Photonic band structures of quadrangular multiconnected networks*

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By means of the network equation and generalized dimensionless Floquet–Bloch theorem, this paper investigates the properties of the band number and width for quadrangular multiconnected networks (QMNs) with a different number of connected waveguide segments (NCWSs) and various matching ratio of waveguide length (MRWL). It is found that all photonic bands are wide bands when the MRWL is integer. If the integer attribute of MRWL is broken, narrow bands will be created from the wide band near the centre of band structure. For two-segment-connected networks and three-segment-connected networks, it obtains a series of formulae of the band number and width. On the other hand, it proposes a so-called concept of two-segment-connected quantum subsystem and uses it to discuss the complexity of the band structures of QMNs. Based on these formulae, one can dominate the number, width and position of photonic bands within designed frequencies by adjusting the NCWS and MRWL. There would be potential applications for designing optical switches, optical narrow-band filters, dense wavelength-division-multiplexing devices and other correlative waveguide network devices.

Keywords: multiconnected network, waveguide, photonic band structure
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1. Introduction

Since the photonic crystals were proposed in 1987,[1,2] photonic band-gap (PBG) materials have attracted much attention.[3−8] Propagation of electromagnetic waves (EM waves) in PBG generated lots of interesting physical phenomena and new fields were opened up in condensed matter physics, optics, material science and so on. The physical concepts and methods related to PBG have been gradually extended to many fields. An incomplete list of fundamental investigations includes PBG structures,[4,9] properties of transmission and reflection,[10,11] photonic waveguides,[12,13] phenomena of negative refraction,[14,15] defect states,[16−18] etc.

As the lattice constant of PBG is on the scale of the wavelength of EM waves, the shorter the work wavelength is, the smaller the characteristic length of PBG is and then the more difficult to fabricate the structures will be, especially for three-dimensional structures. However, for the periodic PBG networks composed of one-dimensional (1D) waveguides, it is experimentally easily realizable because of their structural flexibility,[19−22] and the phase, amplitude and wave function at each node can also be easily accessible. Moreover, strong scattering can be easily introduced in a unit cell to produce large full gaps in any direction.[22] Based on the aforementioned advantages, it is very convenient for the periodic networks composed of 1D waveguides to accomplish experimental investigations. As a result, there have been extensive investigations on this kind of structure, such as defect state mode,[23] random scattering,[24] slow light effect,[25] band gap structure,[21] nonlinear optical effect[26] and so on.

The networks mentioned above are connected by 1D waveguides between adjacent nodes. Only monomode propagation of EM waves needs to be considered and can be regarded as a linear combination of two opposite traveling plane waves. The wave function between nodes i and j satisfies

$$\psi_{ij} = \alpha_{ij} \mathrm{e}^{\mathrm{i}\,kx} + \beta_{ij} \mathrm{e}^{-\mathrm{i}\,kx}, \tag{1}$$

where k=ω/c, ω and c are, respectively, angular frequency and the speed of EM waves. Moreover, at any node, the EM waves satisfies the energy flux conser-

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$$\sum_j \frac{1}{\mu\omega}\psi_{ij}\frac{\partial\psi_{ij}}{\partial x}\,A_{ij}|_{x=0} = 0. \tag{2}$$

Therefore, one can obtain the following network equation:[19,21,22,27]

$$-\psi_i \sum_j \cot(kl_{ij}) + \sum_j \psi_j \csc(kl_{ij}) = 0, \tag{3}$$

where ψ_i (ψ_j) denotes the wave function at node i (j), l_{ij} is the length of the waveguide segment between nodes i and j, the summation is over all of the nodes linked directly to i. When there exists one or more segments between adjacent nodes and the network structure is strictly spatial periodic, one can obtain the band structures by solving a set of coupled equations of Eq. (3) by means of Floquet–Bloch theorem.

It is known that introducing some loops in a network can create band gaps and the loops will produce resonances at certain frequencies.[19,22,23] Adding more segments between adjacent nodes is equivalent to introducing more loops in a network, so the possibility of producing resonances and/or antiresonances will be greatly increased and the band gap structure and localization of the propagation in waveguide segments can be affected remarkably. For networks composed of several segments between adjacent nodes (multiconnected), Floquet–Bloch theorem can be applied to those where there exists absolutely spatial periodicity, but when the periodicity is broken this theorem will be disabled. Recently, we have proposed a dimensionless Floquet–Bloch theorem, which can be expressed as follows:[22]

$$\psi_{\boldsymbol{K}}(\boldsymbol{N}+\boldsymbol{T}) = \psi_{\boldsymbol{K}}(\boldsymbol{N})\mathrm{e}^{\mathrm{i}\,\boldsymbol{K}\cdot\boldsymbol{T}}, \tag{4}$$

where $\boldsymbol{T}$ is the configuration translation vector, $\boldsymbol{N}$ is the node index vector, $\boldsymbol{K}$ is the configuration 'wavevector', and they are all dimensionless. By use of the generalized dimensionless Floquet–Bloch theorem we studied the square multiconnected networks, where the spatial periodicity is broken but there exists configuration translation periodicity, and found extremely large attenuations even though no absorption networks and wide stopbands and narrow passbands are created by resonances and antiresonances. Further, one would like to know, what are the concrete rules of the number, width and position of these wide stopbands and narrow passbands? And which parameters will control these properties? These are interesting problems and the answers would be useful for designing correlative waveguide network devices.

As one of the series works of our group, in this paper, following our published work[22] we study in detail the influences of the number of connected waveguide segments (NCWS), the parity of matching ratio of waveguide length (MRWL) and the broken of the integer attribute of MRWL on the band structures of quadrangular multiconnected networks (QMNs) shown in Fig. 1, and obtain a series of formulae of the number and width of photonic bands. It is found that adjusting NCWS and MRWL one can effectively control the band structure of these networks, especially the number, width and position of the bands near the centre of band structure. It would be useful for designing optical switches, optical narrow-band filters, dense wavelength-division-multiplexing devices and other correlative waveguide network devices.

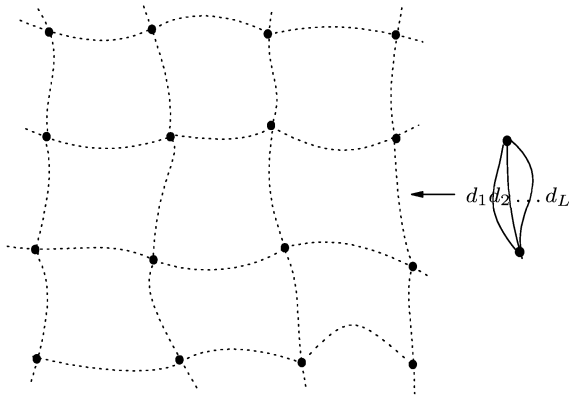


Fig. 1. The QMN, where each dash line connecting adjacent nodes denotes L types of waveguide segment.

This paper is organized as follows. In Section 2, we mainly study the influence of the parity of the MRWL on the photonic band structures for two-segment-connected networks (2SCNs) and three-segment-connected networks (3SCNs). Section 3 is devoted to the investigations on the band variations of 2SCNs and 3SCNs after the integer attribute of MRWL is broken. The summary of the paper is given in Section 4.

2. Influence of the parity of MRWLs

It is known that adding more segments between adjacent nodes is equal to introducing more loops in a network, comparedwith the periodic PBG networks

connected by one segment the resonances and/or antiresonances created in multiconnected PBG networks will be varied. In this section, we discuss the influence of odd and even MRWL on band structures for 2SCNs and 3SCNs, respectively.

2.1. The 2SCN

For the 2SCN with the MRWL of $d_2 : d_1 = n : 1$, it is found that there only exist wide bands in the band structure, where the number of wide stopbands

$$M_{\mathrm{WS}} = n - 1 \tag{5}$$

and the number of wide passbands

$$M_{\mathrm{WP}} = n. \tag{6}$$

The width of the wide band near the centre of band structure $\omega{=}\pi c/d_1$ is

$$\Delta\omega_{\mathrm{WB-CBS}} = \frac{2}{n} \cdot \frac{\pi c}{d_1}, \tag{7}$$

while the widths of other wide bands are identical and can be expressed as follows:

$$\Delta\omega_{\mathrm{WB-others}} = \frac{1}{n} \cdot \frac{\pi c}{d_1}. \tag{8}$$

Further, when the MRWL is odd, the stopband near the centre of band structure is a wide passband; otherwise, that will be a wide stopband. As examples of the cases with equal, odd and even MRWL, we calculate the networks with n=1, 5 and 6, respectively. The results are shown in Fig. 2. (i) When n=1, the lengths of the two segments between adjacent nodes are the same. Compared with the one-segment-connected network (1SCN), we find that although many new loops are introduced in the network, the loops are completely identical. Consequently, in the case of photonic quantum effect, the 2SCN connected by two identical segments is equal to the 1SCN. The only difference between them is the magnitude of EM waves traveling from one node to another.[22] There is no resonance in the network and no gap in the band structure. Band structure of the network connected by two identical segments is shown in Fig.2(a). The symmetry points Γ, X and M correspond to the configuration 'wavevectors' $\boldsymbol{K} = (K_1, K_2) = (0, 0), (0, \pi)$ and $(\pi,\ \pi)$, respectively. Indeed, there is no gap in the band structure and the whole frequency area is covered with n=1 wide passband. Equations (5)–(7) are confirmed by numerical simulations. (ii) When $n = 5$, the lengths of the two segments between adjacent nodes are different, and it corresponds to adding asymmetrical loops to the 1SCN. Not only antiresonances but also resonances are created, and there are both passbands and stopbands in the band structure. Band structure for 2SCN with the MRWL of $d_2 : d_1 = 5 : 1$ is shown in Fig.2(b). One can see that, there are $n - 1 = 4$ wide stopbands and $n = 5$ wide passbands, the widths of the other 8 wide bands are all $0.200\pi c/d_1$ except that the width of the wide passband near the centre of band structure is $0.400\pi c/d_1$. Calculations are accordant with Eqs. (5)–(8). (iii) When $n = 6$, the MRWL is $d_2 : d_1 = 6 : 1$. From Fig. 2(c) one can see that, there exist $n - 1 = 5$ wide stopbands and $n = 6$ wide passbands. The width of the wide stopband near the centre of band structure is $0.333\pi c/d_1$ while the widths for the other wide bands are all $0.167\pi c/d_1$. Calculations accord with Eqs. (5)–(8).

Based on the aforementioned results one knows that the network connected by segments with different lengths corresponds to adding new asymmetrical loops to the 1SCN, then in the network both antiresonances and resonances are produced, and in the band structure both passbands and stopbands exist. When the MRWL is integer, wave path difference of EM waves traveling in segments is integral multiple of wavelength, then the phase variation producing destructive and instructive interferences is slow. Therefore, all of the photonic bands are wide. However, when the integer attribute of MRWL is broken, wave path difference of EM waves will decrease and some narrow bands will be created from wide bands. The smaller the broken of the integer attribute of the MRWL is, the smaller the wave path difference will be, and the larger the phase variation is, the narrower the new bands will be. It can be seen in Section 3. Adjusting the broken of the integer attribute of MRWL, one can effectively control the width of thenarrow bands. There would be potential applications for designing optical switches, optical narrow-band filters, dense wavelength-division- multiplexing devices and other correlative waveguide network devices.

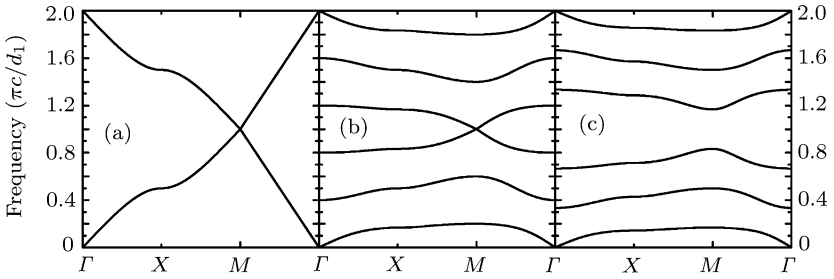


Fig. 2. Band structures of the quadrangular 2SCN with the MRWLs: (a) $d_2 : d_1 = 1 : 1$; (b) $d_2 : d_1 = 5 : 1$ and (c) $d_2 : d_1 = 6 : 1$.

2.2. The 3SCN

Compared with 1SCN and 2SCN, the band structure of 3SCN is quite different. As examples, we calculate the networks with the MRWL of $d_3 : d_2 : d_1 = 5 : 5 : 1$ and $d_3 : d_2 : d_1 = 5 : 1 : 1$, shown in Figs.3(a) and 3(b). One can see that the two networks both possess 9 wide passbands and 8 wide stopbands, but the band shapes are obviously different from each other. For example, the shape of the wide passband within (0.640–0.700)$\pi c/d_1$ for the former network is depressed, but that for the latter network is raised. It is mainly caused by the interactions of two-segment-connected quantum subsystems (2SCQSs).

In the case of quantum effect, every two waveguide segments between adjacent nodes in the network construct a quantum subsystem, which depends on each other and is called as 2SCQS. As a result, for the network with the MRWL of $d_3 : d_2 : d_1 = 5 : 5 : 1$ there exist one 5:5 and two 5:1 2SCQSs, but that with the MRWL of $d_3 : d_2 : d_1 = 5 : 1 : 1$ possesses two 5:1 and one 1:1 2SCQSs. Obviously, the 2SCQSs of the two networks are not the same. This is the reason that makes the band structures of the two networks different from each other. Furthermore, the interactions among 2SCQSs make the subsystem composed of two equal segments not equivalent to 1SCN, which is different from the network with the MRWL of $d_2 : d_1 = 1 : 1$ discussed in subsection 2.1. Therefore, the two networks with the MRWL of $d_3 : d_2 : d_1 = 5 : 5 : 1$ and $d_3 : d_2 : d_1 = 5 : 1 : 1$ will not be equal to the 2SCN with the MRWL of $d_2 : d_1 = 5 : 1$ any more. Comparing with Figs. 2(b), 3(a) and 3(b) one can see that the position, shape, and even number of the photonic bands for the three kinds of network possess remarkable differences.

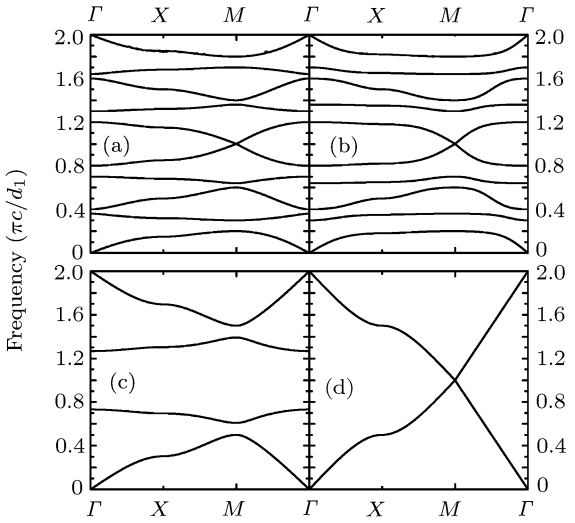


Fig. 3. Band structures of the quadrangular 3SCN with MRWLs: (a) $d_3 : d_2 : d_1 = 5 : 5 : 1$; (b) $d_3 : d_2 : d_1 = 5 : 1 : 1$; (c) $d_3 : d_2 : d_1 = 2 : 2 : 1$ and (d) $d_3 : d_2 : d_1 = 1 : 1 : 1$.

Generally, when the NCWS is not less than 3, i.e., $L \geq 3$, there exist more aforementioned 2SCQSs and the influence among subsystems will be much more complex. In this subsection we study the 3SCNs with the MRWL of $d_3 : d_2 : d_1 = n : n : 1$ and find that when n is odd, the number of wide stopbands is

$$M_{\mathrm{WS-odd}} = 2n - 2, \tag{9}$$

and the number of wide passband is

$$M_{\mathrm{WP-odd}} = 2n - 1. \tag{10}$$

The band near the centre of band structure is a wide passband. When n is even, the number of wide stopbands is

$$M_{\mathrm{WS-even}} = 2n - 1, \tag{11}$$

and the number of wide passbands is

$$M_{\mathrm{WP-even}} = 2n. \tag{12}$$

The band near the centre of band structure is a wide stopband.

As examples of the cases with odd, even and equal MRWL, we calculate the networks with $n = 5$, 2 and 1, respectively. The results are shown in Figs. 3(a), 3(c) and 3(d). (i) In Fig. 3(a) the MRWL is $d_3 : d_2 : d_1 = 5 : 5 : 1$ and n is odd, there exist $2 \times 5 - 2 = 8$ wide stopbands and $2 \times 5 - 1 = 9$ wide passbands. The band near the centre of band structure is a wide passband. Calculations accord with Eqs. (9) and (10). (ii) In Fig. 3(c) the MRWL is $d_3 : d_2 : d_1 = 2 : 2 : 1$ and n is even, there exist $2 \times 2 - 1 = 3$ wide stopbands and $2 \times 2 = 4$ wide passbands. The band near the centre of band structure is a wide stopband. Calculations accord with Eqs. (11) and (12). (iii) In Fig. 3(d) the MRWL is $d_3 : d_2 : d_1 = 1 : 1 : 1$, there exist three 1 : 1 2SCQSs with identical MRWL. Being completely identical the three 2SCQSs can be considered as one 2SCQS with identical MRWL. As the case of system $d_2 : d_1 = 1 : 1$ in subsection 2.1, the 3SCN with $d_3 : d_2 : d_1 = 1 : 1 : 1$ is equal to a 1SCN, and the only difference among the three networks is the magnitude of EM waves traveling from one node to another. Compared with Figs. 2(a) and 3(d), we find that the structures, shapes, numbers and positions of the frequency bands for two networks are completely the same, i.e., in the range of $0 \sim 2\pi c/d_1$ their band structures are both covered with an identical wide passband. Calculations accord with Eqs. (9) and (10).

3. Influence of the parity broken of MRWLs

For actual waveguide networks it would be difficult to make the MRWL integral. Influenced by environment temperature, pressure, material shape differences and other factors, the integer attribute of MRWL would be easily broken. When the brokenness happens, there is a mutation in phase difference, the wave path difference of EM waves traveling in waveguide segments will not be integral multiple of the wavelength, and the resonances (antiresonances) created by destructive (instructive) interference mutates to strong antiresonances (resonances). Consequently, narrow passbands (narrow stopbands) are created from the wide stopband (wide passband) near the centre of band structure. The smaller the brokenness is, the faster the phase difference changes, and the narrower the narrow passbands (narrow stopbands) created near the centre of band structure will be. In this section we investigate the changes of band structures of 2SCN and 3SCN after the integer attribute of MRWLs is broken and obtain a series of width formulae under this condition, where we make a high order approximation.

3.1. Two 2SCN

The band structure for 2SCN mutates when the integer attribute of MRWL is broken. In this subsection we study the system with the MRWL of

$$d_2 : d_1 = (n \pm \Delta d) : 1. \tag{13}$$

It is found that when n is odd (even), a narrow stopband (narrow passband) is produced from the wide passband (wide stopband) near the centre of band structure and the width of the narrow band is

$$\Delta\omega_{\mathrm{N1}} \approx \frac{\Delta d}{n} \cdot \frac{\pi c}{d_1}. \tag{14}$$

In Eq. (13) if one chooses '+', another narrow stopband can be created near the top of band structure $\omega = 2\pi c/d_1$ and its width is

$$\Delta\omega_{\mathrm{N2}} \approx \frac{2\Delta d}{n} \cdot \frac{\pi c}{d_1}. \tag{15}$$

As examples of the cases with odd and even MRWL, we calculate the networks with $n = 5$ and 6, respectively. The results are shown in Fig. 4. (i) When $n = 5$ and $\Delta d = 0.16$, the network can be regarded as the system mutated from the network with $d_2 : d_1 = 5 : 1$. From Fig. 2(b) one can see that, for the network with $d_2 : d_1 = 5 : 1$ there is a wide passband near the centre of band structure in the range of $(0.800\text{–}1.200)\pi c/d_1$. From Fig. 4(a) one can see that, for the network with $d_2 : d_1$=5.16:1 the wide passband near the centre of band structure in the range of $(0.774\text{–}1.163)\pi c/d_1$ is separated into 2 wide passbands by a narrow stopband in the range of $(0.968\text{–}1.000)\pi c/d_1$ and the width of the narrow stopband is $0.032\pi c/d_1$. In addition, another narrow stopband near the top of band structure with the width of $0.064\pi c/d_1$ is created. From Fig. 4(b) one can see that, for the network with $d_2 : d_1$=4.84:1 the wide passband near the centre of band structure in the range of $(0.826\text{–}1.240)\pi c/d_1$ is separated into 2 wide passbands by a narrow stopband in the range of $(1.000\text{–}1.032)\pi c/d_1$ and the width of the narrow

stopband is also $0.032\pi c/d_1$. The widths of narrow stopbands accord with Eqs. (14) and (15). (ii) When n=6 and Δd=0.06, the network can be regarded as the system mutated from the network with $d_2 : d_1$=6:1. From Fig.2(c) one can see that, for the network with $d_2 : d_1$=6:1 there is a wide stopband near the centre of band structure in the range of (0.833–1.167)$\pi c/d_1$. From Fig. 4(c) one can see that, for the network with $d_2 : d_1 = 6.06 : 1$ the wide stopband near the centre of band structure in the range of (0.825–1.155)$\pi c/d_1$ is separated into 2 wide stopbands by a narrow passband in the range of (0.990–1.000)$\pi c/d_1$ and the width of the narrow stopband is $0.010\pi c/d_1$. In addition, another narrow stopband near the top of band structure with the width of $0.020\pi c/d_1$ is created. From Fig. 4(d) one can see that, for the network with $d_2 : d_1 = 5.94 : 1$ the wide stopband near the centre of band structure in the range of (0.842–1.178)$\pi c/d_1$ is separated into 2 wide stopbands by a narrow passband in the range of (1.000–1.010)$\pi c/d_1$ and the width of the narrow stopband is also $0.010\pi c/d_1$. The widths of narrow stopbands and narrow passbands accord with Eqs. (14) and (15).

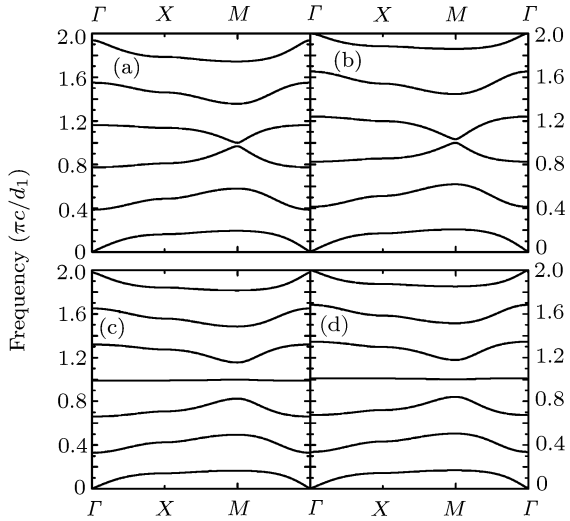


Fig. 4. Band structures of the quadrangular 2SCN with the MRWLs: (a) $d_2 : d_1 = 5.16 : 1$; (b) $d_2 : d_1 = 4.84 : 1$; (c) $d_2 : d_1 = 6.06 : 1$ and (d) $d_2 : d_1 = 5.94 : 1$.

In a word, the brokenness of the integer attribute of MRWL leads to the phase difference of EM waves no longer being an integral multiple of 2π. It makes the resonances (antiresonances) mutate to strong antiresonances (resonances) and for networks with odd (even) MRWL a narrow stopband (narrow passband) will be produced from the wide passband (wide stopband) near the centre of band structure, where the widths of these two kinds of narrow bands all satisfy Eq. (14). By the use of Eq. (14) one can control the width of the narrow band by adjusting the MRWL parameter n and broken factor Δd. There would be potential applications to improving sensitivity of optical switches, precision of optical narrow-band filters and design of other correlative waveguide network devices.

3.2. The 3SCN with 1 brokenness of matching ratio

In this subsection we investigate the network with the MRWL of

$$d_3 : d_2 : d_1 = (n_1 \pm \Delta d) : n_2 : 1. \tag{16}$$

Compared with the 2SCN defined by Eq. (13), the band structure for the 3SCN defined by Eq. (16) is not the same. The differences between them are shown as follows. (i) Near the centre of band structure, a narrow passband is produced in the latter network, while in the former network a narrow band is also created but the sort of the narrow band depends on the parity of the MRWL (see subsection 3.1). (ii) Near the top of band structure, another narrow passband is produced when one chooses '+' in Eq. (16), while another narrow stopband is created when '+' is chosen in Eq. (13).

Here we obtain the width formulae of the narrow passbands near the centre of band structure and the top of band structure for three kinds of network defined by Eq. (16). (i) When n_1=n_2=n and n is even, the width of the narrow passband near the centre of band structure is

$$\Delta\omega_{\mathrm{NP1}} \approx \frac{\Delta d}{2n} \cdot \frac{\pi c}{d_1}. \tag{17}$$

(ii) When n_1=n_2=1, the width of the narrow passband near the centre of band structure is

$$\Delta\omega_{\mathrm{NP2}} \approx \frac{\Delta d}{3} \cdot \frac{\pi c}{d_1}, \tag{18}$$

and the width of the narrow passband near the top of band structure is

$$\Delta\omega_{\mathrm{NP3}} \approx \frac{2\Delta d}{3} \cdot \frac{\pi c}{d_1}. \tag{19}$$

(iii) When n_1 is odd and bigger than 1, $n_2 = 1$, the width of the narrow passband near the centre of band structure is

$$\Delta\omega_{\mathrm{NP4}} \approx \frac{\Delta d}{6(n_1 - 1)} \cdot \frac{\pi c}{d_1}, \tag{20}$$

and the width of the narrow passband near the top of band structure is

$$\Delta\omega_{\mathrm{NP5}} \approx \frac{\Delta d}{3(n_1 - 1)} \cdot \frac{\pi c}{d_1}. \tag{21}$$

As examples of the cases with odd and even MRWL, we calculate two types of networks with $n_1 = n_2 = 2$, $\Delta d = 0.04$ and $n_1 = 3$, $n_2 = 1$, $\Delta d = 0.09$, respectively. The results are shown in Fig. 5. (i) Figures 5(a) and 5(b) are the band structures of the networks with MRWLs of $d_3 : d_2 : d_1 = 2.04 : 2 : 1$ and $d_3 : d_2 : d_1 = 1.96 : 2 : 1$, respectively. One can see that in both networks 1 narrow passband with the width of $0.01\pi c/d_1$ is produced from the wide stopband near the centre of band structure. Calculated results accord with Eq. (17). Comparing with the network of integer MRWL of $d_3 : d_2 : d_1 = 2 : 2 : 1$, we find that the band structure will change remarkably after the integer attribute of MRWL is broken. From Figs. 3(c), 5(a) and 5(b) one can see that the brokenness leads to many produced narrow bands, which are mainly caused by the comprehensive interactions among 2SCQSs. For a 3SCN (e.g. 2.04:2:1), a brokenness of integer MRWL (e.g. 2.04) makes two brokennesses of integer MRWL (e.g. 2.04:2 and 2.04:1) of 2SCQS. So, different from the 2SCN discussed in subsection 3.1, where 1 brokenness creates 1 narrow band, for 3SCN or more-segment-connected network (MSCN) 1 brokenness causes several brokennesses of 2SCQSs and consequently, leads to more than 1 produced narrow band. It should be pointed out, the number of new produced narrow passbands near the centre of band structure is always equal to the broken number, not the broken number of 2SCQSs.

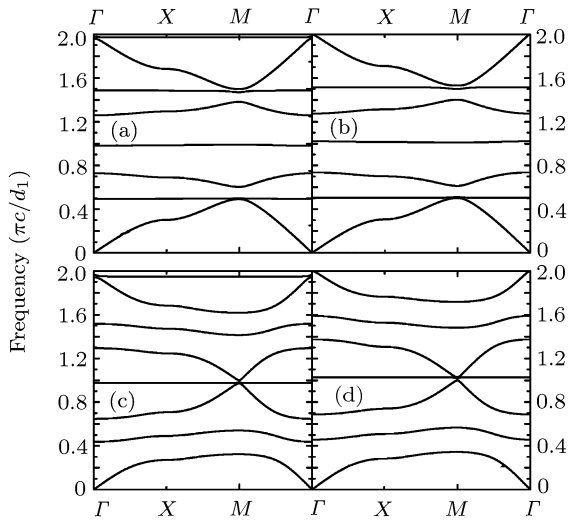


Fig. 5. Band structures of the quadrangular 3SCN with the MRWLs: (a) $d_3 : d_2 : d_1 = 2.04 : 2 : 1$; (b) $d_3 : d_2 : d_1 = 1.96 : 2 : 1$; (c) $d_3 : d_2 : d_1 = 3.09 : 1 : 1$ and (d) $d_3 : d_2 : d_1 = 2.91 : 1 : 1$.

(ii) Figures 5(c) and 5(d) are the band structures of the networks with MRWLs of $d_3 : d_2 : d_1 = 3.09 : 1 : 1$ and $d_3 : d_2 : d_1 = 2.91 : 1 : 1$, respectively. One can see that in both networks 1 narrow passband with the width of $0.080\pi c/d_1$ is produced from the wide stopband near the centre of band structure. In addition, another narrow passband of the width of $0.015\pi c/d_1$ is produced near the top of band structure. Calculated results accord with Eqs. (20) and (21).

3.3. The 3SCNs with 2 brokennesses of matching ratio

In this subsection we investigate the network with the MRWL of

$$d_3 : d_2 : d_1 = (n_1 \pm \Delta d_1) : (n_2 \pm \Delta d_2) : 1. \quad (22)$$

It is found that 2 narrow passbands are produced from the wide stopband near the centre of band structure and at the same time 2, 1 and no narrow passband(s) will be created near the top of band structure when two, one and no '+' are/is chosen in Eq. (22), respectively.

In order to simplify our expressions we order $\Delta d_1 < \Delta d_2$. If $n_1 = n_2 = n$, n is even, and two '+' or two '−' are chosen in Eq. (22), the widths of the 2 narrow passbands near the centre of band structure are, respectively,

$$\Delta\omega_{\mathrm{NP1}} \approx \frac{\Delta d_2 - \Delta d_1}{2n} \cdot \frac{\pi c}{d_1}, \quad (23)$$

and

$$\Delta\omega_{\mathrm{NP2}} \approx \frac{\Delta d_1}{n} \cdot \frac{\pi c}{d_1}. \quad (24)$$

When two '+' are chosen in Eq. (22), the widths of the 2 narrow passbands near the top of band structure are, respectively,

$$\Delta\omega_{\mathrm{NP3}} \approx \frac{\Delta d_2}{2n} \cdot \frac{\pi c}{d_1}, \quad (25)$$

and

$$\Delta\omega_{\mathrm{NP4}} \approx \frac{2\Delta d_1}{2n+1} \cdot \frac{\pi c}{d_1}. \quad (26)$$

For the network with the MRWL of $d_3 : d_2 : d_1 = (n + \Delta d_2) : (n - \Delta d_1) : 1$, the widths of the 2 narrow passbands near the centre of band structure are, respectively,

$$\Delta\omega_{\mathrm{NP5}} \approx \frac{\Delta d_1 + \Delta d_2}{2n} \cdot \frac{\pi c}{d_1} \quad (27)$$

and

$$\Delta\omega_{\mathrm{NP6}} \approx \frac{\Delta d_1}{n} \cdot \frac{\pi c}{d_1}. \quad (28)$$

As examples, we calculate the networks with $n_1 = n_2 = 2$, $\Delta d_1 = 0.07$ and $\Delta d_2 = 0.16$. Band structures of networks with the MRWL of $d_3 : d_2 : d_1 = 2.16 : 2.07 : 1$, $d_3 : d_2 : d_1 = 1.93 : 1.84 : 1$ and $d_3 : d_2 : d_1 = 2.16 : 1.93 : 1$ are shown in Fig. 6. From Fig. 6 one can see that 2 narrow passbands are all produced from the wide stopband near the centre of band structure in these three kinds of networks. In Figs. 6(a) and 6(b), the widths of the 2 narrow passbands are equal to $0.025\pi c/d_1$ and $0.035\pi c/d_1$, respectively. The widths of the 2 narrow passbands in Fig. 6(c) are not equal to those in Figs. 6(a) and 6(b) and equal to, respectively, $0.054\pi c/d_1$ and $0.036\pi c/d_1$. Near the top of band structure, the widths of the 2 narrow passbands in Fig. 6(a) are equal to $0.026\pi c/d_1$ and $0.036\pi c/d_1$, respectively. The 1 narrow passband is produced in Fig. 6(c) but its width formula is not obtained. Calculated results accord approximately with Eqs. (23)–(28), and there exist some deviations. The main reason is that in order to distinguish narrow passbands clearly, we choose bigger Δd_1 and Δd_2, but the formulae in this subsection are all obtained under high order approximation. So the bigger the Δd_1 and Δd_2 are, the larger the relative deviation between calculations and analytical results will lead to.

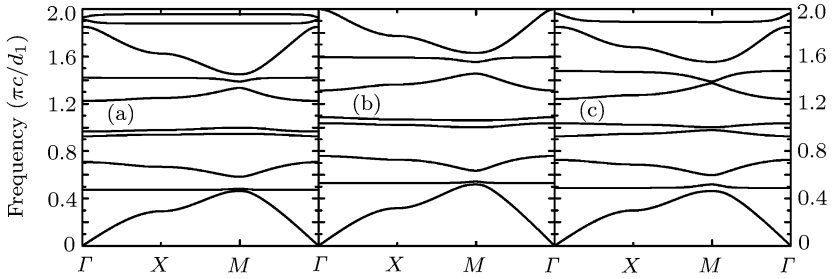


Fig. 6. Band structures of the QMNs with $L = 3$. The MRWL for (a)–(c) are $d_3 : d_2 : d_1 = 2.16 : 2.07 : 1$, $d_3 : d_2 : d_1 = 1.93 : 1.84 : 1$, and $d_3 : d_2 : d_1 = 2.16 : 1.93 : 1$, respectively.

On the other hand, from the results in Section 3 one can also see that the number of newly produced narrow passbands is equal to the broken number of the MRWL and does not increase with the increment of the NCWS. For instance, in the network with the MRWL of $d_3 : d_2 : d_1 = 2.16 : 2.07 : 1$ (see Fig. 6(a)) there are 2 brokennesses of MRWL and then 2 narrow passbands are produced near the centre of band structure. In the network with the MRWL of $d_3 : d_2 : d_1$=2.04:2:1 (see Fig.5(a)) there is 1 brokenness of MRWL and only 1 narrow passband is created. It means that the number of new produced narrow passbands is equal to the broken number of the MRWL. Additionally, in the network with the MRWL of $d_2 : d_1$=6.06:1 (see Fig.4(c)) there is 1 brokenness of MRWL and only 1 narrow passband is created. Comparing with Figs.4(c) and 5(a) one can see that the number of new produced narrow passbands does not increase with the increment of the NCWS. Generally, the width, number and distance of new produced narrow passbands near the center of band structure can be controlled by adjusting the NCWS and the brokenness of the MRWL. Due to the complexity of the interactions among 2SCQSs, it is very difficult to control the widths and distances of all narrow passbands at the same time, especially for the complex networks with a large NCWS. Therefore, simply by means of this kind of network it is very hard to obtain a large amount of narrow passbands with identical width and distance, even the optical frequency comb-like effect.

4. Conclusion

In this paper, by means of the network equation and generalized dimensionless Floquet–Bloch theorem we investigate the properties of the band number and width for QMNs with different NCWS and various MRWL.

It is found that for the QMNs with integer MRWL, the phase differences of EM waves are all integral multiple of 2π and all bands are wide bands. For the 2SCN, we obtain the formulae of the number and width of wide bands. For the 3SCN and MSCN, we propose the concept of 2SCQS and analyse the complexity of band structures by use of the interactions among 2SCQSs. When the network is connected by more than two segments between adjacent nodes, the interactions among 2SCQSs make the band widths different from each other and uniform width formulae can not be obtained. When adjacent nodes are connected

by more than two identical segments, the 2SCQSs are all the same, then the influence of 2SCQSs does not exist and this kind of equal-length-segment-connected network can be equal to the 1SCN.

When the integer attribute of MRWL is broken, the phase difference of EM waves is no longer integral multiple of 2π, resonances (antiresonances) mutate to strong antiresonances (resonances), narrow bands are produced from wide bands and the number of narrow bands near the centre of band structure is equal to the broken number of the MRWL. For the 2SCN there is only 1 brokenness of the MRWL, so only 1 narrow band can be created from the wide band near the centre of band structure and the width formula of the narrow band is obtained. For the 3SCN and MSCN with 1 brokenness of the MRWL, the new produced bands are all narrow passbands and we obtain the width formulae of these narrow passbands. For the 3SCN with 2 brokennesses of the MRWL, the interactions among 2SCQSs are very complex, we mainly study the network with the MRWL of $d_3 : d_2 : d_1$=$(n_1 \pm \Delta d_1)$:$(n_2 \pm \Delta d_2)$:1 and identical evens n_1, n_2, and obtain the formulae of the band number and width near the centre and top of band structure. It is found that even for the networks with 2 brokennesses of the MRWL, it is very difficult to control the width and distance of all narrow passbands at the same time. For the multiconnected networks with more brokennesses of the MRWL, one can imagine how difficult it would be to control the width and distance of all narrow passbands at the same time. Thus, simply by this kind of network it is very hard to obtain a number of narrow passbands with identical widths and distances.

In short, after obtaining these quantitative rules, one can control the number, width and position of photonic bands in the designed frequency range. There would be potential applications for designing optical switches, optical narrow-band filters, dense wavelength-division-multiplexing devices and other correlative waveguide network devices.

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