Determination of the parameters of a linear-viscoelastic thin layer using the normally-incident ultrasonic waves*

Yao Gui-Jin(姚桂锦)[a)†], Lü Wei-Guo(吕卫国)[b)], Song Ruo-Long(宋若龙)[b)], Cui Zhi-Wen(崔志文)[b)], Zhang Xiang-Lin(张香林)[a)], and Wang Ke-Xie(王克协)[b)]

[a)] *The School of Communication Engineering, Jilin University, Changchun* 130012, *China*
[b)] *The School of Physics, Jilin University, Changchun* 130023, *China*

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This paper proposes a method of simultaneous determination of the four layer parameters (mass density, longitudinal velocity, the thickness and attenuation) of an immersed linear-viscoelastic thin layer by using the normally-incident reflected and transmitted ultrasonic waves. The analytical formula of the layer thickness related to the measured transmitted transfer functions is derived. The two determination steps of the four layer parameters are developed, in which acoustic impedance, time-of-flight and attenuation are first determined by the reflected transfer functions. Using the derived formula, it successively calculates and determines the layer thickness, longitudinal velocity and mass density by the measured transmitted transfer functions. According to the two determination steps, a more feasible and simplified measurement setups is described. It is found that only three signals (the reference waves, the reflected and transmitted waves) need to be recorded in the whole measurement for the determination of the four layer parameters. A study of the stability of the determination method against the experimental noises and the error analysis of the four layer parameters are made. This study lays the theoretical foundation of the practical measurement of a linear-viscoelastic thin layer.

Keywords: ultrasonic determination, normally-incident reflected/transmitted waves, layer parameters, linear-viscoelastic thin layer

PACC: 4335, 4385D, 4385G

1. Introduction

There is a great technological need for the ultrasonic measurement of a thin layer in manufacturing, interfacial adhesives in structural materials, composite materials, protective surface coatings on automobile bodies and aerospace structure, and so forth. Here 'a thin layer' means a situation where two successive echoes from the layer cannot be separated in the time domain, i.e. the ultrasonic pulse duration exceeds the round trip time. Various ultrasonic technologies including acoustic microscopy, ultrasonic spectroscopy and Lame waves technique and laser-ultrasonic technique have been developed for the measurement of the thin layer.[1−18]

In practice, the measurement of the independent layer parameters of a thin layer has received a great deal of attention and study. At normal incidence, it is desirable to simultaneously determine mass density, longitudinal velocity, the thickness and attenuation, while the reflected or transmitted transfer functions depend on two compound parameters (acoustical impedance and time-of-flight) and attenuation, other three layer parameters cannot be decoupled with sufficient accuracy. In the traditional broadband reflection/transmission methods,[6−9] through the elaborate designs of the measurement setups of the pulse echo and through transmission, the coupling problem could be solved and the simultaneous determination of the layer parameters was performed. However, the methods were all required to involve the use of the non over lapping reflected and transmitted pulses from the front and back interfaces, and broke down for measuring a thin layer. A variety of the inversion methods in the time or frequency domain had also been developed to determine the layer parameters of the thin layer.[10−17] Although the over lapping problem of the pulses does not need to be considered in these methods, solving the multi-parameter inversion problems became complicated and difficult and was unable to

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†Corresponding author. E-mail: yaogj@jlu.edu.cn
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derive the intrinsic relations of the related parameters. Recently, in order to reduce the number of the layer parameters to be determined, a two-step determination method had been developed as the ultrasonic measurements of an embedded layer and an isotropic thin layer on a stiff plate were performed.[14−16] Although the complete set of layer parameters was simultaneously calculated from the determined non-dimensional parameters, the two-step method essentially belonged to the multi-parameter inversion problem as the non-dimensional parameters were solved from the nonlinear inversion equations in the first step. Yao had derived the analytical formulas of acoustic impedance, time-of-flight and attenuation of a linear-viscoelastic thin layer related to the transfer functions of the normally-incident reflected ultrasonic waves.[18] By using the analytical formulas, three parameters could be directly calculated and determined, however the coupling problem was not yet solved and the four layer parameters could not be simultaneously determined.

The simultaneous determination of the layer parameters is not a new subject in ultrasonic measurement. Although a variety of the measurement setups or determination methods have been developed until now, there still remain the interesting basic problems are not solved or solved well, such as how to design the more feasible and simplified measurement setups and determination methods and what is the corresponding theoretical basis, and so forth. Therefore from the viewpoint of the theory and the experimental measurement, it is significant and necessary to study these basic problems related to the ultrasonic measurement of the layer parameters. In this paper we propose a simultaneous determination method of the four layer parameters of an immersed linear-viscoelastic thin layer as the transfer functions of the normally-incident reflected and transmitted ultrasonic waves are given. The analytical formula of the layer thickness related to the transmitted transfer functions is derived. According to the four analytical formulas of the related parameters, the two determination steps of the four layer parameters, and the corresponding more simplified measurement setups are described and analysed, respectively. The stability of the determination method against the experimental noises and the error analysis of the four layer parameters are numerically investigated.

2. The determination of the four layer parameters using the reflected and transmitted transfer functions

Here considering a steady-state, time-harmonic plane longitudinal waves as the normally-incident waves, imposing the continuity of the displacements and the stresses at the front and back interfaces of a homogeneous isotropic linear-viscoelastic thin layer in the immersed water, through the theoretical derivation, we obtain the reflected and transmitted transfer functions without dispersion at normal incidence as[10]

$$H_{\rm r}(\omega_i) = \frac{R_{21}\left[1-\exp\left(-\dfrac{\beta_2}{2\pi}2\omega_i\psi_2\right)\exp(-{\rm j}2\omega_i\psi_2)\right]}{1-R_{21}^2\exp\left(-\dfrac{\beta_2}{2\pi}2\omega_i\psi_2\right)\exp\left(-{\rm j}2\omega_i\psi_2\right)}, \tag{1}$$

$$H_{\rm t}(\omega_i) = \frac{(1-R_{21}^2)\exp\left(-\dfrac{\beta_2}{2\pi}\omega_i\psi_2\right)\exp[-{\rm j}\omega_i(\psi_2-\psi_1)]}{1-R_{21}^2\exp\left(-\dfrac{\beta_2}{2\pi}2\omega_i\psi_2\right)\exp(-{\rm j}2\omega_i\psi_2)}, \tag{2}$$

where ω_i is the discrete angular frequency in the useful ultrasonic bandwidth; $R_{21} = (z_2 - z_1)/(z_1 + z_2)$, $\psi_1 = h_2/c_1$, and $\psi_2 = h_2/c_2$; $z_i = \rho_i c_i$ $(i = 1, 2)$ are the acoustic impedances of the immersed water and the thin layer with ρ_i and c_i being mass density and the longitudinal velocity; h_2 is the layer thickness. Following Kinra and Lyer,[10] a dimensionless measure of attenuation, namely, β_2 is a material constant related to the quality factor Q by $Q \cong \pi/\beta_2$ as β_2 is small.

From the simultaneous equation of the theoretical and the measured transmitted transfer functions $H_{\rm t}(\omega_i)$ and $H_{\rm t}^*(\omega_i)$ (for discrimination, the superscript $(^*)$ refers to the measured quantity), after the condition that the phase-shift difference $\Delta\omega\psi_1 = (\omega_{i+1} - \omega_i)\psi_1$ being not greater than 2π is introduced to eliminate the ambiguity

of phase-shift $\omega_i\psi_1$,[18] by the derivation, the following expression is obtained:

$$\mathrm{j}\Delta\omega\psi_1 = \ln[f(\omega_{i+1},\omega_i,z_2,\psi_2,\beta_2)] + \mathrm{j}\Delta\omega\psi_2\left[1-\mathrm{j}\frac{\beta_2}{2\pi}\right], \tag{3}$$

where

$$f(\omega_{i+1},\omega_i,z_2,\psi_2,\beta_2) = \frac{H_{\mathrm{t}}^*(\omega_{i+1})\left[1-R_{21}^2\exp\left(-\dfrac{\beta_2}{2\pi}2\omega_{i+1}\psi_2\right)\exp(-2\mathrm{j}\omega_{i+1}\psi_2)\right]}{H_{\mathrm{t}}^*(\omega_i)\left[1-R_{21}^2\exp\left(-\dfrac{\beta_2}{2\pi}2\omega_i\psi_2\right)\exp(-2\mathrm{j}\omega_i\psi_2)\right]}, \tag{4}$$

$f(\omega_{i+1},\omega_i,z_2,\psi_2,\beta_2)$ is regarded as a known function of the measured transmitted transfer functions $H_{\mathrm{t}}^*(\omega_i)$ and the three parameters z_2, ψ_2 and β_2. Then

$$h_2 = \mathrm{Re}\left\{\frac{c_1}{\mathrm{j}\Delta\omega}\ln[f(\omega_{i+1},\omega_i,z_2,\psi_2,\beta_2)] + \psi_2 c_1\left(1-\mathrm{j}\frac{\beta_2}{2\pi}\right)\right\}. \tag{5}$$

Equation (5) is the analytical formula of the thickness of an immersed linear-viscoelastic thin layer that is related to $H_{\mathrm{t}}^*(\omega_i)$ and the three parameters z_2, ψ_2 and β_2. Equation (5) is suitable for the calculation of the thickness of a solid or fluid thin layer at the normal incidence.

It is shown by Eq. (5) that the layer thickness h_2 can be decoupled from the compound parameters z_2 and ψ_2 using the transmitted transfer functions. If z_2, ψ_2 and β_2 are given, then by using Eq. (5), the thickness h_2 can be calculated and determined, the other two layer parameters c_2 and ρ_2 are also simultaneously determined. Fortunately, it was proved that the three parameters z_2, ψ_2 and β_2 can be completely determined only by using the reflected transfer functions $H_{\mathrm{r}}^*(\omega_i)$.

From Eq. (1), the three analytical formulas related the three determined parameters z_2, ψ_2 and β_2 to the input acoustic impedance $z_{2e}^I(\omega_i)$ at the front interface of the thin layer are expressed as[18]

$$z_2 = \sqrt{\frac{2z_{2e}^I(\omega_{i+1})z_{2e}^I(\omega_i)z_{2e}^I(\omega_{i-1}) - [z_{2e}^I(\omega_i)]^2 z_{2e}^I(\omega_{i+1}) - [z_{2e}^I(\omega_i)]^2 z_{2e}^I(\omega_{i-1})}{z_{2e}^I(\omega_{i+1}) - 2z_{2e}^I(\omega_i) + z_{2e}^I(\omega_{i-1})}}, \tag{6}$$

$$\psi_2 = \frac{-1}{2\Delta\omega}\mathrm{Im}\left\{\ln\frac{[z_{2e}^I(\omega_{i+1})-z_2][z_{2e}^I(\omega_i)+z_2]}{[z_{2e}^I(\omega_{i+1})+z_2][z_{2e}^I(\omega_i)-z_2]}\right\}, \tag{7}$$

$$\frac{\beta_2}{2\pi} = \frac{-1}{2\Delta\omega\psi_2}\mathrm{Re}\left\{\ln\frac{[z_{2e}^I(\omega_{i+1})-z_2][z_{2e}^I(\omega_i)+z_2]}{[z_{2e}^I(\omega_{i+1})+z_2][z_{2e}^I(\omega_i)-z_2]}\right\}. \tag{8}$$

Where $\Delta\omega = \omega_{i+1} - \omega_i$ is the angular frequency interval. The $z_{2e}^I(\omega_i)$ can be calculated by the acoustic impedance algorithm at normal incidence.[19]

$$z_{2e}^I(\omega_i) = -\frac{[H_{\mathrm{r}}^*(\omega_i)+1]}{H_{\mathrm{r}}^*(\omega_i)-1}z_1. \tag{9}$$

The $z_{2e}^I(\omega_i)$ is regarded as the known measured data. By using Eqs. (6)–(8), three parameters z_2, ψ_2 and β_2 can be calculated and determined successively by the reflected transfer functions.

According to Eqs. (5)–(9), the proposed simultaneous determination method of the four layer parameters using the normally-incident reflected and transmitted waves consists of the following two steps: first, by using Eqs. (6)–(8), the three parameters z_2, ψ_2 and β_2 are calculated and determined by the reflected transfer functions $H_{\mathrm{r}}^*(\omega_i)$; next, by using Eq. (5), the layer thickness h_2 is calculated and determined by the transmitted transfer functions $H_{\mathrm{t}}^*(\omega_i)$ and the determined parameters z_2, ψ_2 and β_2 in the first step, and the other two layer parameters c_2 and ρ_2 are also determined. The determination steps are consistent with those in the determination of the complete set of acoustical and geometrical parameters of a thin layer using the normal and oblique incidence proposed in Refs. [14]–[16], while the analytical formulas of the layer parameters related to the measured data and the corresponding intrinsic relations have not been derived.

In the proposed determination method, by using the four analytical formulas and the measured data, all the related parameters must be calculated or deter-

mined in a certain orderly way, which is an interesting fact that was not noticed before. In addition, here it is unnecessary to solve the nonlinear inversion equations and study the problems of the initial guesses and the convergence zones of the unknown parameters that are the required steps in the multi-parameter inversion methods.

3. The design of the measurement setups

To simultaneously determine the four layer parameters, the measured reflected and transmitted transfer functions $H_{\rm r}^*(\omega_i)$and $H_{\rm t}^*(\omega_i)$ need to be given. The schematic diagram of the measurement setups is shown in Fig. 1. The two independent measurement steps are performed to obtain $H_{\rm r}^*(\omega_i)$ and $H_{\rm t}^*(\omega_i)$. First, the short-duration pulse $f(t)$ (the reference waves) produced by the ultrasonic transducer 1 is recorded by the same transducer 2 as the thin layer is removed from the setups. Next, with the thin layer in place, the normally-incident reflected and transmitted waves $g_{\rm r}(t)$ and $g_{\rm t}(t)$ are recorded by the transducers 1 and 2, respectively, after the short-duration pulse is emitted by the transducer 1.

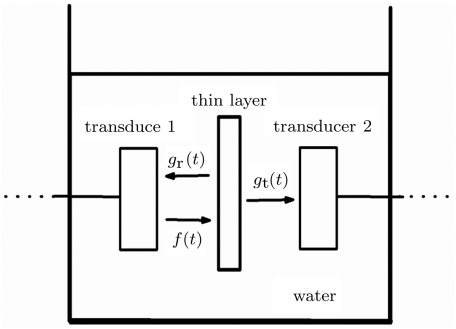


Fig. 1. A schematic diagram of the measurement setups of a linear-viscoelastic thin layer.

Here suppose the dispersion and attenuation of the immersed water are negligible. Since the time decays of $f(t)$ and $g_{\rm r}(t)$ are all in the water. To avoid carrying the extra phase-shift to the reflected transfer functions $H_{\rm r}^*(\omega_i)$in the calculation, the waves $f(t)$ and $g_{\rm r}(t)$ should be shifted to the same origin of time $(t = 0)$, we let $F'(\omega_i)$ and $G'_{\rm r}(\omega_i)$ represent the discrete Fourier transforms (DFTs) of the two shifted pulses. Then $H_{\rm r}^*(\omega_i)$ is equal to $G'_{\rm r}(\omega_i)/F'(\omega_i)$. Let $F(\omega_i)$ and $G_{\rm t}(\omega_i)$ represent the DFTs of $f(t)$ and $g_{\rm t}(t)$. Then $H_{\rm t}^*(\omega_i)$ is equal to $G_{\rm t}(\omega_i)/F(\omega_i)$. $H_{\rm r}^*(\omega_i)$ and $H_{\rm t}^*(\omega_i)$ are regarded as the measured quantities obtained from the ultrasonic experiment and used to determine the four layer parameters.

It is theoretically found that only three signals (the reflected and transmitted waves and the referenced waves) need to be recorded in the whole measurement setups and procedure as shown in Fig. 1. Compared with the traditional broadband reflection/transmission setups, it has the advantages of the least number of the recorded signals and the absence of the requirement of the non over lapping pulses from the front and back layer interfaces, which simplifies and optimizes the measurement setups and procedure.

4. Error analysis of the layer properties

Since the determination method of three parameters z_2, ψ_2 and β_2 with the reflected transfer functions in the first step was studied in Ref. [18]. Here we analyse the effect of the changes of the three determined parameters z_2, ψ_2 and β_2 on the determination of the four layer parameters. Suppose the relative errors of the estimated values $\hat{z}_2$, $\hat{\psi}_2$, $\hat{\beta}_2$ and $\hat{h}_2$ are $E_{\hat{z}_2}$, $E_{\hat{\psi}_2}$, $E_{\hat{\beta}_2}$ and $E_{\hat{h}_2}$, respectively. The relative error $E_{\hat{h}_2}$ is controlled by $E_{\hat{z}_2}$, $E_{\hat{\psi}_2}$ and $E_{\hat{\beta}_2}$. The relative errors of the estimated values $\hat{c}_2$ and $\hat{\rho}_2$ are derived and given as

$$\begin{aligned} E_{\hat{c}_2} &= \frac{(1+E_{\hat{h}_2})}{(1+E_{\hat{\psi}_2})} - 1, \\ E_{\hat{\rho}_2} &= \frac{(1+E_{\hat{z}_2})(1+E_{\hat{\psi}_2})}{(1+E_{\hat{h}_2})} - 1. \end{aligned} \tag{10}$$

For the small $E_{\hat{z}_2}$, $E_{\hat{\psi}_2}$ and $E_{\hat{h}_2}$, then

$$E_{\hat{c}_2} \cong E_{\hat{h}_2} - E_{\hat{\psi}_2}, \quad E_{\hat{\rho}_2} \cong E_{\hat{z}_2} + E_{\hat{\psi}_2} - E_{\hat{h}_2}. \tag{11}$$

It is shown by Eq. (10) that the errors $E_{\hat{c}_2}$ and $E_{\hat{\rho}_2}$ depend on $E_{\hat{h}_2}$ and $E_{\hat{z}_2}$, $E_{\hat{\psi}_2}$ and $E_{\hat{\beta}_2}$. Here the two special cases are discussed: (1) if $E_{\hat{z}_2} \neq 0$, $E_{\hat{\psi}_2} = 0$, then $E_{\hat{c}_2} = E_{\hat{h}_2}$; (2) if $E_{\hat{z}_2} = 0$, $E_{\hat{\psi}_2} \neq 0$, then $E_{\hat{c}_2} = -E_{\hat{\rho}_2} = E_{\hat{h}_2} - E_{\hat{\psi}_2}$.

5. Numerical simulation of the determination method

The advantage and convenience of the derived analytical formulas are that they make possible to numerically simulate and analyse the determination

method of the four layer parameters of the thin layer before the ultrasonic experiment. We first simulate the stability of the determination method of the layer thickness against the experimental noises. Acoustic and geometrical parameters of the constituent layers are listed in Table 1. An investigation procedure used in Ref. [14] has been applied here. Figure 2 show the typical 'noisy' synthetic reference waves $f(t)$, transmitted waves $g_{\rm t}(t)$ with the random noises level of 5% of the signal amplitude and the corresponding amplitude spectra $|F(\omega_i)|$ and $|G_{\rm t}(\omega_i)|$ and the calculated transfer functions $H_{\rm t}^*(\omega_i)$, where the waves $g_{\rm t}(t)$ are numerically generated by using a set of the layer parameters in Table 1. The transmitted transfer functions are calculated by the formula $H_{\rm t}^*(\omega_i) = G_{\rm t}(\omega_i)/F(\omega_i)$. Since the waves $g_{\rm t}(t)$ and $f(t)$ contain the 5% added random noises which is above the noise level in the typical experimental data. The calculated transmitted transfer functions $H_{\rm t}^*(\omega_i)$ are not equal to the original values and can be regarded as the measured quantities obtained from the ultrasonic experiment and used to investigate the stability of the method against the noises. To determine the layer thickness more accurately, the selected useful bandwidth is required to be symmetrical about the centre frequency, the absolute value of $H_{\rm t}^*(\omega_i)$ is less than 1.0 as far as possible. In Fig. 2 the centre frequency of the ultrasonic transducer is 5.0 MHz with the frequency interval of the two adjacent data points $\Delta f = 50$ kHz. The useful bandwidth is selected as 3.0–7.0 MHz. The calculated transmitted transfer functions $H_{\rm t}^*(\omega_i)$ with the two different layer thicknesses of 480 μm and 90 μm are plotted in Figs. 3(a) and 3(b). In Fig. 3, the discrepancies between the real and imaginary parts of $H_{\rm t}^*(\omega_i)$ and the corresponding original values are large even in the selected useful bandwidth, and become more worse in the two sides.

Table 1. Acoustic and geometrical parameters of the constituent materials.

materials	ρ/g · cm^{-3}	c/mm · μs^{-1}	h/μm	β
immersed water	0.997	1.49		
thin layer	1.175	2.630	(1) 480	0.126
(plexiglas)			(2) 90	

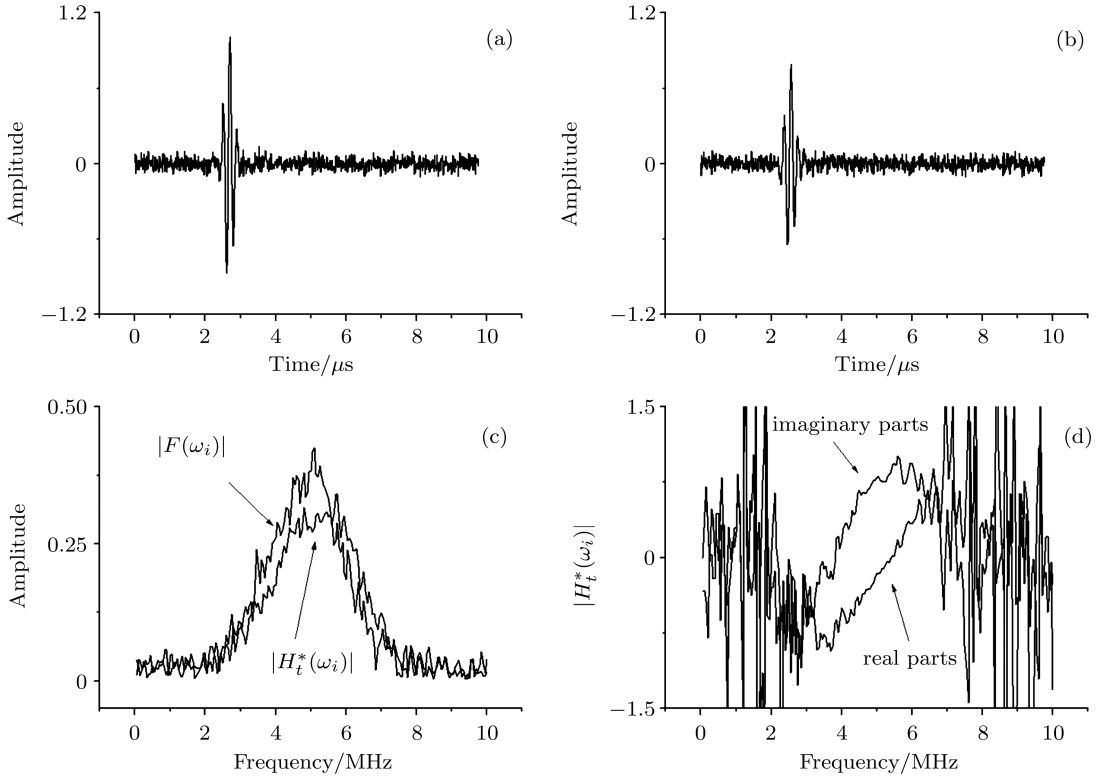


Fig. 2. Typical 'noisy' synthetic reference waves (a), transmitted waves (b) at normal incidence with the 5% random noises, the corresponding amplitude spectra (c) and the calculated transmitted transfer functions (d).

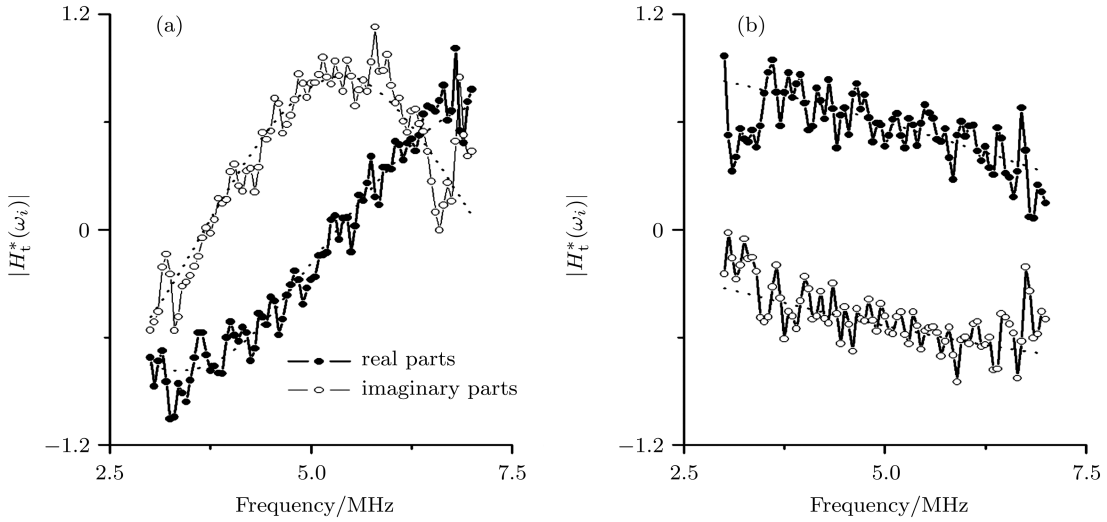


Fig. 3. The calculated transmitted transfer functions in the useful bandwidth. The layer thicknesses are 480 μm in Fig. 3(a) and 90 μm in Fig. 3(b). The dotted lines correspond to the transfer functions without noises.

From Eq. (5), we can see, one calculated value of the layer thickness needs two data points, while the selectable frequency intervals between the two used data points can change from Δf to $(n-1)\Delta f$, where n is the number of the data points in the useful bandwidth. For a given frequency interval of $m\Delta f$, we can successively change the subscript i in Eq. (5) and obtain $(n-m)$ calculated values. Figure 4 shows the calculated results of the two layer thicknesses with two frequency intervals of $20\Delta f = 1.0$ MHz and $40\Delta f = 2.0$ MHz. As shown in Fig. 4, for two different layer thicknesses 480 μm and 90 μm, the variation magnitude and the number of the calculated results all decrease as the frequency interval increases. The numbers of the calculated results are 61 and 41 for the two frequency intervals with $n = 81$. It is numerically shown that the effect of the noises on the calculated results is weakened with the large frequency intervals. However, using too large frequency intervals may lead to the more worse calculated results, because the used two data points are only located at the two sides of the useful bandwidth where the large discrepancies of the two used data from the corresponding original values may occur and the effect of the two bad data points counteracts that of using the large frequency intervals on the calculated results. Therefore the selection of the suitable frequency bandwidth and intervals is of great importance to obtain the abundant high-qualify calculated data.

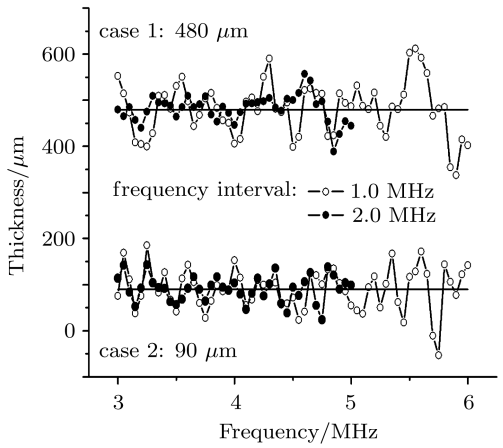


Fig. 4. The calculated results of the two layer thicknesses with two different frequency intervals of 1.0 MHz and 2.0 MHz. The thin solid lines correspond to the true values.

To show the statistical characteristic of the calculated results, choosing 41 frequency intervals increasing from $15\Delta f$ to $55\Delta f$, 1886 calculated results are obtained in total for two layer thicknesses and plotted in Fig. 5. The magnitude of the calculated results is large with the small frequency intervals and is very much reduced as the frequency intervals increase. Besides a few large-deviation calculated results in the small frequency intervals all symmetrically scatter around, and most are very close to the respective true values. Due to the characteristic of symmetrical scatter of all the calculated data, similar to the determination method of three parameters z_2, ψ_2 and

β_2 described in Ref. [18], here we perform the averaging operation for all the calculated results in Fig. 5 to minimise the effect of the random noises. Two average values $\bar{h}_2$ are 481.1 μm and 91.2 μm, and the relative errors $E_{\bar{h}_2}$ are only 0.2% and 1.3%, respectively. It is numerically shown that the determination method of the layer thickness by using the multiple frequency intervals and performing the averaging operation for all the calculated results can minimise the effect of the noises and obtain the more accurate average values.

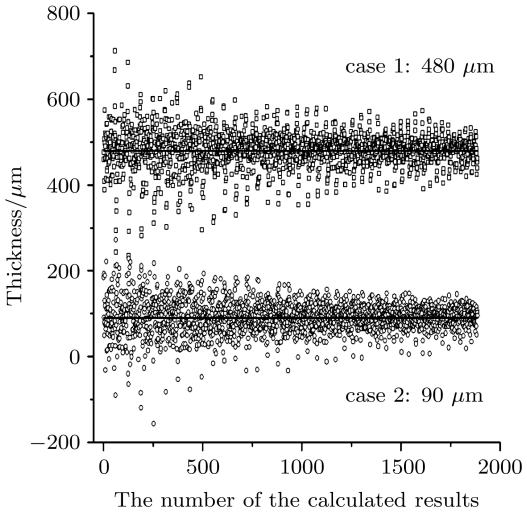


Fig. 5. The calculated results of the two layer thicknesses using the multiple frequency intervals. The used 41 frequency intervals are from 0.75 MHz to 2.75 MHz with $\Delta f = 0.05$ MHz. The thin solid lines correspond to the true values.

To reflect the total features of the effect of the noises on the average values of the layer thickness, Fig. 6 shows the relative errors of the twenty average values for two different layer thicknesses. The two thicknesses are same as those in Fig. 5. Using the same determination procedure mentioned above, we obtain each average value from each 'noisy' synthetic ultrasonic signals with the noise level of 5%. The useful bandwidth and the frequency intervals are also same as those in Fig. 5. Generally speaking, there exists an evident decrease in the relative errors of the average values as the layer thickness increases. For the layer thickness of 480 μm, with the exception of one value, all the relative errors do not exceed $\pm 3.0\%$, for 90 μm, the maximum value is up to about $\pm 10.0\%$. It shows that the proposed determination method is more suitable for the relatively thick thin layer, while more simplified than the traditional broadband reflection/transmission method. In addition, in practice, to obtain the more accurate estimated values of the layer thickness, an effective way is to perform the averaging operation for the average values again. The relative errors of the estimated values $\hat{h}_2$ are only 0.12% and 0.86% for the layer thicknesses of 480 μm and 90 μm.

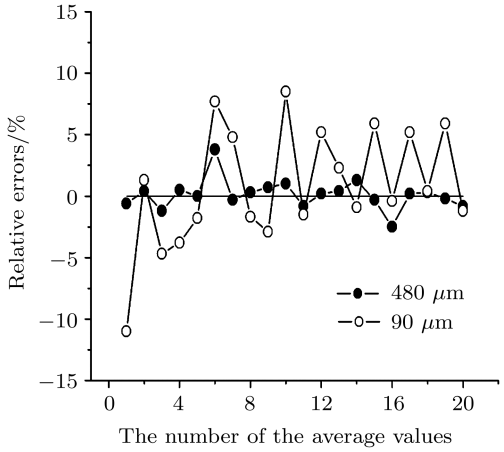


Fig. 6. The relative errors of the average values of the calculated results of the two layer thicknesses.

6. Conclusion

A method of simultaneous determination of the four layer parameters of an immersed linear-viscoelastic thin layer is proposed with the transfer functions of the normally-incident reflected and transmitted ultrasonic waves. Using the four derived analytical formulas of the related parameters, we calculate and determine the four layer parameters, including mass density, longitudinal velocity, the thickness and attenuation in a certain orderly way from the two steps. The determination method avoids the complexities of multi-parameter inversion. A more simplified measurement setup and procedure is described, in which only three signals (the reference waves, the reflected and transmitted waves) need to be recorded for the determination of the four layer parameters. It is numerically shown that the determination method can minimize the effect of the experimental noises by using the multiple frequency intervals in the useful bandwidth and performing the averaging operation for all the calculated results. This study lays the foundation of the practical measurement of a linear-viscoelastic thin layer.

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