studied by both the experimental and theoretical means in recent years. A simple system is the so-called $J_1 - J_2$ model which has antiferromagnetic super-exchange interactions between spins at a pair of the nearest- and the next-nearest-neighbouring sites. In practice, this model is widely used to describe the quantum phase transition in the magnetic $SrCuO_2$ chain that has a unique structure of spin ladder consisting of Cu ions.[1,2] Intuitively, it can be thought of as a ladder. In other words, the system can also be described by two coupled Heisenberg chains with the intra super-exchange interaction along the chain direction. In addition, the spins on different chains are coupled by the weak frustrating interactions in both transverse and diagonal directions.

Many physicists have made great efforts to uncover the phase diagram of this system.[3−15] For instance, an earlier investigation by the bosonization approach concluded that the phase diagram of this model at zero temperature consists of only two parts, the rung-singlet (RS) phase and the Haldane phase.[8] Qualitatively, this conclusion is consistent to the results derived by series expansion[7] and the density-matrix renormalization group (DMRG)[11] numerical calculations.

However, this conclusion was challenged by other authors. Recently, Starykh and Balents[16] investigated the phase diagram of the same system by applying a reformulated weak-coupling field theory.[17,18] They found that, in a weak coupling regime, it is impossible to fine-tune rung and diagonal exchange interactions in order to eliminate all the relevant interchain couplings at all orders. Therefore, a direct transition between the Haldane phase and the RS phase cannot occur. Instead, they predicted the existence of an intermediate spontaneously-dimerized phase between the RS phase and the Haldane phase in this regime. Naturally, this interesting observation rekindles physicists' interest in the spin ladder system. Very recently, Azzouz and Ramakko confirmed this conclusion by the so-called bond mean-field theory.[19] However, other physicists disagree. Hung *et al.* re-calculated the columnar dimerization order parameter, the spin correlations on rung and the string order parameters for this model. They also scaled its excitation gaps by using both DMRG technique and exact diagonalization method.[20] But, they failed to find any evidence which supports the existence of a columnar dimerized phase in its ground state.

*Project supported by the National Natural Science Foundation of China (Grant No. 10874003) and Ministry of Science and Technology of China (Grant No. 2006CB921300).
†Corresponding author. E-mail: shidong238@gmail.com
© 2010 Chinese Physical Society and IOP Publishing Ltd

In a very recent work,[21] we re-investigated this issue by the DMRG numerical method. In order to determine the phase diagram of the spin ladder model more precisely, we studied the behaviours of both the conventional dimerization order parameters and the two-rung localizable entanglement. We found that, although it is very difficult to observe, an intermediate dimerized phase does exist between the RS phase and the Haldane phase in the spin ladder model. As a matter of fact, our data on the order parameter is actually inconclusive, as noticed previously in Ref. [20]. However, the critical behaviour of localizable entanglement reveals clearly that quantum transitions occur in a very narrow region of parameters.

Then, a natural question that one would like to ask is what happens if anisotropy is introduced into the system. For instance, we would like to know whether the dimerized phase still exists if two spins on different chains are only coupled in a specific diagonal direction. Another interesting question is how reliable the bosonization method is. As we mentioned above, it seems that the conclusions derived by this approach are often in contradiction with the ones obtained by numerical techniques, such as the DMRG calculations. In the following, we shall address these problems. More precisely, we shall apply the bosonization method developed in Ref. [16] to uncover the phase diagram of an anisotropic spin ladder. Then, we study again the issue with the DMRG calculation. Interestingly, we reach the same conclusion by both approaches. Namely, the dimerized phase vanishes after the spatial anisotropy is introduced into the system.

This paper is organized as follows. In Section 2, we introduce first the anisotropic spin ladder model. Then, we study its phase diagram by the bosonization method; In Section 3, we shall investigate its phase diagram again with the DMRG calculation; Finally, in Section 4, we summarize our main results in the present paper.

2. The spin ladder model and its phase diagram

The Hamiltonian of the spin ladder model, which we are interested in, is of the following form:

$$\begin{aligned}\hat{H} = & J\sum_{i=1,2}\sum_{n} S_{i,n}\cdot S_{i,n+1} \\ & +\frac{1}{2}J_{\perp}\sum_{n}\left(S_{1n}\cdot S_{2n}+S_{1,n+1}\cdot S_{2,n+1}\right) \\ & +\frac{1}{2}J_{\times}\sum_{n}\left(S_{1n}\cdot S_{2,n-1}+S_{1,n+1}\cdot S_{2,n}\right), \quad (1)\end{aligned}$$

where $S_{i,n}$ denotes the spin at site n of the ith chain, $J>0$ is the interaction between nearest-neighbouring spins along the chain, $J_{\perp}>0$ represents the interaction between nearest-neighbour spins in each rung and $J_{\times}$ is the interaction between the next-nearest neighbouring spins. Intuitively, this model can be depicted as Fig. 1.

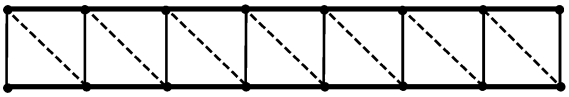


Fig. 1. The structure of the spin ladder model. In this figure, the thick solid line along the direction of each chain represents the coupling constant $J>0$. Similarly, each thin solid line and dashed line stands for interactions $J_{\perp}>0$ and $J_{\times}>0$, respectively.

As stated in Section 1, we would like to determine whether a dimerized phase still exists in this model by using the bosonization method. Following Ref. [16], we start with the case of $J_{\perp}=J_{\times}=0$. Consequently, the system is reduced into two independent one-dimensional Heisenberg spin chains whose Hamiltonian is given by

$$\begin{aligned}\hat{H}^0=\hat{H}_1^0+\hat{H}_2^0 = & J\sum_{n} S_{1,n}\cdot S_{1,n+1} \\ & +J\sum_{n} S_{2,n}\cdot S_{2,n+1}. \quad (2)\end{aligned}$$

Next, for applying the standard bosonization procedure,[22] we take the continuum limit. In other words, we decompose the spin operator $S_{i,n}$ as follows:

$$S_{i,n}\to a\left(J_{i,\mathrm{R}}(x)+J_{i,\mathrm{L}}(x)+(-1)^n N_i(x)\right). \quad (3)$$

Here, $J_{i,\mathrm{R}}(x)$ and $J_{i,\mathrm{L}}(x)$ represent the right- and the left-moving SU(2) currents in the ith chain of the ladder, respectively. Their scaling dimensions are equal to unity. The $N_i(x)$ is the staggered spin magnetization whose scaling dimension is 1/2. The parameter a is the lattice constant and the coordinate $x=na$. Substituting Eq. (3) into Eq. (2), we obtain the Hamiltonian density of the ith chain[16]

$$\mathcal{H}_i^0(x) = \frac{2\pi v}{3}\left(J_{i,\mathrm{R}}(x)\cdot J_{i,\mathrm{R}}(x) + J_{i,\mathrm{L}}(x)\cdot J_{i,\mathrm{L}}(x)\right) - g_{\mathrm{bs}} J_{i,\mathrm{R}}(x)\cdot J_{i,\mathrm{L}}(x), \quad (4)$$

where the parameter $v = \pi J a/2$ is the spin velocity. In the field-theoretic language, this Hamiltonian describes the $\mathrm{SU}_1(2)$ Wess–Zumino–Novikov–Witten (WZNW) model[21] perturbed by a marginally irrelevant back-scattering interaction represented by g_{bs}.

Similarly, applying decomposition Eq. (3) of the spin operator, we can rewrite the interaction Hamiltonian density $\hat{\mathcal{V}}(x)$ in Hamiltonian (1) as

$$\begin{aligned}\hat{\mathcal{V}}(x) = {} & (J_\perp + J_\times) J_1\cdot J_2 + (J_\perp - J_\times) N_1\cdot N_2 \\ & + \frac{1}{2} J_\perp a^2 (\partial_x J_1\cdot\partial_x J_2 + \partial_x N_1\cdot\partial_x N_2) \\ & - \frac{1}{2} J_\times a (J_1\cdot\partial_x J_2 + N_2\cdot\partial_x N_1) \\ & + \frac{1}{2} J_\times a (J_2\cdot\partial_x J_1 + N_1\cdot\partial_x N_2). \end{aligned} \quad (5)$$

There are two possibilities which should be separately considered.

First, if the difference $J_\perp - J_\times \neq 0$, then the term $N_1\cdot N_2$ in $\hat{V}$ is dominant and determines which phase the system is in. This is due to the fact that the scaling dimension of this term is one. Therefore, it is the most relevant interaction in the sense of renormalization group.[22] More precisely, the spin ladder is an RS if $J_\perp - J_\times > 0$ and it is in the Haldane phase when $J_\perp - J_\times < 0$.[16]

Next, we consider the more interesting case of $J_\perp - J_\times = 0$. Now, the interaction Hamiltonian density is reduced to

$$\begin{aligned}\hat{\mathcal{V}}(x) \approx {} & 2J_\perp J_1\cdot J_2 + \frac{1}{2} J_\perp a^2 \partial_x N_1\cdot\partial_x N_2 \\ & - \frac{1}{2} J_\perp a (J_1\cdot\partial_x J_2 + N_2\cdot\partial_x N_1) \\ & + \frac{1}{2} J_\perp a (J_2\cdot\partial_x J_1 + N_1\cdot\partial_x N_2). \end{aligned} \quad (6)$$

Notice that we also drop out the term which is proportional to $\partial_x J_1\cdot\partial_x J_2$ in $\hat{V}$. By the scaling analysis, this term is not important in the sense of renormalization group.

We would like to emphasize that the interaction Hamiltonian density Eq. (6) is merely a special case of the following general Hamiltonian density

$$\begin{aligned}\hat{\mathcal{H}}_V(x) = {} & g_1 N_1\cdot N_2 + g_2 J_1\cdot J_2 \\ & + g_3 \partial_x N_1\cdot\partial_x N_2 + g_4 \epsilon_1\epsilon_2 \\ & + g_5 (J_1\cdot\partial_x J_2 + N_2\cdot\partial_x N_1) \\ & + g_6 (J_2\partial_x J_1 + N_1\cdot\partial_x N_2), \end{aligned} \quad (7)$$

which is allowed by the internal symmetries of the spin ladder model. In $\hat{H}_V$, $\epsilon_i(x = na) \sim (-1)^n S_i(na)\cdot S_i((n+1)a)$ is the staggered dimerization operator with scaling dimension 1/2. We notice that, although the corresponding term of g_4 does not appear in $\hat{\mathcal{V}}(x)$ given in Eq. (6), it may be produced by the quantum fluctuations as the renormalization group procedure is carried out. Indeed, it is this term which determines whether a dimerized phase exists in the system. More precisely, such a phase occurs if and only if a nonzero value of g_4 is derived from the renormalization process.

Now, we are ready to derive the flow equations satisfied by the coupling constants in Eq. (7) by applying the operator product expansion (OPE) technique, which allows us to systematically handle the higher-order terms in the Hamiltonian. Following Ref. [22], we expand first the partition function of the spin ladder model to the second order of the interaction Hamiltonian $\hat{V} = \int \mathrm{d}x \hat{\mathcal{V}}(x)$. We have

$$\begin{aligned}\frac{Z}{Z_0} = {} & \frac{1}{Z_0}\int \mathcal{D}\Phi \mathrm{e}^{-S_0}\mathrm{e}^{-\hat{V}} = \langle \mathrm{e}^{-\hat{V}}\rangle_0 \\ \approx {} & 1 - \langle\hat{V}\rangle_0 + \frac{1}{2}\langle\hat{V}^2\rangle_0. \end{aligned} \quad (8)$$

Next, we apply the following OPE formulas

$$\begin{aligned} J^\alpha_{\mathrm{R/L}}(x,\tau) N^\beta(x',\tau') &= \frac{\mathrm{i}\,\epsilon^{\alpha\beta\gamma} N^\gamma(x,\tau) \mp \mathrm{i}\,\delta^{\alpha\beta}\epsilon(x,\tau)}{4\pi[v(\tau-\tau') \mp i(x-x')]}, \\ J^\alpha_{\mathrm{R/L}}(x,\tau)\epsilon(x',\tau') &= \frac{\pm\mathrm{i}\,N^\alpha(x,\tau)}{4\pi[v(\tau-\tau') \mp \mathrm{i}(x-x')]}, \\ J^\alpha_{\mathrm{R/L}}(x,\tau) J^\beta_{\mathrm{R/L}}(x',\tau') &= \frac{\delta^{\alpha\beta}}{8\pi^2[v(\tau-\tau') \mp \mathrm{i}(x-x')]^2} + \frac{\mathrm{i}\,\epsilon^{\alpha\beta\gamma} J^\gamma_{\mathrm{R/L}}(x,\tau)}{2\pi[v(\tau-\tau') \mp i(x-x')]} \end{aligned} \quad (9)$$

to each product of the right- and left-moving currents in the last term of Eq. (8). In these expansions, the upper signs on the right-hand side correspond to the right-moving currents. Similarly, for the left-moving currents, the lower signs in each equation should be taken. The indices α, β and γ stand for the spin space directions S_x, S_y, S_z and $\epsilon^{\alpha\beta\gamma}$ is the totally antisymmetric tensor. We would like to point out that only the singular

part of each expansion is preserved. These formulas are established by using the bosonization method and their derivation can be found in the standard textbook.[22] Then, by integrating variable $|z| = |v(\tau - \tau') - \mathrm{i}(x - x')|$ from a to $a\mathrm{e}^{dl}$, the third term $\frac{1}{2}\langle \hat{V}^2 \rangle_0$ in Eq. (8) can be put into a one-loop form. Together with the the second term $\langle \hat{V} \rangle_0$ in the same equation, we can derive the one-loop renormalization equations, which describe change of the coupling constants with respect to scaling of the short-distance cutoff ($a \to a\mathrm{e}^{dl}$).

After a lengthy but otherwise straightforward calculation, we obtain the following flow equations satisfied by the coupling constants of the Hamiltonian density $\hat{\mathcal{H}}_V(x)$:

$$\begin{aligned}
\frac{\mathrm{d}g_1}{\mathrm{d}l} &= \frac{g_1 g_2}{4\pi v} + \frac{g_2 g_3}{4\pi v} - \frac{g_2 g_4}{16\pi v} - \frac{g_5^2}{4\pi v} + \frac{g_5 g_6}{2\pi v} - \frac{g_6^2}{4\pi v}, \\
\frac{\mathrm{d}g_2}{\mathrm{d}l} &= 0, \quad \frac{\mathrm{d}g_3}{\mathrm{d}l} = -g_3, \\
\frac{\mathrm{d}g_4}{\mathrm{d}l} &= -\frac{3g_1 g_2}{8\pi v} - \frac{3g_2 g_3}{8\pi v} + \frac{3g_5^2}{8\pi v} - \frac{3g_5 g_6}{4\pi v} + \frac{3g_6^2}{8\pi v}, \\
\frac{\mathrm{d}g_5}{\mathrm{d}l} &= 0, \quad \frac{\mathrm{d}g_6}{\mathrm{d}l} = 0.
\end{aligned} \tag{10}$$

From the expression of interaction density $\hat{\mathcal{V}}(x)$ given in Eq. (6), we have the following initial values for these coupling constants: $g_1 = g_4 = 0, g_2 = 2J_\perp, g_3 = J_\perp a^2/2, g_5 = -J_\perp a/2, g_6 = J_\perp a/2, v = \pi J a/2$. Substituting them into the flow equations, we obtain

$$\frac{\mathrm{d}g_1}{\mathrm{d}l} = \frac{J_\perp^2 a}{2\pi J}, \quad \frac{\mathrm{d}g_4}{\mathrm{d}l} = 0. \tag{11}$$

In particular, the second equation implies that a nonzero coupling g_4 cannot be produced by the quantum fluctuations in the system if it is initially absent. As we said above, it is equivalent to saying that the frustrated spin ladder does not have an intermediate dimerized phase.

On the other hand, the first equation in Eq. (11) tells us that g_1 will become positive by renormalization. Therefore, the system will actually be in the Haldane phase at $J_\perp = J_\times$. This conclusion is consistent with the above statement.

3. The DMRG calculation

In the above section, we showed that the spin ladder model, whose Hamiltonian is given by Eq. (1), does not have the dimerized phase. Our analysis is based on the bosonization method and the field-theoretic renormalization group theory. However, both techniques rely on the continuum limit approximation and the weak-coupling expansion. As a result, errors caused by these approximations cannot be avoided. Therefore, one would naturally like to check these conclusions by other independent approach. In this section, we shall re-investigate the phase diagram of Hamiltonian Eq. (1) by the DMRG method,[23,24] which is an unbiased and accurate numerical technique in computing the ground-state properties of one-dimensional quantum spin systems. Recently, we applied successfully this method to uncover the existence of the columnar dimerized phase in the $J_1 - J_2$ spin ladder model.[21]

To start with, we introduce definitions of the dimerization order parameter and the two-rung localizable entanglement. These quantities will be calculated in this section to re-investigate the existence of the columnar dimerized phase in the spin ladder model.

Following the previous work in Ref. [25], we define the columnar dimerization order parameter O_{CD} by

$$\begin{aligned}
O_{\mathrm{CD}} = \frac{1}{2}\Big| \sum_{i=1,2} \langle \Psi_0 | \hat{S}_{i,n} \cdot \hat{S}_{i,n+1} \\
- \hat{S}_{i,n+1} \cdot \hat{S}_{i,n+2} | \Psi_0 \rangle \Big|,
\end{aligned} \tag{12}$$

where Ψ_0 denotes the ground state of the system and n is an arbitrary site of the lattice. Similarly, we define the two-rung localizable entanglement entropy S_N by Ref. [21]

$$S_N = -\mathrm{Tr}\, \widetilde{\boldsymbol{\rho}}_A \,\mathrm{lb}\, \widetilde{\boldsymbol{\rho}}_A = -\mathrm{Tr}\, \widetilde{\boldsymbol{\rho}}_B \,\mathrm{lb}\, \widetilde{\boldsymbol{\rho}}_B. \tag{13}$$

Here, A represents a subset of the localized spins, which occupy the four sites of two rungs in the middle of the ladder, and B is the subset of the other spins in the system. Then, the corresponding reduced density matrix $\widetilde{\boldsymbol{\rho}}_A$ or $\widetilde{\boldsymbol{\rho}}_B$ of each subsystem can be produced by integrating out the degrees of freedom of B or A

in the density matrix $\hat{\boldsymbol{\rho}}$ of the whole system, respectively. In particular, when $S_N = 0$, subsets A and B are called untangled.

Before proceeding to calculate of these quantities, we would like to explain briefly why we introduce their definitions.

Conceptually, it is easy to see that one should compute directly the order parameter $O_{\rm CD}$ to establish numerically the existence of the dimerized phase in the spin ladder model. However, due to the strong quantum fluctuations in the low-dimensional system, this quantity has an extremely small value in the dimerized phase. Therefore, it makes a reliable judgment whether such a phase exists very difficultly in practice. For that reason, one needs an alternative measure, which should have a more dramatic change at the quantum phase transition points, to reveal the existence of an intermediate phase. Recent researches show that entanglement is an ideal substitution.[26,27]

From Eq. (13), we see that entanglement between subsystems A and B are determined by the eigenvalues of $\widetilde{\boldsymbol{\rho}}_A$ or $\widetilde{\boldsymbol{\rho}}_B$. On the other hand, it is well known that the ground state energy E_0 of Hamiltonian Eq. (1) can be explicitly written as $E_0 = {\rm Tr}(\hat{\rho}\hat{H})$, which is nonanalytic at quantum phase transition points. Therefore, one will naturally speculate that the entanglement behaves in a singular way at these points, too Ref. [28]. This is the basis for the usefulness of the localizable entanglement in study of the quantum phase transition.

Moreover, from the physical point of view, it is also unnecessary to take a sublattice whose size is much larger than some characterizing lengths, such as the interaction range between spins in the system when we compute its localizable entanglement. In fact, as far as quantum entanglement is concerned, the reduced matrix of a small cluster should be sufficient to exhibit it. To make this point more clear, let us recall a well-known fact in optics. By recording interference pattern of two laser beams reflected from a specific object, the whole image of it can be recovered from a small piece of holograph, although the resolution may be reduced. In other words, due to interference of electromagnetic waves, the main global information of the object can be reserved in a quite small spatial area. Similarly, for a quantum many-body system, its reduced density matrix $\widetilde{\boldsymbol{\rho}}_A$ contains information not only on the subset itself, but also on the correlation between this part and the rest of the system, as dictated by the superposition principle of quantum mechanics. Therefore, we need only to calculate a small-size sample to determine very accurately the phase diagram of the entire lattice.[29,30]

For convenience, we impose the open boundary condition on the spin ladder whose size is eventually increased up to 160 rungs. Furthermore, in each renormalization step, we keep 200 states for both the system and environment blocks. In order to suppress further finite size effects, we choose $n = N/2$ in the definition of order parameter $O_{\rm CD}$. Similarly, for the two-rung entanglement, we take a pair of nearest-neighbouring spins at the middle of the sample as the finite subsystem. In addition, we set the coupling constant J to be unity.

First, we compute the order parameter $O_{\rm CD}$. To compare with our previous result in Ref. [21], we choose $J_\times = 0.5$. As shown in Fig. 2, indeed, $O_{\rm CD}$ takes on a nonzero value in the vicinity of $J_\perp \approx J_\times$. However, its value decreases fast as the length of ladder increases. To be more precisely, we inset a small figure in the diagram. It shows the behaviour of $O_{\rm CD}$ at its maximal point. One can clearly see that $O_{\rm CD}$ approaches to zero as the ladder becomes longer. This result is consistent with our conclusion derived by the bosonization method in Section 2.

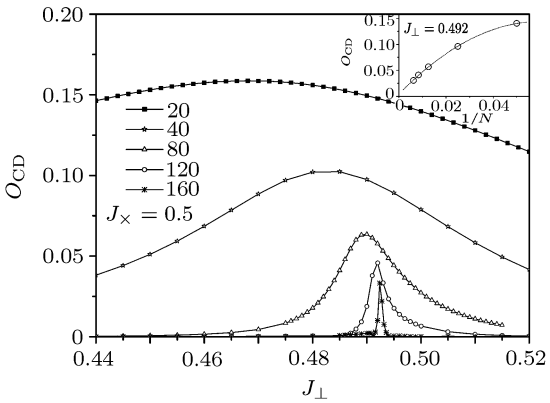


Fig. 2. The order parameter $O_{\rm CD}$ versus $J_\perp$ with $J_\times = 0.5$ for various system sizes. The inset is the scaling of $O_{\rm CD}$ at $J_\perp = 0.492$ as the ladder length increases.

Next, we calculate the two-rung entanglement S_N of the ground state. Our data are shown in Fig. 3. We find that the entanglement is continuous and takes a maximum in the vicinity of $J_\perp \cong J_\times$ as the length of spin ladder tends to infinity. However, as we pointed out in Ref. [21], this maximal value has nothing to do with the existence of the intermediate dimerized phase. It is due to the fact that the phase transition between the RS phase and the dimerized phase should be of the second order if the latter phase does exist, according to Starykh and Balents.[16] However, as we

argued in Ref. [21], such a transition can only be characterized by the change of the first-order derivative of S_N.

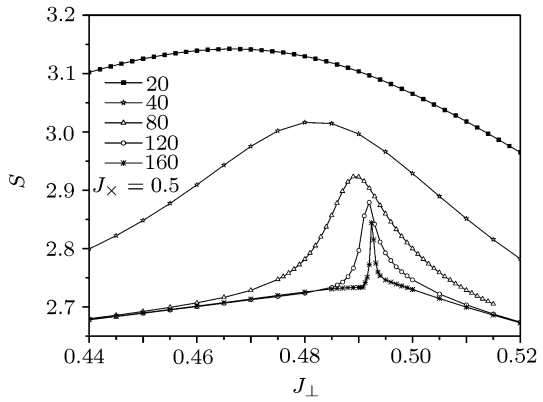


Fig. 3. Two-rung localizable entanglement versus $J_\perp$ for various system sizes.

Indeed, as shown in Fig. 4, the curve of the first-order derivative of the entanglement has two extreme pints as $J_\perp$ varies. These points will signal the existence of the intermediate dimerized phase if they approach to different limits by the finite-size scaling. In this case, one of them is the transition point between the RS phase and the intermediate dimerized phase. The other gives the transition point between the Haldane phase and the dimerized phase. However, after carrying out the finite-size scaling, we find that both points tend actually to the same limit, as shown in the inserted figure of the diagram. Therefore, we conclude that the intermediate dimerized phase is absent in this system. This conclusion confirms the result derived in the above section by the bosonization method.

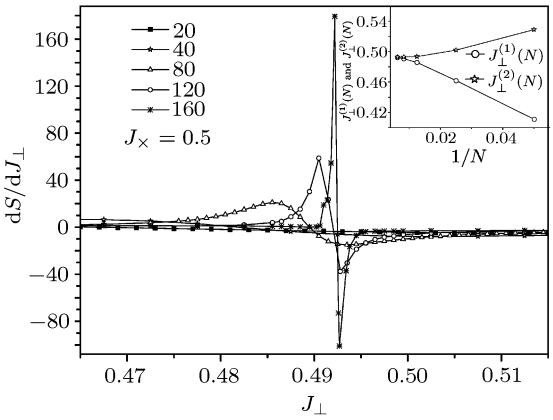


Fig. 4. The first order derivative of two-rung localizable entanglement versus $J_\perp$ for various system sizes. The inset shows the scaling of the extreme points of $\mathrm{d}S_N/\mathrm{d}J_\perp$ as the ladder length increases.

4. Summary

In this paper, we investigate the phase diagram of a frustrated spin-1/2 ladder model by using both the bosonization method and the DMRG calculation. It is a continuation of our previous work in Ref. [21].

We find that, if the frustrating interaction weakens and becomes anisotropic, then the intermediate dimerized phase, which was found in the so-called $J_1 - J_2$ model,[20] disappears. More precisely, for this model, one can fine-tune its rung and diagonal exchange interactions to eliminate all the relevant interchain couplings at all orders in the weak-coupling regime. Therefore, a direct transition between the Haldane phase and the RS phase is allowed. Furthermore, we also find that the extreme points of the first-order derivative of the two-rung entanglement tend to the same limit by the finite-size scaling. It also implies the absence of an intermediate phase. Therefore, we conclude that the intermediate dimerized phase found is completely destroyed by the spatial anisotropy.

References

[1] Motoyama N, Eisaki H and Uchida S 1996 *Phys. Rev. Lett.* **76** 3212

[2] Matsuda M, Katsumata K, Kojima K M, Larkin M, Luke G M, Merrin J, Nachumi B, Uemura Y J, Eisaki H, Motoyama N, Uchida S and Shirane G 1997 *Phys. Rev.* B **55** 11953

[3] Gelfand M P 1991 *Phys. Rev.* B **43** 8644

[4] Bose I and Gayen S 1993 *Phys. Rev.* B **48** 10653

[5] Xian Y 1995 *Phys. Rev.* B **52** 12485

[6] Kitatani H and Oguchi T 1996 *J. Phys. Soc. Jpn.* **65** 1387

[7] Zheng W H, Kotov V and Oitmaa J 1998 *Phys. Rev.* B **57** 11439

[8] Allen D, Essler F H L and Nersesyan A A 2000 *Phys. Rev.* B **61** 8871

[9] Kim E H, Fáth G, Sólyom J and Scalapino D J 2000 *Phys. Rev.* B **62** 14965

[10] Honecker A, Mila F and Troyer M 2000 *Eur. Phys. J.* B **15** 227

[11] Wang X Q 2000 *Mod. Phys. Lett.* B **14** 327

[12] Fáth G, Legeza Ö and Sólyom J 2001 *Phys. Rev.* B **63** 134403

[13] Hakobyan T, Hetherington J H and Roger M 2001 *Phys. Rev.* B **63** 144433

[14] Nersesyan A A and Tsvelik A M 2003 *Phys. Rev.* B **67** 024422

[15] Nakamura M 2003 *Physica* B **329–333** 1000
[16] Starykh O A and Balents L 2004 *Phys. Rev. Lett.* **93** 127202
[17] Tsuchiizu M and Furusaki A 2004 *Phys. Rev. Lett.* **88** 056402
[18] Tsuchiizu M and Furusaki A 2004 *Phys. Rev.* B **69** 035103
[19] Azzouz M and Ramakko B W cond-mat/0708.2926
[20] Hung H H, Gong C D, Chen Y C and Yang M F 2006 *Phys. Rev.* B **73** 224433
[21] Liu G H, Wang H L and Tian G S 2008 *Phys. Rev.* B **77** 214418
[22] Gogolin A O, Nersesyan A A and Tsvelik A M 1998 *Bosonization and Strongly Correlated Systems* (Cambridge: Cambridge University Press) p219
[23] White S R 1992 *Phys. Rev. Lett.* **69** 2863
[24] Wang X and Xiang T 1997 *Phys. Rev.* B **56** 5061
[25] Hijii K, Qin S J and Nomura K 2003 *Phys. Rev.* B **68** 134403
[26] Osterloh A, Amico Luigi, Falci G and Fazio R 2002 *Nature* **416** 608
[27] Osborne T J and Nielsen M A 2002 *Phys. Rev.* A **66** 032110
[28] Wu L A, Sarandy M S and Lidar D A 2004 *Phys. Rev. Lett.* **93** 250404
[29] Shan C J, Cheng W W, Liu T K, Huang Y X and Li H 2008 *Chin. Phys.* B **17** 0794
[30] Zheng Q, Zhang X P, Zhi Q J and Ren Z Z 2009 *Chin. Phys.* B **18** 3210