# Experimental study of regional fractal speckle produced in large angle scattering*

Song Hong-Sheng(宋洪胜)[a)b)], Cheng Chuan-Fu(程传福)[a)†], Liu Yun-Yan(刘云燕)[a)c)], Liu Gui-Yuan(刘桂媛)[a)b)], and Teng Shu-Yun(滕树云)[a)]

[a)] *College of Physics and Electronics, Shandong Normal University, Jinan* 250014, *China*
[b)] *School of Science, Shandong Jianzhu University, Jinan* 250101, *China*
[c)] *School of Science, Shandong University of Technology, Zibo* 255049, *China*

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Using the interference of speckle and the spherical reference wave, we extract the real and the imaginary parts of the complex amplitudes of the speckle in large angle scattering. By calculating the spatial correlation functions of intensity, we find that the speckle has a property of regional fractality, i.e. the fractal exponent equals 1 in a small spatial region and it becomes less than 1 in a bigger region. The empirical analytic expression of the intensity correlation function is gained. The probability density distributions of the intensities and the complex amplitudes show that the regional fractal speckle still obeys zero-mean circular complex Gaussian statistics.

**Keywords:** regional fractal, speckle, Gaussian distribution
**PACC:** 4225H, 0555, 4230K

## 1. Introduction

It is well known that the concept of fractals plays an important role in describing and understanding a large number of complex geometries and phenomena in various areas of science.[1−5] Recently, the fractal of speckle is reported in many papers.[6,7] When coherent light is scattered by a rough surface, a random intensity distribution known as speckle is produced in an observation plane in the diffraction region. If the number of independent scattering elements contributing to a given point in the observation plane is infinite, the random phase of the speckle ranges over $2\pi$ uniformly, and the speckle can be described by zero-mean circular complex Gaussian statistics.[8] The fractal theory[1,2] indicates that a typical and important type of correlation function having a long lasting tail is a power function. This indicates that if random distributions of intensities with fractal properties are realized, their spatial correlations will obey a power function. Such an optical field will provide an alternate type of unconventional intensity distributions which have long spatial correlation lengths. As is well known, if the ordinary rough surface is illuminated with a single Gaussian beam, a speckle distribution without fractal structure is produced in the observation plane, this is because the property of speckle phenomenon is analogous to what is predicted by the van Cittert–Zernike theorem,[8] i.e. the intensity correlation function of the speckle is determined by the intensity distribution of the illumination rather than the statistics of microstructures of the rough surface. Uno *et al.*[9−12] produced fractal speckles with various forms of correlation function by changing the illuminating condition. They generated experimentally fractal speckle by illuminating an ordinary rough surface with speckle produced by random fractal objects, and the properties of the fractal speckle were investigated in detail. The doubly scattered speckle is sometimes called speckled speckle.[13]

Usually, the experimental research on speckle is limited to the near-axis scattering. With the increase of the scattering angle, the intensity of scattered light decreases rapidly, and it is difficult to measure the speckle in large angle scattering due to its weak intensity distribution. In this paper, we use a higher-power laser produced by semiconductor pump laser Verdi V-5 (the maximum power is 5 W) to enhance

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†Corresponding author. E-mail: chengchuanfu@sdnu.edu.cn
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the weak intensity in large angle scattering and record them with a charge-coupled device (CCD) camera. The complex amplitudes of the speckle are extracted experimentally according to the technique of the interference of the speckle and the spherical reference wave.[14,15] By studying the intensity correlation function of the speckle, we find that the fractal property of the speckle is related to spatial separation distance $\rho$. In the area of small value of $\rho$, the fractal exponent $\alpha = 1$, but the fractal exponent $\alpha$ becomes less than 1 when $\rho$ increase to a certain value. Such a speckle is named regional fractal speckle by us. We deduce an empirical analytic expression of the regional fractal speckle intensity correlation function, and calculate fractal exponent $\alpha$ by fitting the experimental curves with an empirical intensity correlation analytic expression. From the probability distribution properties of the intensities and the complex amplitudes, we demonstrate that the regional fractal speckle still obeys the Gaussian distribution.

## 2. Digital extraction of speckle in large angle scattering

The schematic diagram of the proposed system for extracting large angle scattering speckle is illustrated in Fig. 1. Linearly polarized light ($\lambda = 532$ nm) from the semiconductor pump laser Verdi V-5 is split into two beams by the beam splitter (BS1, the intensity ratio of transmission to reflection is $T : R = 9 : 1$): reflected light and transmitted light. After being filtered and expanded by spatial pinhole filter (SPF) the reflected light is used as a reference wave. The speckle produced behind the random surface (RS) serves as an object beam and the RS is illuminated perpendicularly by transmitted light. The interferogram is recorded by a charge-coupled device (CCD) camera (Cascade-1K, with $1004 \times 1002$ pixels and pixel size $8\ \mu\text{m} \times 8\ \mu\text{m}$). Speckle at every scattering angle $\theta$ can be recorded by changing the position and the direction of BS2 ($T : R = 1 : 1$) and CCD. The adjustment of RS and SPF requires special care since the distance from them to CCD should match to gain the parallel interference patterns. The tropism and the density of the interference patterns are controlled by adjusting BS2 horizontally and perpendicularly. The equipment (A) in Fig. 1 denotes attenuator. The speckles at different scattering angles, i.e. $\theta = 65°$, $71°$ and $84°$, are recorded separately, because of their similar properties, the speckle at $\theta = 71°$, as an example, is mainly discussed in this paper and the results of the other two cases will be given and discussed later. After the interference data as shown in Fig. 2 have been recorded, the speckle can be extracted digitally from the following calculations.

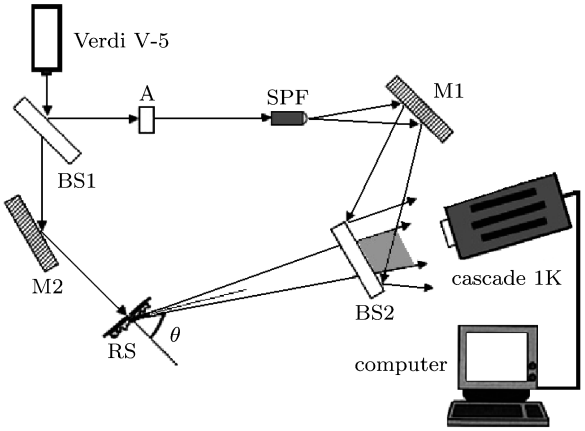



**Fig. 1.** Experimental setup for generation and detection of interferogram.

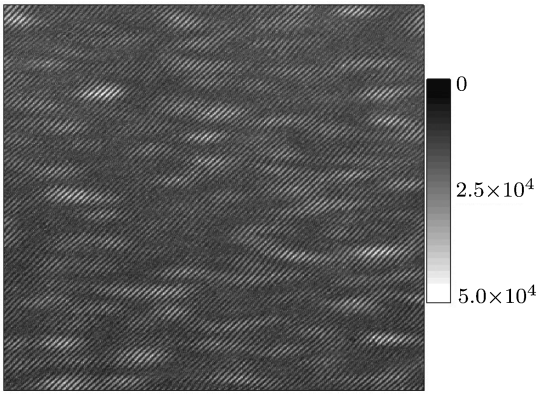



**Fig. 2.** Interferogram recorded by CCD.

The speckle field $R(x, y)$ and the reference wave $U(x, y)$ can be written as

$$U(x,y) = A(x,y)\exp[\mathrm{j}\varphi(x,y)] = A_\mathrm{r}(x,y) + \mathrm{j}A_\mathrm{i}(x,y), \quad (1)$$

$$R(x,y) = \exp[\mathrm{j}2\pi(f_{0x}x + f_{0y}y)], \quad (2)$$

where $A_\mathrm{r}(x, y)$ and $A_\mathrm{i}(x, y)$ are the real part and the imaginary part of the speckle complex amplitude, respectively, $\varphi(x, y) = \arctan[A_\mathrm{i}(x, y)/A_\mathrm{r}(x, y)]$ is the phase with a range of $[-\pi, \pi)$,[8] $f_{0x}$ and $f_{0y}$ are the spatial frequencies of the reference wave, their values lie in the propagation direction of reference wave, and

determine the tropism and the density of the interference fringes. The intensity of the interferogram is

$$\begin{aligned} I_1(x,y) &= |U(x,y)+R(x,y)|^2 \\ &= U(x,y)U^*(x,y)+R(x,y)R^*(x,y) \\ &\quad +U(x,y)R^*(x,y)+U^*(x,y)R(x,y), \quad (3) \end{aligned}$$

where $|\cdots|^2$ denotes modulus square and * denotes a complex conjugate. After Eq. (3) has been Fourier transformed with respect to $x$ and $y$, we obtain

$$\begin{aligned} I_{\rm f}(f_x,f_y) &= B_{\rm f}(f_x,f_y)+U_{\rm f}(f_x,f_y) \\ &\quad \otimes\delta(f_x+f_{0x},f_y+f_{0y}) \\ &\quad +U_{\rm f}^*(f_x,f_y)\otimes\delta(f_x-f_{0x},f_y-f_{0y}) \\ &= B_{\rm f}(f_x,f_y)+U_{\rm f}(f_x+f_{0x},f_y+f_{0y}) \\ &\quad +U_{\rm f}^*(f_x-f_{0x},f_y-f_{0y}), \quad (4) \end{aligned}$$

where $\otimes$ denotes convolution; $f_x$ and $f_y$ are the spatial frequencies in the $x$ and the $y$ directions, respectively; $B_{\rm f}$, $U_{\rm f}$ and $U_{\rm f}^*$ denote Fourier spectra, and they are separated by the carrier frequencies $f_{0x}$ and $f_{0y}$; and the zero order spectrum $B_{\rm f}(f_x,f_y)$ locates at the origin; the positive and the negative first-order spectra $U_{\rm f}(f_x+f_{0x},f_y+f_{0y})$ and $U_{\rm f}^*(f_x-f_{0x},f_y-f_{0y})$ are symmetric distributions about the origin, and the distance between them is $2\sqrt{f_{0x}^2+f_{0y}^2}$. We translate $U_{\rm f}(f_x+f_{0x},\ f_y+f_{0y})$ by $\sqrt{f_{0x}^2+f_{0y}^2}$ in the frequency surface toward the origin to obtain $U_{\rm f}(f_x,f_y)$. Again using the Fourier transform algorithm, we calculate the inverse Fourier transform of $U_{\rm f}(f_x,f_y)$ with respect to $f_x$ and $f_y$, and obtain the complex amplitude $U(x,y)$ of the speckle. The resulting $U(x,y)$ is denoted with real part $A_{\rm r}(x,y)$ and imaginary part $A_{\rm i}(x,y)$ as shown in Eq. (1), because the real part and the imaginary part are calculated separately when we use the Fourier transform algorithm. The intensity of speckle is obtained according to equation $I(x,y)=|A_{\rm r}(x,y)|^2+|A_{\rm i}(x,y)|^2$. Takeda *et al.*[14] have extracted random wave field by using the Eqs. (1)–(4).

The extracted speckle intensity is shown in Fig. 3(b). Figure 3(a) shows the corresponding speckle intensity recorded by CCD in the condition without reference wave. Figures 3(c) and 3(d) show the real part and the imaginary part of the speckle complex amplitude, respectively, and all the figures are obtained at scattering angle $\theta=71^\circ$.

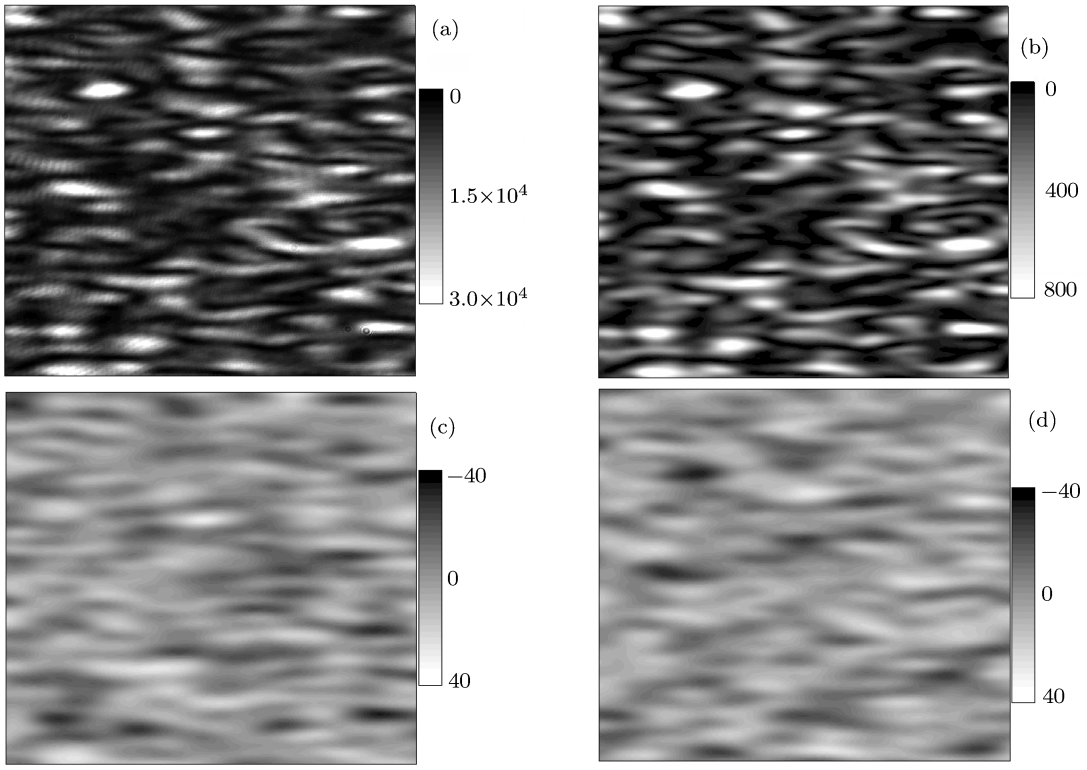



**Fig. 3.** Grey maps at scattering angle $\theta=71^\circ$, showing (a) speckle intensity distribution recorded by CCD, (b) speckle intensity distribution extracted digitally, (c) real part of complex amplitude, and (d) imaginary part of complex amplitude.

Unavoidable noise, especially in the high frequency part, produced in the CCD camera, will interfere with the speckle and lead to ambiguous results. This high frequency part noise can be filtrated by controlling the size of the frequency surface when we use the Fourier transform algorithm. For example, there are many high frequent dots in Figs. 2 and 3(a) and most of them are produced by CCD, but they are no longer present in Fig. 3(b). More importantly, the complex amplitude and the phase of the complex optic fields, which cannot be recorded in common experiment, can be experimentally extracted by using the interference technique. The method introduced in this paper can be used to analyse other properties of the complex optic fields. The relative intensities of the reference wave and the speckle as well as the units used in the Fourier transform will affect the absolute values of the extract results, but they will not affect the relative distributions of the results. We adopt an arbitrary unit, so the grey scale in Fig. 3(b) is different from that in Fig. 3(a), however, their relative intensity distributions are the same. On the other hand, in the Fourier transform algorithm, the operational results of a given field point depend on all the data in the neighbourhood of its corresponding point in the object plane, which will result in data distortion of four boundaries in every field. In order to ensure the accuracy of the results, we delete 100 pixels' data of the four boundaries in every field.

Figure 3(b) shows that the typical intensity structure of the speckle is strongly anisotropic. With the increase of $\theta$, the sizes of speckle particles are widened along the parallel direction of $\theta$. For convenience, this direction is set as $x$-axis. Some speckle particles, especially the bright particles, are linked with each other to form a wider particle. The difference in size between the speckle particles along the $x$-axis becomes more and more remarkable, so the speckle intensity spatial distribution rule is more complicated than that of ordinary speckle.

## 3. Study of regional fractal speckle based on intensity correlation function

Based on the height–height correlation function of rough surfaces,[16] intensity spatial correlation function $\mu_{\rm I}(\rho)$ of speckle can be written as

$$\mu_{\rm I}(\rho) = \langle [I(r_0+\rho) - I(r_0)]^2 \rangle, \tag{5}$$

where the symbol $\langle \cdots \rangle$ denotes average, $\rho$ is the difference coordinate. According to the fractal theory, for the ordinary fractal structure, $\mu_{\rm I}(\rho)$ is given by the power law when $\rho$ is small[2]

$$\mu_{\rm I}(\rho) \propto \rho^{2\alpha}, \tag{6}$$

the fractal exponent satisfies the condition $0 < \alpha \leq 1$, and the curve is a straight line with a $2\alpha$ slope in the logarithm coordinate. The normalized intensity correlation function along the $x$-axis is calculated at scattering angle $\theta = 71^\circ$, and the result is shown in Fig. 4. In this figure, the inset (b) is the magnification of the boxed part. The solid lines 1 and 2 are the linear fitting of the data in the regions of $\rho < 64$ $\mu$m and 216 $\mu$m $< \rho <$ 344 $\mu$m, respectively. The slope in the region of $\rho < 64$ $\mu$m is 2 and $\alpha_1 \approx 1$. In the region of 216 $\mu$m $< \rho <$ 344 $\mu$m the slope is 1.5834 and $\alpha_2 \approx 0.7917$. This result indicates that the fractal exponent is 1 for small values of $\rho$, however, it will be less than 1 in a long region. That is to say, the spatial intensity distribution of this speckle has a particular character. We call it regional fractal speckle.

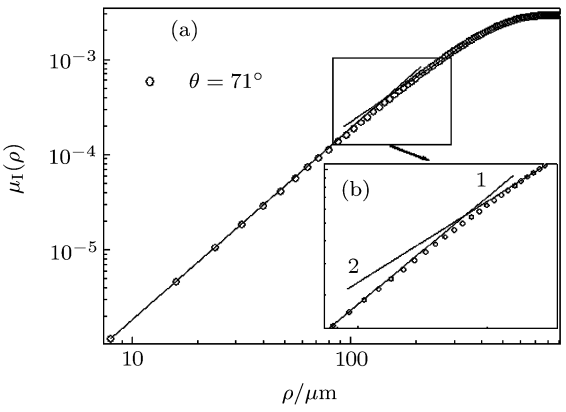



**Fig. 4.** Experimental curves of normalized intensity correlation function.

In order to obtain an analytic expression of the intensity correlation function, we defined two intensity correlation length $d$ and $D$ along $x$-axis, which denote the correlation length of small and big speckle particles, respectively. In the region of $\rho < d$, the intensities in small and big speckle particles are correlative and satisfy the Gaussian distribution, so the correlation function $\mu_{\rm I}(\rho) \propto \rho^2$ and there is no fractal character. Figure 5 gives a one-dimensional intensity profile along the $x$-axis at the scattering angle $\theta = 71^\circ$, which reveals that some small speckle particles and a big speckle particle are formed. It is obvious that the intensities are correlative in the two gray regions with

the widths $\rho_1$ and $\rho_2$ respectively. The intensity is still correlative and satisfies the Gaussian distribution in big speckle particles when $d < \rho < D$, such as in gray region $A$ with the width $\rho_2$. But it is no longer correlative in small speckle particles because $\rho$ has exceeded their correlation length $d$. The gray region $B$ in Fig. 5 indicates this condition. So in small speckle particles, the correlation function $\mu_{\rm I}(\rho) \propto \rho^{2\alpha'}(\alpha' < 1)$. Combine the two different distributions in big and small speckle particles, the correlation function satisfies $\mu_{\rm I}(\rho) \propto \rho^{2\alpha}$ $(\alpha' < \alpha < 1)$. This indicates that in the region $d < \rho < D$ the fractal character has come into being.

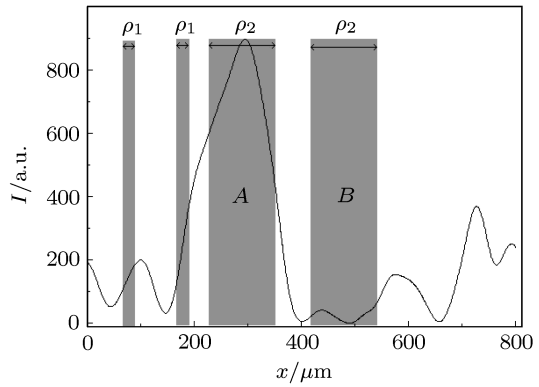



**Fig. 5.** 1-dimension intensity distribution along $x$-axis. From left to right, width of gray region is $\rho_1$, $\rho_1$, $\rho_2$, $\rho_2$; and $\rho_1 < d$, $d < \rho_2 < D$.

Frankenthal and Whitman[17] studied the scintillations due to multiscale phase screens. They gave a multiscale phase screen characterized by an inner scale $l$ and an outer scale $L$ and obtained its correlation function. The fractal property of the phase screen is similar to that of the regional fractal speckle. We care only about the range $\rho < D$ when studying the fractal character, so a similar expression of the intensity correlation function is obtained by considering the results of Frankenthal

$$\mu_{\rm I}(\rho) = \omega\left\{1 - \exp\left[-(d/D)^{2\alpha}\left(\left\{1+\rho^2/d^2\right\}^{\alpha} - 1\right)\right]\right\} \quad (\rho < D), \tag{7}$$

where $\omega$ is an empirical constant. The asymptotic expressions for $\mu_{\rm I}(\rho)$ at small and long values of difference coordinate $\rho$ can be obtained separately by expanding Eq. (7) appropriately. They are as follows:

$$\mu_{\rm I}(\rho) \approx \frac{\omega\alpha d^{2(\alpha-1)}}{D^{2\alpha}}\rho^2 \quad (\rho << d), \tag{7a}$$

$$\mu_{\rm I}(\rho) \approx \frac{\omega}{D^{2\alpha}}\rho^{2\alpha} \quad (d << \rho << D). \tag{7b}$$

These expressions are consistent with the property of regional fractal. Expression (7a) represents $\mu_{\rm I}(\rho) \propto \rho^2$ for small values of $\rho$, and the fractal exponent $\alpha = 1$ in this region; expression (7b) reveals that $\mu_{\rm I}(\rho) \propto \rho^{2\alpha}$, $0 < \alpha < 1$ in the region of $d << \rho << D$, and the speckle has fractal characters.

Take $d = 150$ $\mu$m and $D = 756$ $\mu$m, the curve shown in Fig. 4 will be fitted based on Eq. (7). We obtain $\alpha = 0.79242$ and $\omega = 0.00682$ as shown in Fig. 6. In the process of fitting curve, the value of $d$ is determined by the midpoint location between the two lines which have different slopes on the curve, and $D$ equals the value of $\rho$ when the curve firstly increases to a maximum.

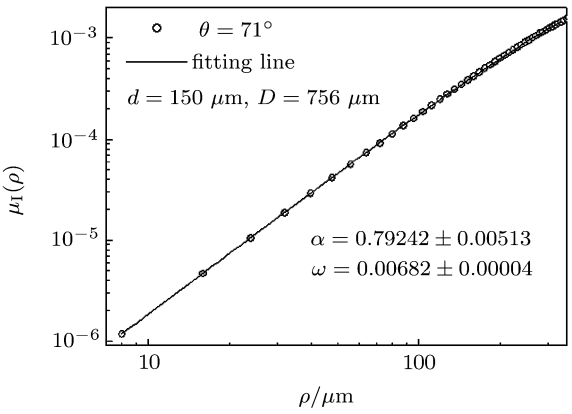



**Fig. 6.** Results of fitting.

For the circular Gaussian statistic speckle, Goodman *et al.* have indicated that the probability density of speckle intensity obeys a negative exponential law and the complex amplitude obeys the Gaussian distribution,[8,18] that is,

$$p_{\rm I}(I) = \begin{cases} \dfrac{1}{\langle I\rangle}\exp\left(-\dfrac{I}{\langle I\rangle}\right), & I \geq 0, \\ 0, \quad \text{otherwise}, \end{cases} \tag{8}$$

$$p_{\rm r}(A_{\rm r}) = \frac{1}{\sqrt{\pi\langle I\rangle}}\exp\left(-\frac{A_{\rm r}^2}{\langle I\rangle}\right), \tag{9a}$$

$$p_{\rm i}(A_{\rm i}) = \frac{1}{\sqrt{\pi\langle I\rangle}}\exp\left(-\frac{A_{\rm i}^2}{\langle I\rangle}\right). \tag{9b}$$

Normalized probability densities of the speckle intensities and complex amplitudes at scattering angle $\theta = 71°$ are calculated and shown in Figs. 7 and 8, respectively. According to Eq. (8) and Gaussian fitting, the solid lines in Figs. 7 and 8 are obtained. All the cases we have examined allow us to conclude that the regional fractal speckle is in the Gaussian regime and obeys zero-mean circular complex Gaussian statistics.

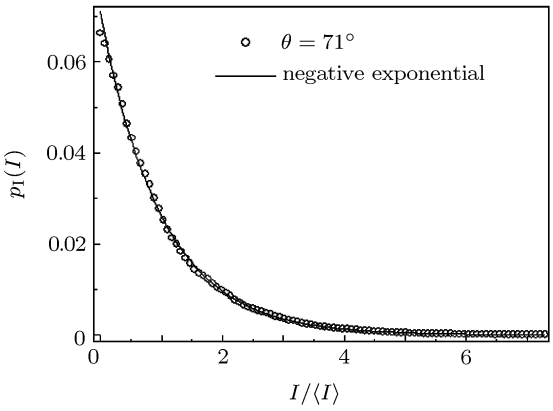



**Fig. 7.** Experimental probability density functions of the speckle intensities, $\theta = 71°$.

Furthermore, the statistic investigating results of the speckle at different scattering angles, i.e. $\theta = 65°$ and 84° show that they also obey zero-mean circular complex Gaussian statistics. The normalized intensity correlation functions and the fitting results are shown in Fig. 9. Comparing Fig. 9 with Fig. 6, we can see that with the increase of scattering angle, the correlation lengths of small speckle particle $d$ and big speckle particle $D$ increase. But the fractal exponent changes inconsiderably. In conclusion, all the speckles at large scattering angles possess the property of spatial regional fractal.

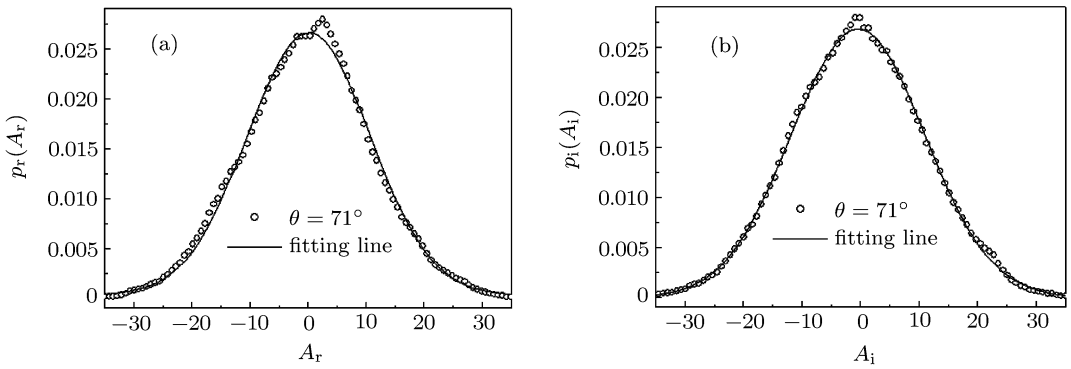



**Fig. 8.** Experimental probability density functions of speckle complex amplitudes at $\theta = 71°$.

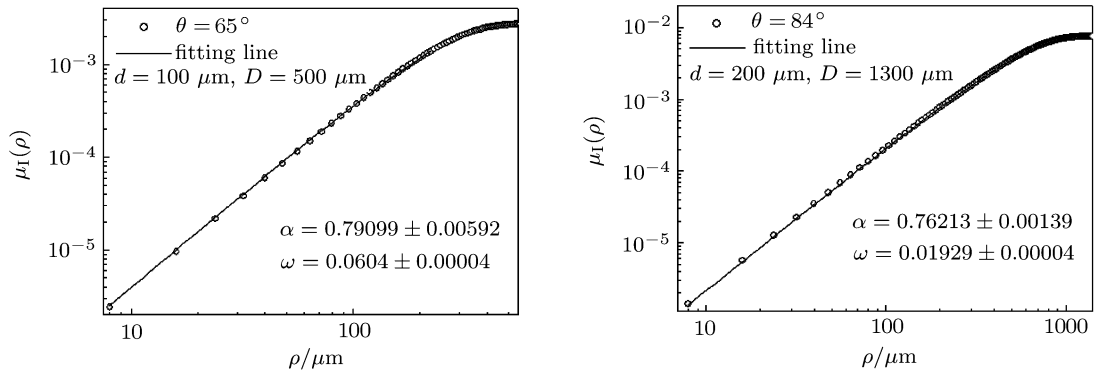



**Fig. 9.** Experimental normalized intensity correlation curves at two angles, i.e. $\theta = 65°$ and 84°.

## 4. Conclusions

Speckle with regional fractal property is generated experimentally at large scattering angles. By the technique of interference, the complex amplitudes and intensities of the speckle are extracted. Through fitting intensity correlation curves to an empirical analytic expression, we obtain the values of speckle fractal exponent at different scattering angles. From the probability distribution properties of the intensities and the complex amplitudes, we demonstrate that the regional fractal speckle obeys the zero-mean circular complex Gaussian statistics. The method and the results presented in this paper have some significance in the generation and the research of fractal speckle.

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