Analytical solution of microwave transition spectral lines for ^{87}Rb atoms in a Hanle configuration*

Hu Zheng-Feng(胡正峰)†, Deng Jian-Liao(邓见辽), Ma Yi-Sheng(马易升), He Hui-Juan(何慧娟), and Wang Yu-Zhu(王育竹)

The Key Laboratory of Quantum Optics, Shanghai Institute of Optics and Fine Mechanics, Chinese Academy of Sciences, Shanghai 201800, *China*

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This paper theoretically investigates the microwave transition spectrum of ^{87}Rb atomic D1 line with specially prepared atomic state in a Hanle configuration. The approximate analytical results have shown that the 0–0 transition spectral line has the highest contrast and can be applied to microwave frequency standards.

Keywords: photon–atom interaction, susceptibility, quantum interference phenomena

PACC: 4250, 3280, 3200

1. Introduction

The microwave hyperfine transition spectral line of Alkali-metal vapour has been applied to the microwave atomic clock for many years.[1] At present the investigations have been focused on the miniature vapour-cell atomic frequency references and the increase of the stability of the atomic clock.[2,3] At the same time, in order to improve the performance of atomic clock, many quantum optics theories and techniques have been applied to atomic reference. For example, apart from the traditional Λ-type coherent population trapping (CPT) microwave atomic clock,[4,5] the four-level configuration atom has been used for the atomic clock.[6] By comparing with the detection of the CPT transparent spectral line, the absorption spectral line is measured in multi-level atomic system. The absorption spectrum of four-level atomic system interacted with coupling, probe and microwave fields has been studied.[7−9] The multi-level microwave transitions and nonlinear magneto-optical rotation in a coated cell have been investigated by using laser optical pumping.[10] In this paper we study the microwave transition spectral lines for the multi-level atomic system. The procedure is that at first we optically pump all the population into a specific state, such as $|F_g = 2, m_F = 2\rangle$ end state. This can be realised by circularly polarised lights. In our system by using an intense coupling light and a weak probe light we can fulfil the optical pumping. In experiment one can obtain the coupling or the probe lights with one laser device by using a Pockels cell. At the end of optical pumping electromagnetically induced transparency (EIT) system[11−16] has been obtained. At the same time, a weak microwave field is swept over all the ground states transitions, such as $|F_g = 2\rangle \leftrightarrow |F_g = 1\rangle$. Although this kind of problem can be solved by numerically computing the motion equations of the density matrix elements, we will give the detailed analysis of the system and by solving the twelve individual four-level atomic systems we will give the analytical expression of the absorption line and the numerical calculations. These results clearly show that the 0–0 microwave transition spectral line has the highest height and contrast which may be used for the microwave atomic clock. This kind of microwave atomic clock uses one laser and microwave device (the probe light can be obtained by using a Pockels cell). Compared with a traditional CPT clock, atomic populations will steadily stay in an energy state due to very weak probe and microwave fields, so we expect that this kind clock will exhibit higher contrast. Because only 0–0 transition has been detected, the zero-order Zeeman effect will be very weak.

2. Theory and simulations

The system is considered as shown in Fig. 1, where $|F = 1\rangle$, $|F = 2\rangle$, $|F' = 1\rangle$ are the hyper-

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†Corresponding author. E-mail: huzhengfeng@tsinghua.org.cn

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fine states of warm ^{87}Rb atom ensemble. The transition of $|F = 2\rangle \leftrightarrow |F' = 1\rangle$ is the D1 line of ^{87}Rb. Under the condition of a weak magnetic field, these hyperfine states will be split into the corresponding Zeeman states. Initially the atoms are optically pumped into the end state $|F = 2, m_F = 2\rangle$. Because the line width of laser light is much larger than the energy shift caused by weak magnetic field, a weak probe laser field can couple the transitions $|F' = 1, m_{F'} = -1, 0, 1\rangle \leftrightarrow |F = 2, m_F = 2, 1, 0\rangle$ and an intense coupling laser field drives the transitions $|F' = 1, m_{F'} = -1, 0, 1\rangle \leftrightarrow |F = 2, m_F = 0, -1, -2\rangle$. That means with respect to laser field the Zeeman states are degenerate.

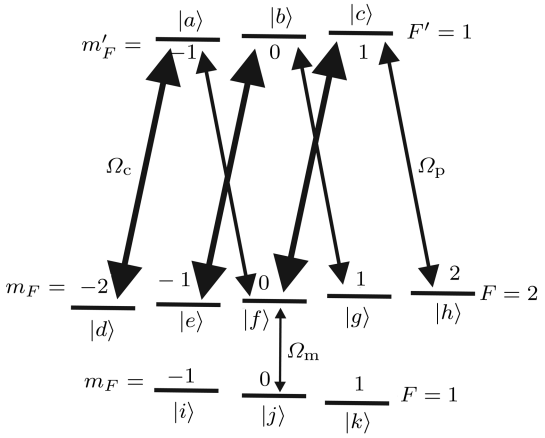


Fig. 1. The system diagram and its related transitions. A coupling laser with frequency ω_c and a probe laser field with frequency ω_p drive the transitions $|F' = 1\rangle \leftrightarrow |F = 2\rangle$. A microwave field with frequency ω_m couples the transitions of the Zeeman states of the ground hyperfine states.

A weak microwave field is scanned and drives the Zeeman states transitions of the ground hyperfine states according to selection rule. The system Hamiltonian can be written as

$$H = H_{\rm atom} + H_{\rm coupling} + H_{\rm probe} + H_{\rm microwave}, \quad (1)$$

where $H_{\rm atom}$ is the atomic Hamiltonian and $H_{\rm coupling}$, $H_{\rm probe}$, $H_{\rm microwave}$ are the interaction parts of the Hamiltonian.

$$\begin{aligned} & H_{\rm atom} \\ = {} & \hbar\omega_b|b\rangle\langle b| + \hbar(\omega_b - \delta')|a\rangle\langle a| \\ & +\hbar(\omega_b + \delta')|c\rangle\langle c| + \hbar\omega_j|j\rangle\langle j| \\ & +\hbar(\omega_j - \delta_1)|k\rangle\langle k| + \hbar(\omega_j + \delta_1)|i\rangle\langle i| \\ & +\hbar\omega_f|f\rangle\langle f| + \hbar(\omega_f - 2\delta_2)|d\rangle\langle d| \\ & +\hbar(\omega_f - \delta_2)|e\rangle\langle e| + \hbar(\omega_f + \delta_2)|g\rangle\langle g| \\ & +\hbar(\omega_f + 2\delta_2)|h\rangle\langle h|, \end{aligned} \quad (2)$$

$$\begin{aligned} & H_{\rm coupling} \\ = {} & -\hbar\left[\Omega_c\,{\rm e}^{-{\rm i}\omega_c t}\,(C_{cf}|c\rangle\langle f| + C_{be}|b\rangle\langle e| \right. \\ & \left. +C_{ad}|a\rangle\langle d|) + {\rm h.c.}\right], \end{aligned} \quad (3)$$

$$\begin{aligned} & H_{\rm probe} \\ = {} & -\hbar\left[\Omega_p\,{\rm e}^{-{\rm i}\omega_p t}\,(C_{af}|a\rangle\langle f| + C_{bg}|b\rangle\langle g| \right. \\ & \left. +C_{ch}|c\rangle\langle h|) + {\rm h.c.}\right], \end{aligned} \quad (4)$$

$$\begin{aligned} & H_{\rm microwave} \\ = {} & -\hbar\left[\Omega_m\,{\rm e}^{-{\rm i}\omega_m t}\,(C_{fj}|f\rangle\langle j| \right. \\ & +C_{ei}|e\rangle\langle i| + C_{gk}|g\rangle\langle k| \\ & +C_{di}|d\rangle\langle i| + C_{ej}|e\rangle\langle j| \\ & +C_{fk}|f\rangle\langle k| + C_{gj}|g\rangle\langle j| \\ & \left. +C_{fi}|f\rangle\langle i| + C_{hk}|h\rangle\langle k|) + {\rm h.c.}\right], \end{aligned} \quad (5)$$

where $\hbar\omega_b$, $\hbar\omega_f$, $\hbar\omega_j$ are the energies with respect to Zeeman states $|F_e = 1, m_F = 0\rangle$, $|F_g = 2, m_F = 0\rangle$, $|F_g = 1, m_F = 0\rangle$ and δ_1, δ_2, δ' are the energy shifts of hyperfine states defined as $\delta = -\mu_{\rm B} g_F m_F B/\hbar$ with the Bohr magneton $\mu_{\rm B}$, g factor g_F, inside a weak magnetic field B. The ω_p, ω_c, ω_m are the angular frequencies corresponding to the probe, coupling and microwave fields respectively. The Ω_c, Ω_p, Ω_m are the Rabi frequencies corresponding to related transitions defined as $\Omega = \mu E/2\hbar$. Here we have considered Clebsch–Gordan coefficients $C_{\mu\nu}(\mu = a, ..., k; \nu = a, ..., k)$ and the $\Omega_i(i = {\rm c, p, m})$ represent the largest transition Rabi frequencies. The other transition Rabi frequencies have been scaled with the largest ones. The μ is the dipole transition matrix element and E is the electric field of laser. At present, the cell containing the ^{87}Rb is filled with buffer gas. So we do not need to consider the Doppler effect. We consider that the magnetic field is very weak, so laser field can not resolve the individual Zeeman state. Therefore the excited and ground Zeeman states are almost degenerated with respect to laser lights. Because there are many atomic states involved in the problem, it may be a little difficult for obtaining the analytical solution by solving all the motion equations of the density matrix elements. In order to solve the problem, firstly we dealt with the laser fields atoms interaction and the results will give the populations of the states. Secondly we solve the interaction of individual four-level atoms and the fields because it is understood that the absorption of the probe field reaches its maximum when all the fields are applied resonantly (three-photon resonant) and the influences of the nonresonant parts are very small due to the very narrow absorption spectral

line. According to this idea all the interactions between the fields and atoms can be divided into several interaction systems between the four-level atoms and the fields. Therefore as long as we solve those four-level atoms fields interaction systems, we can obtain the absorption spectra.

In order to obtain the populations of the atomic states for the usage of solving individual four-level system, for the interaction of the laser fields with atoms, under the condition of degeneration of the Zeeman states, we can solve the problem by using Schrödinger equation,

$$\mathrm{i}\hbar\frac{\mathrm{d}\psi}{\mathrm{d}t} = H\psi. \tag{6}$$

The state wave function can be written as

$$\begin{aligned}|\psi\rangle = {} & \mathrm{e}^{-\mathrm{i}\omega_b t}\,(A_c|c\rangle + A_b|b\rangle + A_a|a\rangle) \\ & + \mathrm{e}^{-\mathrm{i}\omega_f t}(A_h|h\rangle + A_g|g\rangle + A_f|f\rangle \\ & + A_e|e\rangle + A_d|d\rangle),\end{aligned} \tag{7}$$

where $A_l(l = a, b, ..., k)$ are the amplitudes of the states. With the Hamiltonian and wave function, we can obtain a set of differential equations:

$$\frac{\mathrm{d}A_a}{\mathrm{d}t} = -\gamma A_a + \mathrm{i}C_{ad}\Omega_\mathrm{c}A_d + \mathrm{i}C_{af}\Omega_\mathrm{p}A_f, \tag{8a}$$

$$\frac{\mathrm{d}A_b}{\mathrm{d}t} = -\gamma A_b + \mathrm{i}C_{be}\Omega_\mathrm{c}A_e + \mathrm{i}C_{bg}\Omega_\mathrm{p}A_g, \tag{8b}$$

$$\frac{\mathrm{d}A_c}{\mathrm{d}t} = -\gamma A_c + \mathrm{i}C_{cf}\Omega_\mathrm{c}A_f + \mathrm{i}C_{ch}\Omega_\mathrm{p}A_h, \tag{8c}$$

$$\frac{\mathrm{d}A_d}{\mathrm{d}t} = \mathrm{i}C_{\mathrm{d}A}\Omega_\mathrm{c}A_a, \tag{8d}$$

$$\frac{\mathrm{d}A_e}{\mathrm{d}t} = \mathrm{i}C_{eb}\Omega_\mathrm{c}A_b, \tag{8e}$$

$$\frac{\mathrm{d}A_f}{\mathrm{d}t} = \mathrm{i}C_{fc}\Omega_\mathrm{c}A_c + \mathrm{i}C_{fa}\Omega_\mathrm{p}A_a, \tag{8f}$$

$$\frac{\mathrm{d}A_g}{\mathrm{d}t} = \mathrm{i}C_{gb}\Omega_\mathrm{p}A_b, \tag{8g}$$

$$\frac{\mathrm{d}A_h}{\mathrm{d}t} = \mathrm{i}C_{hc}\Omega_\mathrm{p}A_c. \tag{8h}$$

Because the probe field is weak, we use adiabatic approximation $A_h^{(0)} \approx 1$, $A_l^{(0)}(l = a, ..., g, i, j, k) \approx 0$. So we can obtain the following populations:

$$\begin{aligned}&\left|A_a^{(1)}\right|^2 = 0, \left|A_b^{(1)}\right|^2 = 0, \left|A_\mathrm{c}^{(1)}\right|^2 = 0, \\ &\left|A_d^{(1)}\right|^2 = \left(\frac{C_{af}C_{ch}\Omega_\mathrm{p}^2}{C_{ad}C_{cf}\Omega_\mathrm{c}^2}\right)^2, \\ &\left|A_e^{(1)}\right|^2 = \left(\frac{C_{bg}C_{ch}\Omega_\mathrm{p}^2}{C_{be}C_{cf}\Omega_\mathrm{c}^2}\right)^2, \\ &\left|A_f^{(1)}\right|^2 = \left(\frac{C_{ch}\Omega_\mathrm{p}}{C_{cf}\Omega_\mathrm{c}}\right)^2, \\ &\left|A_g^{(1)}\right|^2 = \left(\frac{C_{ch}\Omega_\mathrm{p}}{C_{cf}\Omega_\mathrm{c}}\right)^2.\end{aligned} \tag{9}$$

Due to $\Omega_\mathrm{c} \gg \Omega_\mathrm{p}$, therefore $|A_f^{(1)}|^2$, $|A_g^{(1)}|^2 \gg |A_d^{(1)}|^2$, $|A_e^{(1)}|^2$. In the following when we consider individual four-level atomic system, we can neglect the $|A_d^{(1)}|^2$, $|A_e^{(1)}|^2$.

As long as we get the populations of the states, then we can obtain the approximate solution of the motion equations of the density matrix elements by considering individual four-level system interacted with two laser fields and one microwave field. There are twelve four-level atomic systems in total. One half are the cases that the microwave and coupling connect a common atomic state and the other half are the cases that the probe and microwave connect a common state. The total absorption is the summation of the all four-level systems. We use density matrix approach to solve the four-level systems to obtain the susceptibility. The slowly varying atomic density matrix element can be described as

$$\boldsymbol{\sigma}_{\mu\nu} = |\mu\rangle\langle\nu|\,\mathrm{e}^{-\mathrm{i}\omega_{\mu\nu}t}. \tag{10}$$

With the above Hamiltonian we can obtain equations of motion for the atomic density matrix elements of individual four-level system under rotating wave approximation. For the cases that the microwave and coupling connect a common state, such as $|c\rangle, |f\rangle, |h\rangle, |j\rangle$ four-level system, we can obtain

$$\begin{aligned}\dot{\sigma}_{cc} = {} & -(\gamma_\mathrm{c} + \gamma_2)\sigma_{cc} + \mathrm{i}\Omega_\mathrm{p}(C_{hc}\sigma_{hc} - C_{ch}\sigma_{ch}) \\ & + \mathrm{i}\Omega_\mathrm{c}(C_{fc}\sigma_{fc} - C_{cf}\sigma_{cf}),\end{aligned} \tag{11}$$

$$\begin{aligned}\dot{\sigma}_{ff} = {} & \gamma_\mathrm{c}\sigma_{cc} + \mathrm{i}\Omega_\mathrm{c}(C_{cf}\sigma_{cf} - C_{fc}\sigma_{fc}) \\ & + \mathrm{i}\Omega_\mathrm{m}(C_{jf}\sigma_{jf} - C_{fj}\sigma_{fj}),\end{aligned} \tag{12}$$

$$\dot{\sigma}_{hh} = \gamma_2\sigma_{cc} + \mathrm{i}\Omega_\mathrm{p}(C_{ch}\sigma_{ch} - C_{hc}\sigma_{hc}), \tag{13}$$

$$\dot{\sigma}_{jj} = \mathrm{i}\Omega_\mathrm{m}(C_{fj}\sigma_{fj} - C_{jf}\sigma_{jf}), \tag{14}$$

$$\begin{aligned}\dot{\sigma}_{cf} = {} & -\Gamma_{cf}\sigma_{cf} - \mathrm{i}C_{fc}\Omega_\mathrm{c}(\sigma_{cc} - \sigma_{ff}) \\ & - \mathrm{i}C_{cj}\Omega_\mathrm{m}\sigma_{cj},\end{aligned} \tag{15}$$

$$\begin{aligned}\dot{\sigma}_{ch} = {} & -\Gamma_{ch}\sigma_{ch} + \mathrm{i}C_{hc}\Omega_\mathrm{p}(\sigma_{hh} - \sigma_{cc}) \\ & + \mathrm{i}C_{fh}\Omega_\mathrm{c}\sigma_{fh},\end{aligned} \tag{16}$$

$$\begin{aligned}\dot{\sigma}_{cj} = {} & -\Gamma_{cj}\sigma_{cj} + \mathrm{i}C_{hj}\Omega_\mathrm{p}\sigma_{hj} + \mathrm{i}C_{fj}\Omega_\mathrm{c}\sigma_{fj} \\ & - \mathrm{i}C_{cf}\Omega_\mathrm{m}\sigma_{cf},\end{aligned} \tag{17}$$

$$\begin{aligned}\dot{\sigma}_{fh} = {} & -\Gamma_{fh}\sigma_{fh} + \mathrm{i}C_{ch}\Omega_\mathrm{c}\sigma_{ch} - \mathrm{i}C_{fc}\Omega_\mathrm{p}\sigma_{fc} \\ & + \mathrm{i}C_{jh}\Omega_\mathrm{m}\sigma_{jh},\end{aligned} \tag{18}$$

$$\dot{\sigma}_{jh} = -\Gamma_{hj}\sigma_{hj} - \mathrm{i}C_{jc}\Omega_\mathrm{p}\sigma_{jc} + \mathrm{i}C_{fh}\Omega_\mathrm{m}\sigma_{fh}, \tag{19}$$

$$\dot{\sigma}_{\mu\nu} = \dot{\sigma}^*_{\nu\mu}, \tag{20}$$

where the generalised decay rates are

$$\Gamma_{cf} = \mathrm{i}\Delta_{\mathrm{c}} + \gamma_{cf}, \tag{21}$$

$$\Gamma_{ch} = \mathrm{i}\Delta_{\mathrm{p}} + \gamma_{ch}, \tag{22}$$

$$\Gamma_{cj} = \mathrm{i}(\Delta_{\mathrm{c}} + \Delta_{\mathrm{m}}) + \gamma_{cj}, \tag{23}$$

$$\Gamma_{fh} = \mathrm{i}(\Delta_{\mathrm{p}} - \Delta_{\mathrm{c}}) + \gamma_{fh}, \tag{24}$$

$$\Gamma_{jh} = \mathrm{i}(\Delta_{\mathrm{p}} - \Delta_{\mathrm{c}} - \Delta_{\mathrm{m}}) + \gamma_{jh}, \tag{25}$$

with $\Delta_{\mathrm{p}} = \omega_{ch} - \omega_{\mathrm{p}}$, $\Delta_{\mathrm{c}} = \omega_{cf} - \omega_{\mathrm{c}}$, $\Delta_{\mathrm{m}} = \omega_{fj} - \omega_{\mathrm{m}}$. Because the probe field and the microwave field are very weak, we keep terms of all orders in the strong coupling field, but only the linear terms in the probe and microwave fields. This is equivalent to use

$$\sigma_{hh}^{(0)} = 1, \sigma_{jf}^{(0)} = 0, \sigma_{fc}^{(0)} = 0, \sigma_{jc}^{(0)} = 0. \tag{26}$$

Thus we get

$$\dot{\sigma}_{ch} = -\Gamma_{ch}\sigma_{ch} + \mathrm{i}C_{ch}\Omega_{\mathrm{p}} + \mathrm{i}C_{fh}\Omega_{\mathrm{c}}\sigma_{fh}, \tag{27}$$

$$\dot{\sigma}_{fh} = -\Gamma_{fh}\sigma_{fh} + \mathrm{i}C_{ch}\Omega_{\mathrm{c}}\sigma_{ch} + \mathrm{i}C_{jh}\Omega_{\mathrm{m}}\sigma_{jh}, \tag{28}$$

$$\dot{\sigma}_{jh} = -\Gamma_{jh}\sigma_{jh} + \mathrm{i}C_{fh}\Omega_{\mathrm{m}}\sigma_{fh}. \tag{29}$$

Therefore, we obtain

$$\sigma_{ch} = \mathrm{i}C_{hc}\Omega_{\mathrm{p}}\frac{\Gamma_{fh}\Gamma_{jh} + C_{jh}C_{fh}\Omega_{\mathrm{m}}^2}{\Gamma_{ch}\Gamma_{fh}\Gamma_{jh} + \Gamma_{ch}C_{jh}C_{fh}\Omega_{\mathrm{m}}^2 + \Gamma_{jh}C_{ch}C_{fh}\Omega_{\mathrm{c}}^2}. \tag{30}$$

When the microwave field is used resonantly to the transition $|f\rangle \leftrightarrow |k\rangle$, similarly to the $|c\rangle, |f\rangle, |h\rangle, |k\rangle$ four-level system, by replacing j with k, we can get σ'_{ch} as

$$\sigma'_{ch} = \mathrm{i}C_{hc}\Omega_{\mathrm{p}}\frac{\Gamma_{fh}\Gamma_{kh} + C_{kh}C_{fh}\Omega_{\mathrm{m}}^2}{\Gamma_{ch}\Gamma_{fh}\Gamma_{kh} + \Gamma_{ch}C_{kh}C_{fh}\Omega_{\mathrm{m}}^2 + \Gamma_{kh}C_{ch}C_{fh}\Omega_{\mathrm{c}}^2}. \tag{31}$$

The differences are the Rabi frequencies caused by the different C–G (Clebsch–Gordan) coefficients and the resonant frequency. We have the similar result for the case $|c\rangle, |f\rangle, |i\rangle, |h\rangle$. For the cases $|b\rangle, |e\rangle, |g\rangle, |j\rangle$; $|b\rangle, |e\rangle, |g\rangle, |i\rangle$, $|a\rangle, |d\rangle, |f\rangle, |i\rangle$; additional difference is the highly occupied state population which has been obtained in the Eq. (9). For example of the cases $|b\rangle, |e\rangle, |g\rangle, |i\rangle$, the density matrix element is

$$\sigma_{bg} = \mathrm{i}\Omega_{\mathrm{p}}\sigma_{gg}^{(1)}C_{bg}\frac{\Gamma_{eg}\Gamma_{ig} + C_{ig}C_{eg}\Omega_{\mathrm{m}}^2}{\Gamma_{bg}\Gamma_{eg}\Gamma_{ig} + \Gamma_{bg}C_{ig}C_{eg}\Omega_{\mathrm{m}}^2 + \Gamma_{ig}C_{bg}C_{eg}\Omega_{\mathrm{c}}^2}, \tag{32}$$

where $\sigma_{gg}^{(1)} = |A_g^{(1)}|^2$. The results for the case $|b\rangle, |e\rangle, |g\rangle, |j\rangle$ and $|a\rangle, |d\rangle, |f\rangle, |i\rangle$ are similar to σ_{bg}. For the cases that the probe and microwave connect a common state, the susceptibility is similar to that of the above discussed four-level atomic system. For example for $|c\rangle, |f\rangle, |h\rangle, |k\rangle$, the density matrix element is given by Eq. (31). For other kind configurations, the differences are the dipole moment, state population and Zeeman level splitting. For example for $|b\rangle, |e\rangle, |g\rangle, |j\rangle$, the result is

$$\sigma'_{bg} = \mathrm{i}\Omega_{\mathrm{p}}\sigma_{gg}^{(1)}C_{gb}\frac{\Gamma_{eg}\Gamma_{ig} + C_{ej}C_{eg}\Omega_{\mathrm{m}}^2}{\Gamma_{bg}\Gamma_{eg}\Gamma_{ig} + \Gamma_{bg}C_{ej}C_{eg}\Omega_{\mathrm{m}}^2 + \Gamma_{ig}C_{bg}C_{eg}\Omega_{\mathrm{c}}^2}. \tag{33}$$

With the above analyses and the results, finally the total susceptibility of the system is the summation of the twelve four-level atomic systems. Some of them are degenerated for the same Zeeman splitting. If we assume that the decay rates from the $|F_e = 1\rangle$ Zeeman states to the $|F_g = 2\rangle$ states are equal to γ_1 and the decay rates among $|F_g = 2\rangle$ are same to γ_2 as well as the relaxations from the Zeeman states of $|F_g = 2\rangle$ to the Zeeman states of $|F_g = 3\rangle$ are equal to γ_3. Then total susceptibility is the sum of the susceptibility of the individual four-level system which is given as

$$\begin{aligned}\chi = \sum_n \mathrm{i}\beta\sigma_{\nu\nu}C_{\nu\mu}\{&\gamma_2[-\mathrm{i}(\Delta_{\mathrm{m}} + n\Delta) + \gamma_3] \\ &+C_{\zeta\mu}C_{\alpha\nu}\Omega_{\mathrm{m}}^2\}/\{\gamma_1\gamma_2[-\mathrm{i}(\Delta_{\mathrm{m}} + n\Delta) + \gamma_3] \\ &+\gamma_1 C_{\zeta\mu}C_{\alpha\nu}\Omega_{\mathrm{m}}^2 + [-\mathrm{i}(\Delta_{\mathrm{m}} + n\Delta) + \gamma_3] \\ &C_{\eta\mu}C_{\kappa\mu}\Omega_{\mathrm{c}}^2\},\end{aligned} \tag{34}$$

where $\beta = (3/8\pi)N\gamma_{ch}\lambda^3$, $n = 0, \pm1, \pm2, \pm3$, Δ is the Zeeman level splitting. With the analytical solution we can obtain the line width of the spectra. The 0–0

clock line width depends mainly on the relaxations of the excited state and the ground states, coupling light and microwave intensities. We numerically computed the susceptibility and plotted the results in Fig. 2.

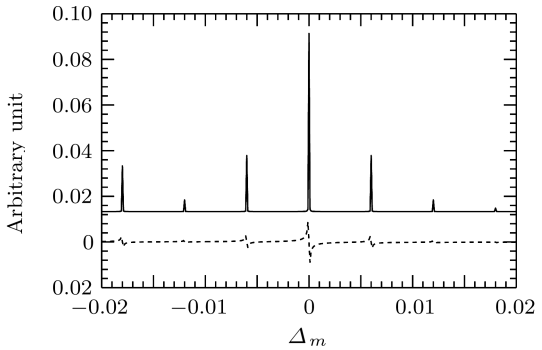


Fig. 2. The susceptibility of the system. The solid line is the absorption, the dashed line is the dispersion. The parameters are $\gamma_1 = 1.0, \gamma_2 = 0.001, \gamma_3 = 10^{-5}, \Omega_c = 1.0, \Omega_p = \Omega_c/\sqrt{5}, \Omega_m = 0.001$. All the parameters are scaled to the decay rate of the excited hyperfine state.

It can be seen that 0–0 transition spectral line is the highest and has the maximum signal-to-noise ratio. Therefore this 0–0 transition spectral line is suitable for microwave frequency standards.

3. Conclusions

In conclusion, by separately considering the individual four-level atoms fields interactions, we have analytically studied the microwave transition spectra for ^{87}Rb D1 line for the first time. The solution can give most characteristics of the spectrum. It has been shown that the 0–0 transition spectral line has the highest contrast and signal-to-noise ratio. The system can be realised with a simple optical system and the 0–0 transition line can be used for atomic frequency standards.

References

[1] Vanier J and Audoin C 1989 In: *The Quantum Physics of Atomic Frequency Standards* (Bristol, England: Adam Hilger)

[2] Knappe S, Gerginov V, Schwindt P D D, Shah V, Robinson H G, Hollberg L and Kitching J 2005 *Opt. Lett.* **30** 2351

[3] Knappe S, Schwindt P D D, Shah V, Hollberg L, Kitching J, Liew L and Moreland J 2005 *Opt. Express* **13** 1249

[4] Arimondo E In: Wolf E (ed) 1996 *Progress in Optics* (Amsterdam: Elsevier Science) **35** p257

[5] Vanier J 2005 *Appl. Phys.* B **81** 421 and references therein

[6] Zibrov S, Novikova I, Phillips D F, Taichenachev A V, Yudin V I, Walsworth R L and Zibrov A S 2005 *Phys. Rev.* A **72** 011801(R)

[7] Lukin M D, Yelin S F, Fleischhauer M and Scully M O 1999 *Phys. Rev.* A **60** 3225

[8] Chen Y C, Liao Y A, Chiu H Y, Su J J and Yu I A 2001 *Phys. Rev.* A **64** 053806

[9] Yelin S F, Sautenkov V A, Kash M M, Welch G R and Lukin M D 2003 *Phys. Rev.* A **68** 063801

[10] Budker D, Hollberg L, Kimball D F, Kitching J, Pustelny S and Yashchuk V V 2005 *Phys. Rev.* A **71** 012903

[11] Harris S E 1997 *Physics Today* **50** 759

[12] Scully M O and Zubairy M S 1997 *Quantum Optics* (Cambridge: Cambridge University Press)

[13] Marangos J P 1998 *J. Mod. Opt.* **45** 471

[14] Fleischhauer M, Imamoglu A and Marangos J P 2005 *Rev. Mod. Phys.* **77** 633

[15] Wang F Y, Shi B S, Lu X S and Guo G C 2008 *Chin. Phys.* B **17** 1798

[16] Li Y Y, Hou X, Bai J T, Yan J F, Gan C L and Zhang Y P 2008 *Chin. Phys.* B **17** 2885