The Wigner function and phase properties of superposition of two coherent states with the vacuum state*

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This paper discusses some statistical properties of the superposition of two coherent states with a vacuum state, such as sub-Poissonian photon statistics and negativity of the Wigner function. Phase probability distribution and phase variance are calculated. Special cases of the constructed superposition states are presented. The results show that depending on the vacuum state coefficient γ and the coherent state coefficient α, it can generate a variety of nonclassical states.

Keywords: coherent states, vacuum state, Wigner distribution function, phase properties

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1. Introduction

Quantum mechanical superposition of coherent states of light results in a new state which displays various nonclassical features.[1] Studying nonclassical effects of quantum superposition of optical fields has recently attracted renewed attention because they may play the key role in understanding the fundamentals of quantum physics, and they have many applications in quantum communication and quantum information technology[2] such as quantum teleportation,[3,4] quantum cryptography,[5] interferometric measurements,[6] quantum computation[7−9] and the nonclassicality is the necessary input for the entangled states. Generating nonclassical light from the superposition of coherent states has been one of the important approaches. Various schemes to generate those nonclassical states from the essentially classical coherent states have been suggested previously.[10−16]

In order to describe more realistic nonclassical quantum optical fields, we construct a new state which adds (or subtracts) the vacuum state to the superposition of the two coherent states with their phase differences $\pi/2$. Such a state is shown to exhibit nonclassical properties like sub-Poissonian photon statistics and negativity of the Wigner function. We also study in detail the phase properties and phase variance for such states. The paper is organised as follows.

In Section 2 we analyse the properties regarding photon statistics, such as oscillation and sub-Poissonian character and explain how to use the probability distribution functions of the photon number to choose the coefficient of the vacuum state. Because we are mainly interested in the nonclassical properties of the state, in Section 3 we use the Wigner function which is a quasiprobability distribution that can take negative values. In Section 4 we study the phase probability distribution and phase variance of the superposition states from the point of view of the Pegg–Barnett (P–B) phase formalism, and compare the properties of the P–B phase distributions with the Wigner phase quasiprobability distributions. We show that depending on the vacuum state coefficient γ and values of different α, one can generate a variety of nonclassical states. Finally a conclusion is given in Section 5.

2. Coefficient of the vacuum state and sub-Poissonian photon statistics

We consider a pure state of light expressed as

$$|\psi\rangle = \frac{\mathcal{N}}{\sqrt{2}}\left(|\alpha\rangle + |\mathrm{i}\alpha\rangle + \gamma|0\rangle\right). \tag{1}$$

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The coefficient $\mathcal{N}$ is then determined by normalising the state $|\psi\rangle$ such that

$$\mathcal{N}^2 = \left[1 + \exp\left(-|\alpha|^2\right)\cos|\alpha|^2 + 2\gamma\exp\left(-\frac{1}{2}|\alpha|^2\right) + \frac{\gamma^2}{2}\right]^{-1}. \tag{2}$$

For the superposition state given by Eq. (1), the photon-count probability of finding n photons is given by

$$P(n,\alpha,\gamma) = |\langle n|\psi\rangle|^2 = \frac{P_{|\alpha\rangle}\left(1+\cos\frac{n\pi}{2}\right) + \frac{\delta(n)}{2}\left(\gamma^2 + 4\gamma\,\mathrm{e}^{-|\alpha|^2/2}\right)}{1 + \exp\left(-|\alpha|^2\right)\cos|\alpha|^2 + 2\gamma\exp\left(-\frac{1}{2}|\alpha|^2\right) + \frac{\gamma^2}{2}}, \tag{3}$$

where $\delta(n)$ has the value 1 for $n = 0$, and the value 0 for $n \neq 0$ and

$$P_{|\alpha\rangle} = \mathrm{e}^{-|\alpha|^2}\frac{|\alpha|^{2n}}{n!} \tag{4}$$

is the Possion distribution of a single coherent state.

The quantum interference between the two coherent states and between the two coherent states with the vacuum state creates the interference terms $1+\cos(n\pi/2)$ and $\delta(n)[\gamma^2\exp(-|\alpha|^2/2)]$, which indicate the oscillatory photon statistics in the state.

From the curves of probability $P(n,\alpha,\gamma)$ of Eq. (3) depicted in Fig. 1 ($n \neq 0$), we can see more detail. This probability distribution function has the main property that $P(n_k) = 0$, $n_k = 4k+2$, and $k = 0, 1, \ldots$. This is sometimes seen as indicating the nonclassicality.[17] The first two moments of the photon number can be calculated by taking into account that the second term of the numerator of Eq. (3) has no contribution to the moments of n. We have

$$\langle n\rangle = \frac{|\alpha|^2\left(1 - \mathrm{e}^{-|\alpha|^2}\sin|\alpha|^2\right)}{1 + \exp\left(-|\alpha|^2\right)\cos|\alpha|^2 + 2\gamma\exp\left(-\frac{1}{2}|\alpha|^2\right) + \frac{\gamma^2}{2}}, \tag{5a}$$

$$\langle n^2\rangle = \frac{|\alpha|^2\left[1 + |\alpha|^2 - \mathrm{e}^{-|\alpha|^2}\left(\sin|\alpha|^2 + |\alpha|^2\cos|\alpha|^2\right)\right]}{1 + \exp\left(-|\alpha|^2\right)\cos|\alpha|^2 + 2\gamma\exp\left(-\frac{1}{2}|\alpha|^2\right) + \frac{\gamma^2}{2}}. \tag{5b}$$

In fact, because the first two moments of the photon number are $|\alpha|^2$ and $|\alpha|^2(1+|\alpha|^2)$ for the coherent state, we derive the values of $\gamma(\alpha)$ from the solutions of the equations (i): $\langle n\rangle - |\alpha|^2 = 0$ and (ii): $\langle n^2\rangle - |\alpha|^2(1+|\alpha|^2) = 0$, where $\langle n\rangle$ and $\langle n^2\rangle$ are given by Eqs. (5). From condition (i) and (ii) we obtain that

$$\gamma_\pm^* = -2\exp\left(-\frac{1}{2}|\alpha|^2\right) \times \left(1 \mp \sqrt{1 - \frac{\sin|\alpha|^2 + \cos|\alpha|^2}{2}}\right), \tag{6a}$$

the condition (ii) leads to

$$\gamma_\pm = -2\exp\left(-\frac{1}{2}|\alpha|^2\right) \times \left(1 \mp \sqrt{1 - \frac{\sin|\alpha|^2 + (1+2|\alpha|^2)\cos|\alpha|^2}{2(1+|\alpha|^2)}}\right). \tag{6b}$$

Obviously, γ depends on α, and $\gamma_\pm(0) = \gamma_\pm^*(0) = -2 \pm \sqrt{2}$.

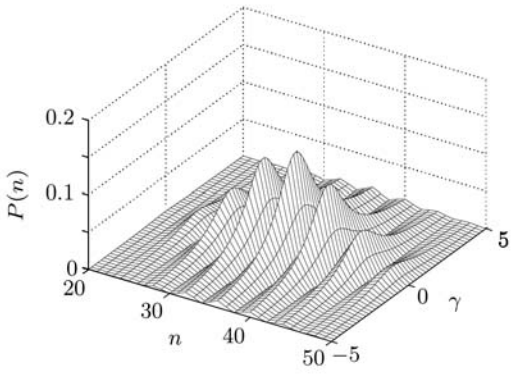


Fig. 1. The probability $P(n,\alpha,\gamma)$ of finding photons in $|\psi\rangle$ of Eq. (1) in its dependence on quantum number n and coefficient of the vacuum state γ. Here we have chosen $|\alpha|^2 = 36$ and $n \neq 0$.

The second order moment is more significant physically, we choose Eq. (6b) to define the dependence of γ on α. We then have $\gamma_- \leq \gamma_+ \leq 0$ ($|\gamma_-| \geq |\gamma_+|$). We can also show that $\gamma_\pm(\alpha \to \infty) \to 0$. The calculation of the variance of the photon number is easily deduced from Eqs. (5a) and (5b),

$$\sigma = 1 - |\alpha|^2 \frac{\mathrm{e}^{-2|\alpha|^2} - 2\sin|\alpha|^2\,\mathrm{e}^{-|\alpha|^2} - \left(1 - \cos|\alpha|^2\,\mathrm{e}^{-|\alpha|^2}\right)\left(2\gamma\,\mathrm{e}^{-\frac{1}{2}|\alpha|^2} + \dfrac{\gamma^2}{2}\right)}{\left(1 - \sin|\alpha|^2\,\mathrm{e}^{-|\alpha|^2}\right)\left(1 + \mathrm{e}^{-|\alpha|^2}\cos|\alpha|^2 + 2\gamma\,\mathrm{e}^{-\frac{1}{2}|\alpha|^2} + \dfrac{\gamma^2}{2}\right)}. \tag{7}$$

The state $|\psi\rangle$ obeys sub-Poissonian statistics whenever $\sigma < 1$. From Eq. (7), we note that the state $|\psi\rangle$ may exhibit sub-Poissonian when the following condition is satisfied:

$$\mathrm{e}^{-2|\alpha|^2} - 2\sin|\alpha|^2\,\mathrm{e}^{-|\alpha|^2} - \left(1 - \cos|\alpha|^2\,\mathrm{e}^{-|\alpha|^2}\right)\left(2\gamma\,\mathrm{e}^{-\frac{1}{2}|\alpha|^2} + \frac{\gamma^2}{2}\right) > 0. \tag{8}$$

3. Wigner distribution

The Wigner distribution function may be defined as

$$W(\beta) = \frac{1}{\pi^2}\int \exp\left(\eta^*\beta - \eta\beta^*\right)\mathrm{tr}[\rho\exp\left(\eta\hat{a}^+ - \eta^*\hat{a}\right)]\,\mathrm{d}^2\eta. \tag{9}$$

Notice that $W(\beta)$ is a real function normalised to unity. Substituting Eq. (1) into Eq. (9), one obtains the Wigner function for the state $|\psi\rangle$,

$$W(\beta) = \frac{\mathcal{N}^2}{2}\Big[W_{|\alpha\rangle}(\beta) + W_{|\mathrm{i}\alpha\rangle}(\beta) + \gamma^2 W_{|0\rangle}(\beta) + W_\nu(\beta) + \gamma W_e(\beta)\Big]. \tag{10}$$

The terms $W_{|\alpha\rangle}(\beta)$, $W_{|\mathrm{i}\alpha\rangle}(\beta)$ and $W_{|0\rangle}(\beta)$ are the Wigner function for a single coherent state and vacuum state respectively, and may be written as

$$W_{|\alpha\rangle}(\beta) = \frac{2}{\pi}\exp\left[-2\left(|\alpha|^2 + |\beta|^2 - \alpha^*\beta - \alpha\beta^*\right)\right], \tag{11}$$

$$W_{|\mathrm{i}\alpha\rangle}(\beta) = \frac{2}{\pi}\exp\left[-2\left(|\alpha|^2 + |\beta|^2 + \mathrm{i}\alpha^*\beta - \mathrm{i}\alpha\beta^*\right)\right], \tag{12}$$

$$W_{|0\rangle}(\beta) = \frac{2}{\pi}\exp\left[-2|\beta|^2\right]. \tag{13}$$

The other terms of Eq. (10) are

$$W_\nu(\beta) = \frac{1}{\pi^2}\int \exp\left(\eta^*\beta - \eta\beta^*\right)\sum_n \langle n|(|\alpha\rangle\langle\mathrm{i}\alpha| + |\mathrm{i}\alpha\rangle\langle\alpha|)\exp\left(\eta\hat{a}^+ - \eta^*\hat{a}\right)|n\rangle\,\mathrm{d}^2\eta, \tag{14}$$

and

$$W_e(\beta) = \frac{1}{\pi^2}\int \exp\left(\eta^*\beta - \eta\beta^*\right)\sum_n \langle n|(|\alpha\rangle\langle 0| + |\mathrm{i}\alpha\rangle\langle 0| + |0\rangle\langle\alpha| + |0\rangle\langle\mathrm{i}\alpha|)\exp\left(\eta\hat{a}^+ - \eta^*\hat{a}\right)|n\rangle\,\mathrm{d}^2\eta, \tag{15}$$

which correspond to the interference between the coherent states and the vacuum state, respectively. We obtain

$$\begin{aligned} W(\beta) = \frac{\mathcal{N}^2}{\pi}\Big\{ & \exp\left\{-2\left[(y_1 - x_1)^2 + (y_2 - x_2)^2\right]\right\} + \exp\left\{-2[(y_1 + x_2)^2 + (y_2 - x_1)^2]\right\} \\ & + \gamma^2\exp\left[-2(y_1^2 + y_2^2)\right] + 2\cos\left[(2x_1 - 2x_2)y_1 + (2x_1 + 2x_2)y_2 - (x_1^2 + x_2^2)\right] \\ & \times \exp\left\{-2\left[\left(y_1 - \frac{x_1 - x_2}{2}\right)^2 + \left(y_2 - \frac{x_1 + x_2}{2}\right)^2\right]\right\} + 2\gamma\{\cos 2(x_2y_1 - x_1y_2)\exp\left[2(x_1y_1 + x_2y_2)\right] \\ & + \cos 2(x_1y_1 + x_2y_2)\exp\left[2(-x_2y_1 + x_1y_2)\right]\}\exp\left[-2\left(y_1^2 + y_2^2 + \frac{1}{4}(x_1^2 + x_2^2)\right)\right]\Big\}, \end{aligned} \tag{16}$$

where we have introduced $\alpha = x_1 + \mathrm{i}x_2$ and $\beta = y_1 + \mathrm{i}y_2$.

The Wigner functions for the state $|\psi\rangle$ are plotted in Figs. 2 and 3, respectively, for different values of α and γ. In Fig. 2, three Gaussian bells represent the two coherent states and the vacuum state, which are triangular distribution each other. The structure halfway between them is the result of quantum interference between the states $|\alpha\rangle$ and $|\mathrm{i}\alpha\rangle$, $|\alpha\rangle$ and $|0\rangle$, $|\mathrm{i}\alpha\rangle$ and $|0\rangle$. One clearly sees that the bell of the vacuum state is located at $(0,0)$ in phase space and the two bells of coherent states are symmetric with respect to the reflection $y_1 \to -y_1$. Any two bells and the interference structure are separated gradually with the increase of α. The frequency of the interference structure increases with the separation. Some examples of the functions $W(\beta)$ given by Eq. (16) for moderate power $\alpha = 1 + \mathrm{i}$ and for different values of γ are displayed in Fig. 3. The main property of the nonclassicality, i.e., the negativity of these functions, is clearly shown for the values of γ_+ and γ_-. It is clear that the Wigner distributions for the state $|\psi\rangle$ by Eq. (1) are clearly enough to reflect the different features of quantum interference with the superposition of two coherent states in Ref. [18].

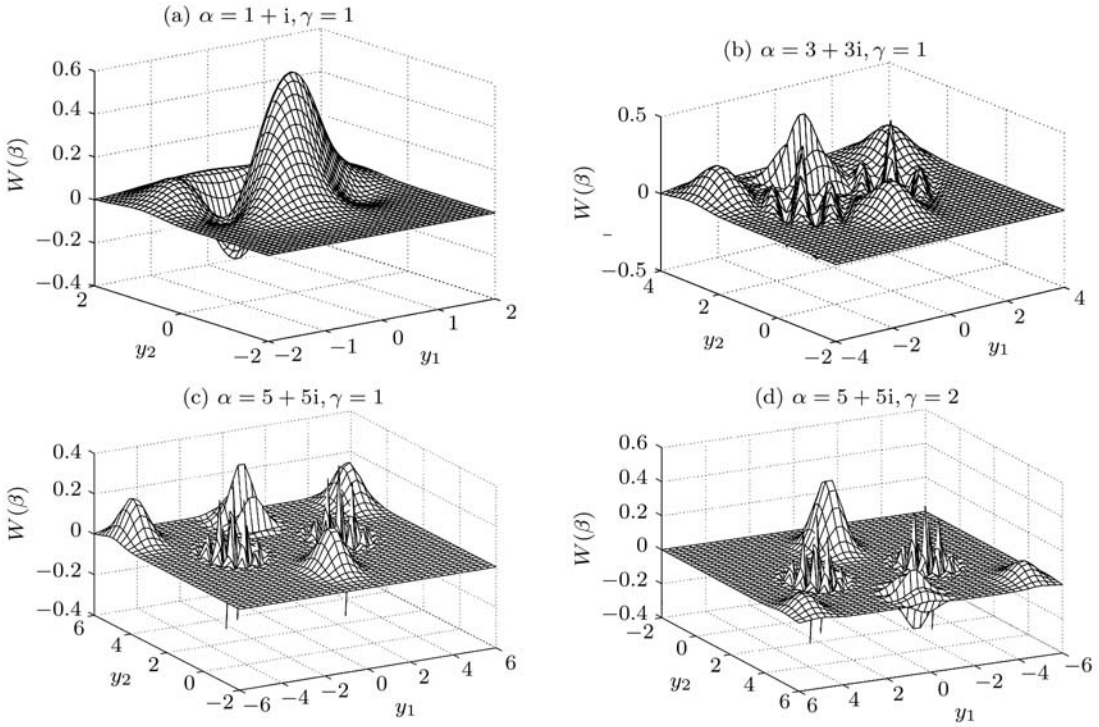


Fig. 2. The Wigner distributions of the state $|\psi\rangle$ corresponding to different values of α and γ.

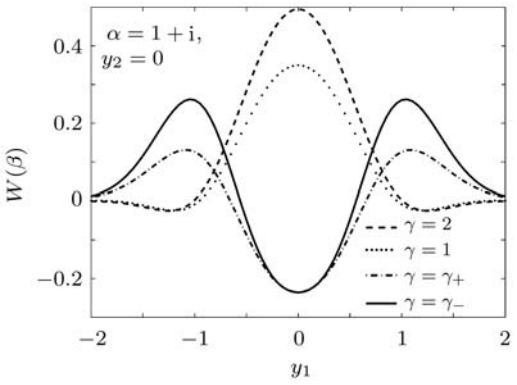


Fig. 3. The Wigner functions $W(\beta)$ are plotted versus the coordinate y_1 for $\alpha = 1 + \mathrm{i}$. The curves are indexed with the value of $\gamma = 2, 1, \gamma_+ = -0.44, \gamma_- = -1.03$.

4. Phase properties

Now we turn to obtain the phase probability distributions from Eq. (1). We adopted the P–B Hermitian phase operator formalism at first which describes the quantum properties of optical phase on the basis of the Hermitian quantum phase operator and its eigenstates.

In terms of the P–B phase operator formalism[19–21] in a finite-dimensional $s+1$ Hilbert space, a set of complete orthonormal phase states $|\theta_m\rangle$ is defined as

$$|\theta_m\rangle = (s+1)^{-\frac{1}{2}} \sum_{n=0}^{s} \exp(\mathrm{i}n\theta_m)|n\rangle, \qquad (17)$$

where $|\theta_m\rangle$ is given by

$$\theta_m = \theta_0 + \frac{2\pi m}{s+1} \qquad (m = 0, 1, 2, \ldots, s). \tag{18}$$

in which the value of θ_0 is arbitrary and indicates a particular basis in the phase space. In this space the Hermitian phase operator $\hat{\Phi}_\theta$ is defined by

$$\hat{\Phi}_\theta = \sum_{m=0}^{s} \theta_m |\theta_m\rangle\langle\theta_m|. \tag{19}$$

For the state $|\psi\rangle$ the expectation value of the phase operator $\hat{\Phi}_\theta$ is given by

$$\langle\psi|\hat{\Phi}_\theta|\psi\rangle = \sum_{m=0}^{s} \theta_m |\langle\theta_m|\psi\rangle|^2, \tag{20}$$

where $|\langle\theta_m|\psi\rangle|^2$ denotes the probability of $|\psi\rangle$ in the phase state $|\theta_m\rangle$. For the state $|\psi\rangle$ the phase probability distribution is expressed as

$$P^{(\mathrm{PB})}(\theta_m) = |\langle\theta_m|\psi\rangle|^2 = \frac{1}{s+1}\left\{1 + \frac{N^2}{2}\sum_{\substack{n,n'=0\\ n\neq n'}}^{\infty} \mathrm{e}^{-\mathrm{i}(n-n')\theta_m}\left[a_n + \gamma\delta(n)\right]\left[b_{n'} + \gamma\delta(n')\right]\right\}, \tag{21}$$

where

$$a_n = \frac{\exp\left(-\frac{1}{2}|\alpha|^2\right)}{\sqrt{n!}}\alpha^n\left(1 + \mathrm{i}^n\right), \qquad b_{n'} = \frac{\exp\left(-\frac{1}{2}|\alpha|^2\right)}{\sqrt{n'!}}\alpha^{*n'}\left[1 + (-\mathrm{i})^{n'}\right]. \tag{22}$$

Since the density of phase states is $(s+1)/2\pi$, thus setting $s \to \infty$ in Eq. (21), we can obtain the continuous phase probability distributions as

$$P^{(\mathrm{PB})}(\theta) = \lim_{s\to\infty}\frac{s+1}{2\pi}|\langle\theta_m|\psi\rangle|^2 = \frac{1}{2\pi}\left\{1 + \frac{N^2}{2}\sum_{\substack{n,n'=0\\ n\neq n'}}^{\infty} \mathrm{e}^{-\mathrm{i}(n-n')\theta}\left[a_n + \gamma\delta(n)\right]\left[b_{n'} + \gamma\delta(n')\right]\right\}. \tag{23}$$

Equation (23) is normalised such that

$$\int_{\theta_0}^{\theta_0+2\pi} P^{(\mathrm{PB})}(\theta)\,\mathrm{d}\theta = 1. \tag{24}$$

For the state $|\psi\rangle$ the expectation value and the phase variance are given by, respectively,

$$\langle\hat{\Phi}\rangle = (\pi + \theta_0) + \frac{N^2}{2}\sum_{\substack{n,n'=0\\ n\neq n'}}^{\infty} \mathrm{i}\,\mathrm{e}^{-\mathrm{i}(n-n')\theta_0}\left[a_n + \gamma\delta(n)\right]\left[b_{n'} + \gamma\delta(n')\right], \tag{25}$$

$$\begin{aligned}\langle(\Delta\hat{\Phi})^2\rangle \;=\; & \frac{1}{3}\pi^2 + N^2\sum_{\substack{n,n'=0\\ n\neq n'}}^{\infty}\frac{\mathrm{e}^{-\mathrm{i}(n-n')\theta_0}}{(n-n')^2}\left[a_n + \gamma\delta(n)\right]\left[b_{n'} + \gamma\delta(n')\right] \\ & + \frac{N^4}{4}\sum_{\substack{n,n'=0\\ n\neq n'}}^{\infty}\frac{\mathrm{e}^{-2\mathrm{i}(n-n')\theta_0}}{(n-n')^2}\left\{\left[a_n + \gamma\delta(n)\right]\left[b_{n'} + \gamma\delta(n')\right]\right\}^2.\end{aligned} \tag{26}$$

Another way to describe the phase properties of the field is to use quasiprobability distribution functions.[22] Since the quasiprobability description of the quantum state of the system can be in some cases associated with realistic measurements performed on the system and contain all information about the quantum states of the field required by a proper phase distribution, so we can integrate the Wigner distributions functions in the Section 3 to find the Wigner phase quasiprobability distributions and compare with the P–B phase distributions.

$$P^{(\mathrm{W})}(\theta) = \int_{\theta_0}^{\infty} rW(r\,\mathrm{e}^{\mathrm{i}\theta})\mathrm{d}r, \quad \beta = r\,\mathrm{e}^{\mathrm{i}\theta} \tag{27}$$

where $P^{(\mathrm{W})}(\theta)$ is the Wigner phase quasiprobability distribution.

To illustrate the phase properties of the state $|\psi\rangle$ in Figs. 4–6 we plot the phase distribution function $P^{(\mathrm{PB})}(\theta)$ and phase variance for different values of θ,

α and γ. From Figs. 4(a) and 4(b) we see that the phase probability distribution splits into two separate peaks with same heights and the peaks appear at the phase angles $\theta = 0$ and $\theta = \pi/2$ which corresponds to contributions from two component states $|\alpha\rangle$ and $|\mathrm{i}\alpha\rangle$. In Fig. 4(a) the width of peaks is the narrowest and the value of the peak is the biggest when $\alpha = 3$ for fixed $\gamma = 2$, namely the peak structure becomes the most remarkable. The values of the peaks are fixed when $\alpha = 0$, namely the state is vacuum state and the curve is beeline. As α increases and γ is positive, the values of amplitude increase. The value of the peak is the biggest when $\alpha = 3.45$, and then decreases. With $\alpha \to \infty$, the distribution asymptotically approaches x axis up to zero distribution. However, when γ is negative in Fig. 4(d), the values of the peaks are very different. As α increases, the values of the peaks present non-regular change. This means that the phase becomes more uncertain. However, as γ increases the variations of the peak structure increase at first and then decrease. For fixed $\alpha = 1 + \mathrm{i}$ and $\theta = \pi$, the most values of the peak appear at $\gamma = -0.78$.

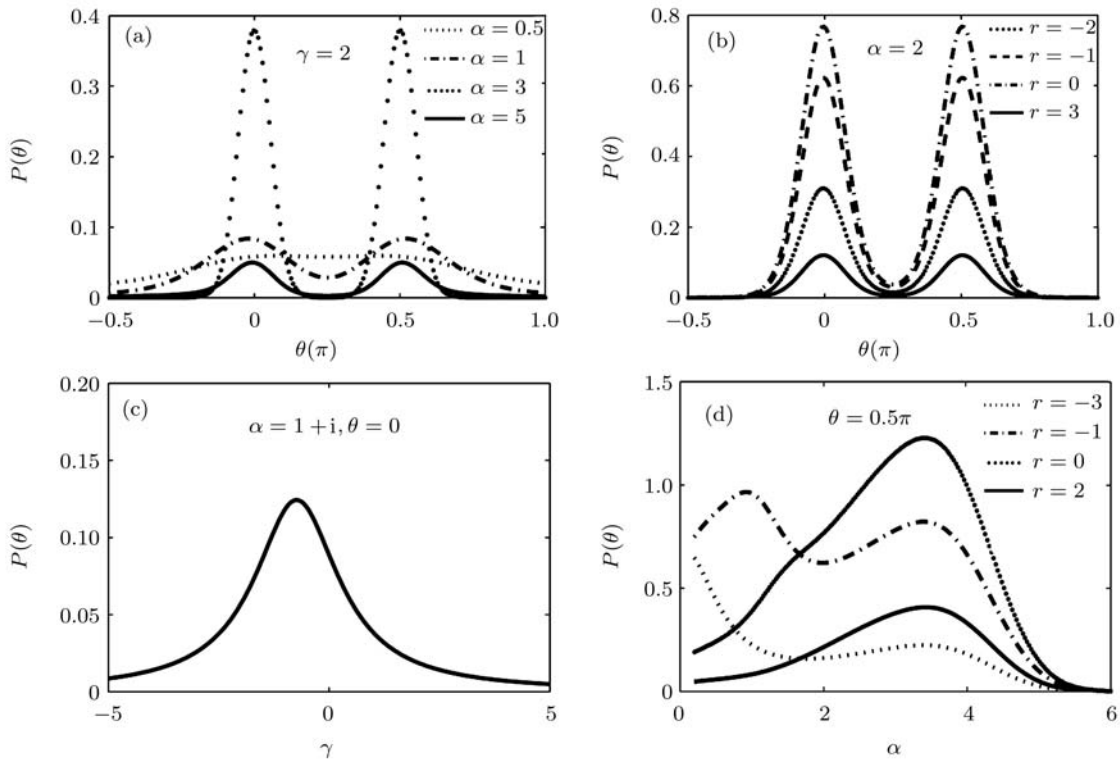


Fig. 4. P–B phase probability distributions of $|\psi\rangle$ for various α and γ.

In Fig. 5 we show the phase distributions of the state $|\psi\rangle$ using two different approaches. We see that two functions, the Wigner quasiprobability phase distributions $P^{(\mathrm{W})}(\theta)$ and the P–B phase distribution $P^{(\mathrm{PB})}(\theta)$, give rather different results. When $\gamma = 0$, the peak structure of the two distributions is similar and all of them have the two peaks with same height, and $P^{(\mathrm{W})}(\theta)$ presents negative values. But when $\gamma \neq 0$, the difference of the Wigner phase distributions and the P–B phase distributions is more obvious. Except $P^{(\mathrm{W})}(\theta)$ has still negative values, the two peaks structure of the Wigner distributions is destroyed and the negative value regions becomes larger. Quantum interference leads to negative values, so these negative values should be interpreted as a signature of the quantum nature of the field state. From this discussion we can conclude that the Wigner phase quasiprobability distribution reflects the quantum interference effects very well when adds (or subtracts) the vacuum state to the superposition of the two coherent states.

From Fig. 6 we find that phase variance starts from $\pi^2/3$ and decreases up to a certain value of α and then again increases to approach asymptotically $\pi^2/3$. Phase variance assumes the minimum value at $\alpha = 1.08$ when $\gamma = -1$. Thus for $\alpha = 1.08$ the phase becomes less uncertain and the best measurement of phase becomes possible around this value of α.

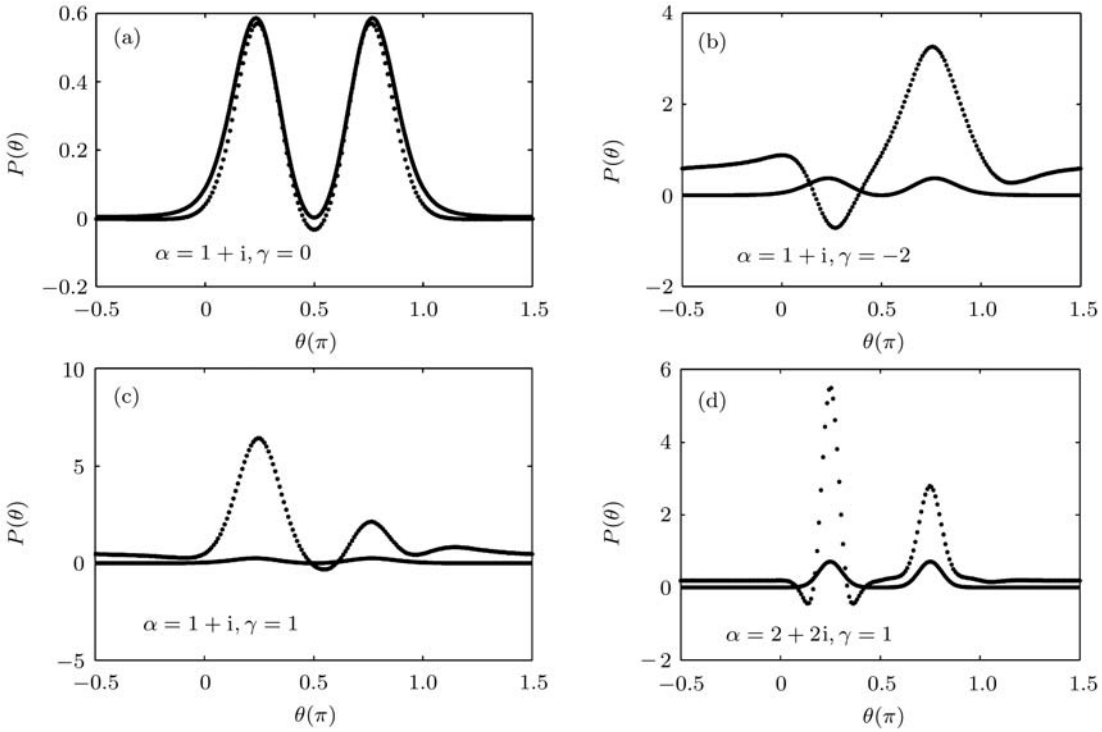


Fig. 5. The Wigner phase quasiprobability distribution $P^{(\mathrm{W})}(\theta)$ (dashed line) and the P–B phase probability distribution $P^{(\mathrm{PB})}(\theta)$ (solid line) of the state $|\psi\rangle$.

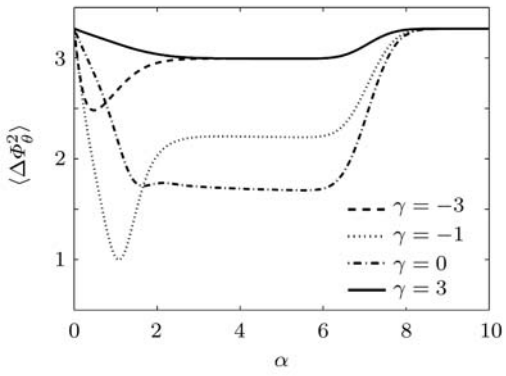


Fig. 6. Phase variance against α for various γ.

5. Conclusion

In summary, we conclude that adding the vacuum state to the superposition of two coherent states generates a variety of classical and nonclassical states which show greatly different features with the superposition of two coherent states. The state may exhibit sub-Poissonian statistics in photon-count probability. There are several values of $n_k = 4k + 2$ $k = 0, 1, \ldots$ yielding $P(n_k) = 0$. The nonclassical indicator, i.e. the negativity of the Wigner function, of the superposition state shows unique quality in their dependence on the amplitude α and coefficient γ. By virtue of the P–B formalism of phase operator we have examined the phase probability distributions of the superposition state of light for the different parameters α and γ. The numerical computation results show that the Wigner distribution function and the phase probability distributions for the state can clearly exhibit the different features of quantum interference and are distinct from those of the ordinary superposition of two coherent states. Present results in this paper may be useful for checking the influence of the noise on the quantum information processes in which the superposition state is used as a resource.

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