Einstein–Podolsky–Rosen entanglement in bad cavity case*

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This paper explores continuous variable entanglement in four-wave mixing when the atomic relaxation time is comparable to and longer than the cavity relaxation time. In this case the atomic memory is included in the field correlations and the entanglement in the output fields can be significantly enhanced. Einstein–Podolsky–Rosen (EPR) entanglement is achievable even in the bad cavity limit. This shows the EPR entanglement generation without need of good cavity.

Keywords: atomic memory effect, EPR entanglement, four-wave mixing
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1. Introduction

In the studies of light–matter interactions one usually assumes that atoms relax into the steady state much more rapidly than the optical fields in the cavity.[1] This assumption makes one eliminate adiabatically atomic variables. The adiabatic elimination makes the theoretical treatment become simple. However, in many situations, the atomic variables do not necessarily relax more rapidly than the cavity fields. For example, a three-level Λ atomic system exhibiting coherent population trapping[2] and electromagnetically induced transparency,[3−5] the two ground states and the coherence between them are long-lived. Even in some cases one involves a bad cavity.[6−9] Instead of the atoms, the cavity fields decay relatively more rapidly. In other words, the atoms can have longer relaxation times than that of the cavity fields. In this case, the atoms have not enough time to evolve into the steady state before the cavity fields reach the steady state. This gives rise to atomic memory to the field dynamics. It was predicted that due to the atomic memory the laser linewidth and the intensity fluctuations are reduced to a certain degree.[6] As it is, this is a very universal case. Even for the two-level atom, the simplest atom model, the ground state does not decay at all although the excited state decays more rapidly than the cavity fields. Carmichael[8] and Reid[9] once presented the nonadiabatic effects on quantum correlations of the absorptive and dispersive optical bistability in the two-level systems.

Here we explore the atomic memory effects in the four-wave mixing. It was demonstrated that the four-wave mixing provides an efficient and important way for the generation of squeezed light.[10−17] Recently, by using a spatially multimode amplifier based on four-wave mixing in a hot vapour, Boyer *et al.*[18] produced twin images exhibited localized entanglement. It has become aware that it is potentially efficient way for continuous variable entanglement.[19−26] The variance sum of a pair of Einstein–Podolsky–Rosen (EPR) like operators (a criterion for intracavity field entanglement) is reduced by 50% below the standard quantum limit.[22,23] Alike, atomic variables are adiabatically eliminated. For an enhancement of entanglement, Pielawa *et al.*[24] proposed a two-step atomic reservoir engineering in the absence of spontaneous emission. In this scheme, both the atoms and the cavity fields have sufficiently long relaxation times, during which the dissipation due to the atomic absorption pulls the cavity fields into EPR state. It is a challenge to perform such a reservoir engineering. In each step, atoms of different frequencies and different initial superposition states are chosen and prepared for the common cavity fields. It is highly disable to obtain an enhancement of the entanglement even in the bad cavity case. In what follows we will verify our expectation. Almost perfect EPR entangled state can even be obtained in the bad cavity case.

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2. Model and equation

We consider an ensemble of N two-level atoms inside a two-mode optical cavity. The atom has the ground state $|1\rangle$ and the excited state $|2\rangle$. The transition between them is electronic dipole-allowed. One external coherent driving field of frequencies $\omega_{\rm d}$ is applied and the two cavity fields of frequencies $\omega_{1,2}$ are generated from two Rabi sidebands of the driven atoms. The master equation for the density operator of the atom–field system is derived in the dipole approximation and an appropriate rotating frame as[1]

$$\dot{\rho} = -\frac{\mathrm{i}}{\hbar}[H_0 + H_{\rm I}, \rho] + L_{\rm a}\rho + L_{\rm c}\rho, \tag{1}$$

where Hamiltonian $H_0 = \sum_{\mu=1}^{N} H_0^{\mu}$ denotes the interaction between the coherent driving fields and the atoms, and $H_{\rm I} = \sum_{\mu=1}^{N} H_{\rm I}^{\mu}$ represents the interaction of the two cavity fields with the atoms,

$$H_0^{\mu} = \hbar\left[\Delta\sigma_{22}^{\mu} + \frac{\Omega}{2}(\sigma_{12}^{\mu} + \sigma_{21}^{\mu})\right], \tag{2}$$

$$H_{\rm I}^{\mu} = \hbar\left(g_1 a_1 \mathrm{e}^{-\mathrm{i}\,\delta_1 t}\sigma_{21}^{\mu} + g_2 a_2 \mathrm{e}^{-\mathrm{i}\,\delta_2 t}\sigma_{21}^{\mu}\right) + \text{H.c.}. \tag{3}$$

In Eqs. (2) and (3), H.c. is the Hermitian conjugate. $\sigma_{kl}^{\mu} = |k^{\mu}\rangle\langle l^{\mu}|$ $(k, l = 1, 2)$ are the projection operators when $k = l$ and the spin-flip operators while $k \neq l$. We have assumed that the wave vector matching is satisfied and the atomic states contain the phase factors due to the randomly orientated dipole moments. The a_l and $a_l^{\dagger}$ are photon annihilation and creation operators for the cavity fields, g_l are the coupling constants. Ω is the Rabi frequency associated with the coherent driving field and is assumed to be real for simplicity. $\Delta = \omega_0 - \omega_{\rm d}$ and $\delta_l = \omega_l - \omega_{\rm d}$ are the detunings of the atomic resonance frequency ω_0 and the cavity field frequencies ω_l from the driving field frequency $\omega_{\rm d}$, respectively. The atomic and cavity relaxation terms are written as

$$L_{\rm a}\rho = \sum_{\mu=1}^{N}\frac{\gamma}{2}\left(2\sigma_{12}^{\mu}\rho\sigma_{21}^{\mu} - \rho\sigma_{21}^{\mu}\sigma_{12}^{\mu} - \sigma_{21}^{\mu}\sigma_{12}^{\mu}\rho\right), \tag{4}$$

$$L_{\rm c}\rho = \sum_{l=1,2}\frac{\kappa_l}{2}\left(2a_l\rho a_l^{\dagger} - \rho a_l^{\dagger}a_l - a_l^{\dagger}a_l\rho\right), \tag{5}$$

where γ represents the atomic relaxation rate due to spontaneous emission from the excited state $|2\rangle$ to the ground state $|1\rangle$, and $\kappa_{1,2}$ are the field relaxation rates due to the cavity leakage.

Since we are interested in Rabi resonances we consider the case when the driving field is strong, i.e., $d = \sqrt{\Delta^2 + \Omega^2} \gg (|\langle g_l a_l\rangle|, \gamma)$. It is convenient to use the dressed atomic states. By diagonalizing the Hamiltonian H_0^{μ} we obtain the dressed states[27]

$$|\tilde{1}^{\mu}\rangle = \cos\theta|1^{\mu}\rangle - \sin\theta|2^{\mu}\rangle, \tag{6}$$

$$|\tilde{2}^{\mu}\rangle = \sin\theta|1^{\mu}\rangle + \cos\theta|2^{\mu}\rangle, \tag{7}$$

where $\tan(2\theta) = \Omega/\Delta$, $0 < \theta < \pi/2$, $|\tilde{1}^{\mu}\rangle$and $|\tilde{2}^{\mu}\rangle$ have their eigenvalue $\lambda_{1,2} = (\Delta \mp d)/2$. The Hamiltonian H_0 takes the form $H_0 = \sum_{\mu=1}^{N}\hbar\lambda_l\tilde{\sigma}_{ll}^{\mu}$, where $\tilde{\sigma}_{kl}^{\mu} = |\tilde{k}^{\mu}\rangle\langle\tilde{l}^{\mu}|$, $k, l = 1, 2$. Since the driving field is strong, the dressed states are well separated from each other. We can choose the single-photon processes between the dressed states by tuning the cavity fields resonant with Rabi sidebands, i.e., $\delta_1 = -\delta_2 = d$. The case for $\delta_1 = -\delta_2 = -d$ is treated in a similar way. Making a unitary transformation with $U = \exp(-\mathrm{i}\,H_0 t/\hbar)$ and neglecting the rapidly oscillating terms such as $\exp(\pm\mathrm{i}\,\mathrm{d}t)$ and $\exp(\pm 2\,\mathrm{i}\,\mathrm{d}t)$, we rewrite the master equation for $\tilde{\rho} = U^{\dagger}\tilde{\rho}U$ in terms of atomic dress states as

$$\frac{\mathrm{d}}{\mathrm{d}t}\tilde{\rho} = -\frac{\mathrm{i}}{\hbar}[\tilde{H}_{\rm I}, \tilde{\rho}] + L_{\rm a}\tilde{\rho} + L_{\rm c}\tilde{\rho}, \tag{8}$$

where Hamiltonian takes the form

$$\tilde{H}_{\rm I} = \sum_{\mu=1}^{N}\hbar(g_1\cos^2\theta a_1 - g_2\sin^2\theta a_2^{\dagger})\tilde{\sigma}_{21}^{\mu} + \text{H.c.}. \tag{9}$$

The cavity damping term $L_{\rm c}\tilde{\rho}$ keeps the original form unchanged except for substitution of $\tilde{\rho}$ for ρ, and the damping term for the atoms is rewritten as

$$\begin{aligned} L_{\rm a}\tilde{\rho} = &\sum_{\mu=1}^{N}\frac{\Gamma}{2}\left(2\tilde{\sigma}_{12}^{\mu}\rho\tilde{\sigma}_{21}^{\mu} - \rho\tilde{\sigma}_{21}^{\mu}\tilde{\sigma}_{12}^{\mu} - \tilde{\sigma}_{21}^{\mu}\tilde{\sigma}_{12}^{\mu}\rho\right) \\ &+\sum_{\mu=1}^{N}\frac{\Lambda}{2}\left(2\tilde{\sigma}_{21}^{\mu}\rho\tilde{\sigma}_{12}^{\mu} - \rho\tilde{\sigma}_{12}^{\mu}\tilde{\sigma}_{21}^{\mu} - \tilde{\sigma}_{12}^{\mu}\tilde{\sigma}_{21}^{\mu}\rho\right) \\ &+\sum_{\mu=1}^{N}\Gamma_p\left(2\tilde{\sigma}_p^{\mu}\rho\tilde{\sigma}_p^{\mu} - \rho\tilde{\sigma}_p^{\mu}\tilde{\sigma}_p^{\mu} - \tilde{\sigma}_p^{\mu}\tilde{\sigma}_p^{\mu}\rho\right), \end{aligned} \tag{10}$$

where $\Gamma = \gamma\cos^4\theta$, $\Lambda = \gamma\sin^4\theta$, $\Gamma_p = \frac{\gamma}{2}\cos^2\theta\sin^2\theta$, and $\tilde{\sigma}_p^{\mu} = \tilde{\sigma}_{11}^{\mu} - \tilde{\sigma}_{22}^{\mu}$.

Comparing the atomic and cavity relaxation times we have two extreme cases. One is the good cavity limit, where the atomic relaxation time is much shorter than the cavity relaxation time, and the other is the bad cavity limit, where the case is just reversed. Between the two extreme cases there is the intermediate case, where the atomic relaxation time is comparable to the cavity relaxation time. Usually, one assumes atomic variables to decay much more rapidly than the fields and then employs the adiabatic approximation to eliminate the atomic variables.[1,28,29]

However, in many situations the atoms do not decay more rapidly than cavity fields, as stated in Section 1. In this case the adiabatic elimination does no longer hold. Here we focus on this case. Following the standard lines,[8,9,30–32] in what follows we calculate the quantum correlations in the mixing interaction without elimination of atomic variables.

By means of the generalized P representation of Drummond and Gardiner[31,32] we first derive a set of Langevin equations from the master equation (8). In order to do so, we define the atomic operators $\tilde{\sigma}_{kl} = (1/N)\sum_{\mu=1}^{N}\tilde{\sigma}_{kl}^{\mu}$, choose a definite order $(a_1^{\dagger}, a_2^{\dagger}, \tilde{\sigma}_{21}, \tilde{\sigma}_{22}, \tilde{\sigma}_{12}, a_2, a_1)$, and use the correspondences between the c-numbers and the operators $\alpha_i \leftrightarrow a_i$, $\alpha_i^* \leftrightarrow a_i^{\dagger}$, $v_{kl} \leftrightarrow \tilde{\sigma}_{kl}$. Then we derive the set of Langevin equations as

$$\dot{\alpha}_1 = -\frac{\kappa_1}{2}\alpha_1 - \mathrm{i}\, g_1 \cos^2\theta v_{12} + F_{\alpha_1}, \tag{11}$$

$$\dot{\alpha}_2 = -\frac{\kappa_2}{2}\alpha_2 + \mathrm{i}\, g_2 \sin^2\theta v_{21} + F_{\alpha_2}, \tag{12}$$

$$\dot{v}_{12} = -\left(\frac{\Gamma}{2} + \frac{\Lambda}{2} + \Gamma_p\right)v_{12} - \mathrm{i}\,\beta v_{11} + \mathrm{i}\,\beta v_{22} + F_{v_{12}}, \tag{13}$$

$$\dot{v}_{22} = \Gamma v_{11} - \Lambda v_{22} + \mathrm{i}\,\beta^* v_{12} - \mathrm{i}\,\beta v_{21} + F_{v_{22}} \tag{14}$$

together with those for the corresponding variables α_1^*, α_2^*, and v_{21}. In Eqs. (13) and (14) we have used $\beta = g_1\cos^2\theta\alpha_1 - g_2^*\sin^2\theta\alpha_2^*$. The populations follow the closure relation $v_{11} + v_{22} = 1$. The F_x are the Langevin fluctuation operators and are assumed to be δ correlated, satisfying $\langle F_x(t)\rangle = 0$, $\langle F_x(t)F_y(t')\rangle = D_{xy}\delta(t-t')$ with $D_{xy} = D_{yx}$, $D_{y^*x^*} = D_{xy}^*$. We assume that there is no pumping of these modes by the external driving fields. Steady state solutions are obtained by setting the time derivatives to zero. The nonzero solutions are

$$\langle v_{11}\rangle = \frac{\cos^4\theta}{\cos^4\theta + \sin^4\theta} \quad \text{and} \quad \langle v_{22}\rangle = \frac{\sin^4\theta}{\cos^4\theta + \sin^4\theta},$$

and the nonzero diffusion coefficients are $D_{v_{21}v_{12}} = \Lambda + 2\Gamma_p\langle v_{22}\rangle$, $D_{v_{22}v_{22}} = 2\Gamma\langle v_{22}\rangle$, and $D_{v_{12}\alpha_2} = -\mathrm{i}\, g\sin^2\theta(2\langle v_{22}\rangle - 1)$.

3. Quantum correlation for the output fields

According to the entanglement criterion proposed by Duan *et al.*,[33] the two fields in the output are entangled if the following inequality is satisfied

$$V = \left\langle(\delta u)^2\right\rangle + \left\langle(\delta v)^2\right\rangle - c^2 - \frac{1}{c^2} < 0, \tag{15}$$

where we have defined a pair of EPR operators $u = cx_1 - \frac{1}{c}x_2$ and $v = cp_1 + p_2/c$ with the quadratures $x_l = (1/\sqrt{2})(a_l^{\mathrm{out}}\mathrm{e}^{\mathrm{i}\,\phi_l} + a_l^{\dagger\mathrm{out}}\mathrm{e}^{-\mathrm{i}\,\phi_l})$, $p_l = (1/\sqrt{2}\,\mathrm{i})(a_l^{\mathrm{out}}\mathrm{e}^{\mathrm{i}\,\phi_l} - a_l^{\dagger\mathrm{out}}\mathrm{e}^{-\mathrm{i}\,\phi_l})$ for the output fields $a_l^{\mathrm{out}} = \sqrt{\kappa_l}\,|\langle a_l\rangle|\,\mathrm{e}^{-\mathrm{i}\,\phi_l}$, $l = 1, 2$. At the steady state, the sum of the respective phases ϕ_l is locked to $\phi_1 + \phi_2 = 0$. The inequality reduces to

$$V = 4\sqrt{\kappa_1\kappa_2}\left(\sqrt{\langle\delta\alpha_1^{\dagger}\delta\alpha_1\rangle\langle\delta\alpha_2^{\dagger}\delta\alpha_2\rangle} - |\langle\delta\alpha_1\delta\alpha_2\rangle|\right) < 0, \tag{16}$$

which indicates that entanglement is obtainable when the nonclassical correlation between two modes satisfies $|\langle\delta\alpha_1\delta\alpha_2\rangle| > \sqrt{\langle\delta\alpha_1^{\dagger}\delta\alpha_1\rangle\langle\delta\alpha_2^{\dagger}\delta\alpha_2\rangle}$. We are interested in the output spectrum. We linearize the Langevin equations (11)–(14) by writing $\delta O = O - \langle O\rangle$ and then write the set of equations in a compact form

$$\frac{\mathrm{d}}{\mathrm{d}t}\delta O(t) = -A\delta O(t) + F(t), \tag{17}$$

where the column vectors are arranged into $\delta O(t) = (\delta\alpha_1,\ \delta\alpha_2,\ \delta\alpha_1^*,\ \delta\alpha_2^*,\ \delta v_{12},\ \delta v_{21},\ \delta v_{22})^{\mathrm{T}}$ and $F(t) = (F_{\alpha_1},\ F_{\alpha_2},\ F_{\alpha_1^*},\ F_{\alpha_2^*},\ F_{v_{12}},\ F_{v_{21}},\ F_{v_{22}})^{\mathrm{T}}$. Both the drift matrix $\boldsymbol{A}$ and the noise correlations $\langle F(t)F^{\mathrm{T}}(t')\rangle = D\delta(t-t')$ can be easily obtained from Eqs. (11)–(14). The spectrum $S(\omega) = \int_{-\infty}^{\infty}\mathrm{e}^{\mathrm{i}\,\omega\tau}\langle\delta O(t)\,\delta O^{\mathrm{T}}(t+\tau)\rangle\mathrm{d}\tau$ takes the form[8,9,21–31]

$$S(\omega) = (A + \mathrm{i}\,\omega I)^{-1}D(A^{\mathrm{T}} - \mathrm{i}\,\omega I)^{-1}. \tag{18}$$

Then the entanglement criterion inequality (15) can be rewritten in terms of the spectrum as

$$\begin{aligned} V(\omega) = {} & \kappa_1 c^2\left[S_{13}(\omega) + S_{31}(\omega)\right] \\ & + \frac{\kappa_2}{c^2}\left[S_{24}(\omega) + S_{42}(\omega)\right] \\ & - \sqrt{\kappa_1\kappa_2}\Big[S_{12}(\omega) + S_{21}(\omega) \\ & + S_{34}(\omega) + S_{43}(\omega)\Big]. \end{aligned} \tag{19}$$

By choosing $c^2 = \sqrt{\dfrac{\kappa_2\langle\delta\alpha_2^*\delta\alpha_2\rangle}{\kappa_1\langle\delta\alpha_1^*\delta\alpha_1\rangle}}$, we have $V_u(\omega) = V_v(\omega) = \frac{1}{2}V(\omega)$, where $V_u(\omega)$ and $V_v(\omega)$ are the fluctuation spectra for the EPR operators u and v. Then we can denote the degree of entanglement by $V(\omega)$. If $V(\omega) = -2$ we have the perfect EPR entanglement.[34]

So far we have derived the correlation spectrum for the criterion of entanglement without adiabatic elimination of atomic variables. Therefore the spectrum holds for arbitrary rates of atomic and cavity relaxation times. The spectrum exists if the steady

state solutions are stable. The stability can be verified by calculating the eigenvalues of the matrix $\boldsymbol{A}$, which is performed by using the computer. When the real parts of all eigenvalues are positive, then the system is stable. In our numerical calculation, we assume that $g_1 = g_2 = g$ and $\kappa_1 = \kappa_2 = \kappa$ for simplicity detunings, Rabi frequencies, coupling constants, and atomic decay rates are scaled in units of κ, the relaxation times in units of κ^{-1}. In Fig. 1 we plot the zero frequency spectrum $V(0)$ as a function of the ratio Δ/Ω for different ratios of atomic and field relaxation times $\gamma^{-1}/\kappa^{-1} = 0.2$ (dotted, a good cavity case), 1 (dashed, an intermediate case), and 10 (solid, a bad cavity case). The parameter is $g^2N = 100$. This figure shows that the dip of the spectrum drops as the atomic relaxation time increases. For the three cases, the minimal values are (a) $V_{\min} \approx -1.639$ at $\Delta/\Omega \approx \pm 0.31$ for $\gamma^{-1}/\kappa^{-1} = 0.2$, (b) $V_{\min} \approx -1.832$ at $\Delta/\Omega \approx \pm 0.21$ for $\gamma^{-1}/\kappa^{-1} = 1$, and (c) $V_{\min} \approx -1.946$ at $\Delta/\Omega \approx \pm 0.12$ for $\gamma^{-1}/\kappa^{-1} = 10$. In the situation (a), the atoms have the much shorter relaxation time than that of the cavity fields, i.e., $\gamma^{-1} \ll \kappa^{-1}$. This corresponds to a good cavity case. In a very short time compared with the cavity relaxation time κ^{-1}, the atoms evolve into the steady state. In this case, usual adiabatic elimination is valid. However, this is no longer valid in cases (b) and (c), where the atomic relaxation time is comparable with or longer than the cavity relaxation time, $\gamma^{-1} \sim \kappa^{-1}$ and $\gamma^{-1} > \kappa^{-1}$. The atoms and cavity fields evolve in the comparable time period. During the cavity relaxation time, the atoms cannot arrive at their steady states. Therefore the spontaneous emission cannot treated as an uncorrelated event. This indicates that the spontaneous emission has a relatively small contribution to the cavity field fluctuations. This is the very memory effect, as pointed out previously,[6,7] which gives rise to a reduction in the laser linewidth and the intensity fluctuation. In the present case, due to the atomic memory, the correlation between two output beams is enhanced when $\gamma^{-1} \sim \kappa^{-1}$ and $\gamma^{-1} > \kappa^{-1}$. In case (c) we have almost perfect EPR entanglement. In addition, we show the correlation spectrum $V(\omega)$ in Fig. 2 for different ratios of atomic and field relaxation times $\gamma^{-1}/\kappa^{-1} = 0.2$ (dotted), 1 (dashed), and 10 (solid). The parameters are chosen as $g^2N = 100$, $\Delta/\Omega = 0.2$. The remarkable feature is the occurrence of side peaks. The positions of the peaks are determined by the eigenfrequencies of the elements v_{12} and v_{21} and the eigenfrequencies are determined by the drift matrix $\boldsymbol{A}$. This reflects the dynamics of the atoms and the cavity fields when the atoms have long relaxation times. For this reason new sideband resonances only appear when $\gamma^{-1} \sim \kappa^{-1}$ and $\gamma^{-1} > \kappa^{-1}$, i.e., when the atomic memory plays its role.

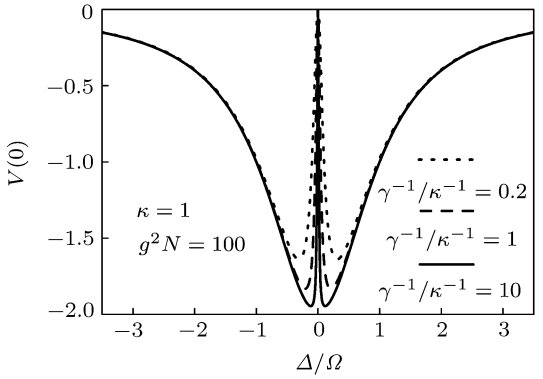


Fig. 1. The zero frequency output spectrum $V(0)$ vs the normalized detuning Δ/Ω for $\gamma^{-1}/\kappa^{-1} = 0.2$ (dotted), 1 (dashed), 10 (solid). The parameter is chosen as $g^2N = 100$.

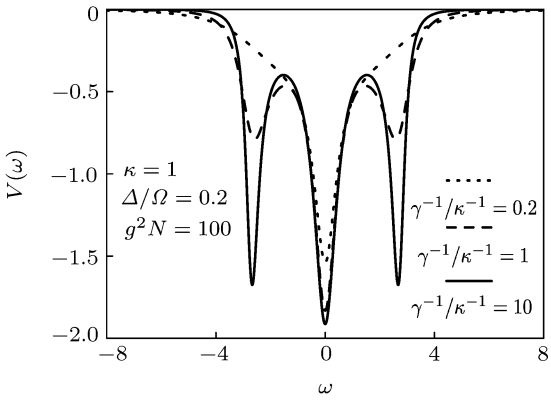


Fig. 2. The output spectrum $V(\omega)$ vs frequency ω for $\gamma^{-1}/\kappa^{-1} = 0.2$ (dotted), 1 (dashed), 10 (solid). The parameters are $g^2N = 100$, $\Delta/\Omega = 0.2$.

4. Conclusion

In conclusion, we have shown that EPR entanglement is possible in the bad cavity case. Since the atomic relaxation time is comparable to or longer than the cavity relaxation time, the atomic memory plays its role in the field correlations. Without the adiabatic elimination of atomic variables we have calculated the correlation in the output fields. It has been shown that the atomic memory reduces the quantum fluctuations, enhances the entanglement, even yields EPR entanglement. This suggests the possibility of generating EPR entanglement without the need of high quality cavity.

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