Relations between chirp transform and Fresnel diffraction, Wigner distribution function and a fast algorithm for chirp transform

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Two physical interpretations of chirp transform related to Fresnel diffraction and Wigner distribution function are given. The chirp transform can be regarded as a Fresnel diffraction observed on a spherical tangent to the diffraction plane, or a rotation and stretching transformation of the Wigner distribution function space. A general fast algorithm for the numerical calculation of chirp transform is developed by employing two fast Fourier transform algorithms. The algorithm, by which a good evaluation can be achieved, unifies the calculations of Fresnel diffraction, arbitrary fractional-order Fourier transforms and other scalar diffraction systems. The algorithm is used to calculate the Fourier transform of a Gaussian function and the Fourier transform, the Fresnel transform, the Fractional-order Fourier transforms of a rectangle function to evaluate the performance of this algorithm. The calculated results are in good agreement with the analytical results, both in the amplitude and phase.

Keywords: chirp transform, Fresnel diffraction, Wigner distribution function, fractional Fourier transform

PACC: 4200, 4230K, 4230, 4230V

1. Introduction

Chirp transform (ChT) expressing the signal transform from x space (signal domain) to u space (response domain) is useful as a mathematical tool for calculating the responses of physical systems, such as chirp on laser pulses,[1] Fresnel transform (FnT)[2−6] near-field optical system,[7,8] fractional Fourier transforms (FrFT)[9−16] and other scalar diffraction systems. Therefore the numerical calculation of ChT is of fundamental importance for evaluating the optical parameters and the optical performances of physical response systems in practical implementation. A number of algorithms[17−21] have been devised for ChT systems, therefore the properties of ChT systems have been investigated mathematically.

In this paper, we relate the ChT to Fresnel diffraction and Wigner distribution function (WDF) and a fast algorithm is proposed to unify the evaluations of ChT systems. Computational complexity of this algorithm, in which two fast Fourier transform algorithms (FFT) with an analytical kernel are employed, is better than that of a fast convolution algorithm. Several applied examples are presented to demonstrate the excellent performance of the algorithm.

2. Chirp transform and Fresnel diffraction

One-dimensional ChT of an arbitrary signal function $f(x)$ is defined as

$$g_{\mathrm{ch}}(u) = C\{f(x)\} = \int_{-\infty}^{+\infty} f(x) \cdot B_{\mathrm{ch}}(u,x)\mathrm{d}x. \quad (1)$$

The analytical kernel is

$$B_{\mathrm{ch}}(u,x) = A(p)\exp\left[\mathrm{j}\pi\left(\alpha x^2 + \beta xu + \gamma u^2\right)\right], \quad u,x \in R, \quad (2)$$

where $A(p)$ is the amplitude of the analytical kernel and generally it is a complex value, $p = (\alpha, \beta, \gamma)$ is the transform parameter, and j is the imaginary unit.

Let A be the incidence plane and O be the coordinate origin. Point M in plane A is located by a vector $x = OM$. Then let B be a plane at distance z from plane A and O' be the origin of plane B. The value of z is positive if plane B is taken to be in the direction of the propagation of light. Vector $x' = O'N$ represents an arbitrary point N in B as shown Fig. 1.

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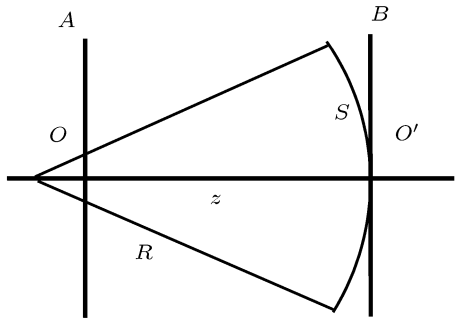


Fig. 1. Schematic diagram of Light propagation, where S is the sphere tangent of plane B with radius $R = 1/(\lambda(\beta^2/((4\alpha) - \gamma)))$.

The wavelength of light propagating in an isotropic and homogeneous medium is λ and λ is fixed from now on. If $U_A(x)$ represents the complex amplitude in plane A and $U_B(x')$ is the diffraction field amplitude at point N in plane B, the FnT from plane A to plane B is expressed as

$$U_B(x') = \frac{\exp(\mathrm{j}kz)}{\lambda z}\int U_A(x) \times \exp\left[\frac{\mathrm{j}\pi}{\lambda z}(x-x')^2\right]\mathrm{d}x. \tag{3}$$

This form is very similar to the mathematical expression of ChT $k = 2\pi/\lambda$ is the wave vector. By introducing scaling factors

$$\begin{cases} f = 1/(\alpha\lambda), \\ a = -\beta/(2\alpha), \\ R = 1/\left(\lambda\left(\beta^2/(4\alpha) - \gamma\right)\right), \end{cases} \tag{4}$$

and

$$x' = au = -\beta u/(2\alpha) \tag{5}$$

into Eq. (3), ChT can be rewritten as

$$g_{\mathrm{ch}}(u) = C\{f(x)\} = A(p)\int f(x) \times \exp\left[\frac{\mathrm{j}\pi}{\lambda z}(x'-x)^2\right]\mathrm{d}x \times \exp\left(-\frac{\mathrm{j}\pi}{\lambda R}u^2\right). \tag{6}$$

The external quadratic phase factor $\exp\left(-\mathrm{j}\pi/\lambda R\cdot u^2\right)$ represents a quadratic approximation to a spherical wave diverging from an illuminant point at distance R from plane B. Let S be circle tangent to plane B with radius R. The concavity or convexity of the circle tangent is relative to the positive or negative radius of curvature, which is taken from vertex to centre. For the point on S located by its projection in plane B, the coordinates in B can be used as coordinates on S approximately. Thus the quadratic factor is compensated if the field amplitude is observed on S rather than in plane B. The field amplitude on S can be expressed as a function of scaling factor u

$$U_S(u) = g_{\mathrm{ch}}(u)\cdot A'(p), \tag{7}$$

where $A'(p) = \mathrm{j}\lambda z/(\exp(jkz)\cdot A(p))$ is constant. Therefore the field amplitude of ChT of arbitrary optical signal can be observed on S except for a constant factor as shown in Fig. 1. This explanation is reasonable when formula (2) is satisfied.

3. Chirp transforms and Wigner distribution function

The Wigner distribution function is defined as[21−26]

$$W(x,\xi) = \int f_{\mathrm{F}}(\xi+\xi'/2)\, f_{\mathrm{F}}^*(\xi-\xi'/2)\exp(2\pi\,\mathrm{i}\,x\xi'/\lambda f_1)\,\mathrm{d}\xi', \tag{8}$$

where $\xi = \lambda f_1\nu$, ν is the spatial frequency and f_1 is an arbitrary fixed focal length.

As is well-known, the associated physical process of a 90° rotation of the WDF in the (x,ξ) space is Fraunhofer diffraction; the passage through a thin lens causes shearing in the ξ direction; and the propagation in free space corresponds to the x shearing of the WDF. The values of $W(x,\xi)$ are invariable under these three operations, which can be rewritten as three affine matrixes.[26]

By substituting scaling factors $f = -2/(2\alpha\lambda + \beta\lambda)$, $z = -2/\beta\lambda$ and $f' = -2/(\beta\lambda + 2\gamma\lambda)$ into Eq. (1), the equation can be rewritten as

$$g_{\mathrm{ch}}(u) = A(p)\,F^{-1}\left\{F\left\{f(x)\exp\left(-\frac{\mathrm{j}\pi}{\lambda f}x^2\right)\right\}\cdot F\left\{\exp\left(\frac{\mathrm{j}\pi}{\lambda z}x^2\right)\right\}\right\}\cdot\exp\left(-\frac{\mathrm{j}\pi}{\lambda f'}u^2\right). \tag{9}$$

Let $f = f_1/Q$, $f' = f_1/Q'$ and $z = Rf_1$, then we will obtain

$$g_{\mathrm{ch}}(u) = A(p)\,F^{-1}\left\{F\left\{f(x)\exp\left(-\frac{\mathrm{j}\pi}{\lambda f_1/Q}x^2\right)\right\}\cdot F\left\{\exp\left(\frac{\mathrm{j}\pi}{\lambda z}x^2\right)\right\}\right\}\cdot\exp\left(-\frac{\mathrm{j}\pi}{\lambda f_1/Q'}u^2\right). \tag{10}$$

Fourier transform of the transforming kernel has a closed form as follows:

$$F\left\{\exp\left(\frac{\mathrm{j}\pi}{\lambda z}x^2\right)\right\} = \sqrt{\mathrm{j}\lambda R f_1}\exp\left(-\frac{\mathrm{j}\pi}{\lambda f_1}R\xi^2\right). \tag{11}$$

Therefore Eq. (10) can be expressed as

$$g_{\mathrm{ch}}(u) = A'(p)\,F^{-1}\left\{F\left\{f(x)\exp\left(-\frac{\mathrm{j}\pi}{\lambda f_1/Q}x^2\right)\right\}\cdot\exp\left(-\frac{\mathrm{j}\pi}{\lambda f_1}R\xi^2\right)\right\}\cdot\exp\left(-\frac{\mathrm{j}\pi}{\lambda f_1/Q'}u^2\right), \tag{12}$$

where $A'(p) = A(p)\cdot\sqrt{\mathrm{j}\lambda R f_1}$ and $\nu = \xi/\lambda f_1$.

The ChT can be realized physically through two lenses and a propagation of free space as plotted in Fig. 2.

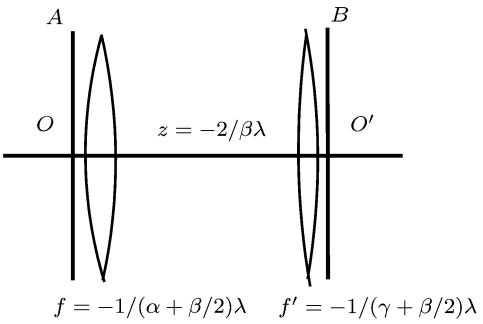


Fig. 2. Optical implemention of ChT with two lenses and a propagation of free space.

Affine transformation matrix of ChT is given by

$$\begin{pmatrix} x \\ u \end{pmatrix} \to \begin{pmatrix} 1-RQ & -R \\ Q'+Q-RQQ' & 1-RQ' \end{pmatrix}\begin{pmatrix} x \\ u \end{pmatrix} = \begin{pmatrix} 2\alpha/\beta & 2/\beta\lambda f_1 \\ \lambda f_1(2\alpha\gamma/\beta-\beta/2) & 2\gamma/\beta \end{pmatrix}\begin{pmatrix} x \\ u \end{pmatrix}. \tag{13}$$

The transform is reversible and one-to-one correspondence for the model of transformation matrix equal to 1. Therefore the ChT can be defined as a rotation and stretching transformation of the WDF space.

If we let $Q = Q' = \tan(\theta/2)$ and $R = \sin\theta$, the transformation matrix is changed into

$$\begin{pmatrix} x \\ u \end{pmatrix} \to \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} x \\ u \end{pmatrix}, \tag{14}$$

which means a rotation of the WDF. Then we obtain

$$g_{\mathrm{FrFT}}(u) = A(p)\int f(x)\exp\left(\frac{\mathrm{j}\pi}{\lambda f_1\tan\theta}x^2 - \frac{2\mathrm{j}\pi}{\lambda f_1\sin\theta}xu + \frac{\mathrm{j}\pi}{\lambda f_1\tan\theta}u^2\right)\mathrm{d}x. \tag{15}$$

A normal fractional-order Fourier transform is obtained. Therefore FrFT, which can be regarded as a rotation of the WDF, is a special case of ChT.

4. Fast chirp transform algorithm

From the explanation mentioned above, a fast and practical algorithm is developed to calculate the ChT. Equation (1) is discretized first by using the equidistant sampling for simplicity. The sampling steps of the signal and the response space are denoted by δx and δu respectively. The sampling numbers N for x space and M for u space must be limited, but they need not equal. The discretization of Eq. (1) is given by

$$\begin{aligned} &x \to n\delta x \quad n = 0, 1, 2, \ldots N-1, \\ &u \to m\delta u \quad m = 0, 1, 2, \ldots M-1, \end{aligned} \tag{16}$$

$$\begin{aligned} g_m(m) &= A(p)\sum_{n=0}^{N-1} f_n \exp\left[\mathrm{j}\pi\left(\alpha+\frac{\beta}{2}\right)(n\delta x)^2\right]\exp\left\{-\mathrm{j}\pi\frac{\beta}{2}\left[(n\delta x)-(m\delta u)\right]^2\right\}\exp\left[\mathrm{j}\pi\left(\gamma+\frac{\beta}{2}\right)(m\delta u)^2\right]\delta x \\ &= A(p)\,F^{-1}\left\{F\left\{f_n\exp\left[\mathrm{j}\pi\left(\alpha+\frac{\beta}{2}\right)(n\delta x)^2\right]\right\}F\left\{\exp\left(-\mathrm{j}\pi\frac{\beta}{2}n^2\delta x^2\right)\delta x\right\}\right\} \\ &\quad\times\exp\left[\mathrm{i}\,\pi\left(\gamma+\frac{\beta}{2}\right)(m\delta u)^2\right]. \end{aligned} \tag{17}$$

The transforming kernel is a generalized Gaussian function whose Fourier transform has a closed form. Suppose that the sampling number in x space is $L \geq N$ and the Fourier transform of the kernel on the u grids is

$$F\left\{\exp\left(-\mathrm{i}\,\pi\frac{\beta}{2}n^2\delta x^2\right)\delta x\right\} = \sqrt{-\frac{2\mathrm{j}}{\beta}}\exp\left[\frac{2\pi\mathrm{j}}{\beta\delta x^2}\left(\frac{k}{L}\right)^2\right], \quad k = 0, 1, 2, \ldots, L-1. \tag{18}$$

The discrete ChT of Eq. (17) can be rewritten as

$$A(p)\,F^{-1}\left\{F\left\{f_n\exp\left[\mathrm{j}\pi\left(\alpha+\frac{\beta}{2}\right)\left(n\frac{\Delta x}{L}\right)^2\right]\right\}\cdot\sqrt{-\frac{2j}{\beta}}\exp\left[\frac{2\pi j}{\beta\delta x^2}\left(\frac{k}{L}\right)^2\right]\right\}\cdot\exp\left[\mathrm{i}\,\pi\left(\gamma+\frac{\beta}{2}\right)(k\delta u)^2\right], \tag{19}$$

where $\Delta x = L\cdot\delta x$. Equation (19) provides a good evaluation if the Nyquist sampling condition is fulfilled. The quadratic phase factor multiplying f_n is considered first. If $|\alpha+\beta/2|$ is too large, the kernel has a too rapid variation with the spatial coordinate, thereby impeding an accurate sampling for numerical calculation. Thus it is easy to obtain the approximate condition as follows:

$$\Delta x^2\left|\alpha+\frac{\beta}{2}\right|\Big/L \leq 1. \tag{20}$$

Done the same way, the range of transforming kernel where the ChT of $f(x)$ achieves evaluable results will be given as

$$\left|\frac{2}{\beta L\delta x^2}\right| \leq 1. \tag{21}$$

Combining the conditions in Eqs.(20) and (21) yields

$$\Delta x^2\left|\alpha+\frac{\beta}{2}\right| \leq L \leq \Delta x^2\left|\frac{\beta}{2}\right|, \tag{22}$$

where $|\alpha+\beta/2|$ and $|\beta/2|$ denote the absolute values of $\alpha+\beta/2$ and $\beta/2$, respectively. However, another condition must be fulfilled, i.e.

$$\delta x = \delta u. \tag{23}$$

Suppose that the original sampling numbers are N and M in x space and in u space respectively. The length of the convolution kernel is $L = N$. The sequence of simulation is summarized, with the constant factors ignored below.

(i) Let $L = N$ for $N \geq M$.

(ii) Then let

$$f'_n(n) = f(n)\exp\left[\mathrm{j}\pi\left(\alpha+\frac{\beta}{2}\right)\left(\frac{n\Delta x}{L}\right)^2\right]. \tag{24}$$

(iii) Perform the fast convolution operation in Eq. (19)

$$C(l) = F^{-1}\left\{F\{f'_n(n)\}\cdot\sqrt{-\frac{2\mathrm{j}}{\beta}}\times\exp\left[\frac{2\pi\mathrm{j}}{\beta\delta x^2}\left(\frac{k}{L}\right)^2\right]\right\}, \quad l = 0, 1, 2, \ldots, L-1. \tag{25}$$

(iv) Discard the first and last $(L-M)/2$ data of $C(L)$, and for $0 \leq k \leq M-1$ let

$$C^p(l) = C(l)\cdot\exp\left[\mathrm{j}\pi\left(\gamma+\frac{\beta}{2}\right)(k\delta u)^2\right], \quad l = 0, 1, 2, \ldots, M-1. \tag{26}$$

If $N \geq M$, we can let $L = M$ and perform the algorithm similarly.[19] The chirp transform of arbitrary signal function $f(x)$ can be computed exactly through the above steps.

The complexity of numerically evaluating Eq. (19) lies in $2L[\log L + O(1)]$. To demonstrate the performance of this fast algorithm, some examples are presented in the next section.

5. Application examples

5.1. Analytical chirp transforms

The ChT possesses many functions that are analytically available, such as the Fourier transform of a Gaussian function and a rectangle function. For simplicity, $\Delta x = \Delta u = \sqrt{N}$, $N = M = 512$ and $L = 512$ are used here. First, the fast algorithm is evaluated with a Gaussian function $f(x) = \exp(-x^2/a^2)$ where $a = \sqrt{\pi}/2$ and $x \in [-\Delta x/2\,,\,\Delta x/2]$. A normalized error measurement $e_{\mathrm{ChT}} = \left|g_k - F\left\{\exp\left(-x^2/a^2\right)\right\}\right|/\max(|g_k^p|)$ between the numerical and the analytical results is used to check the accuracy. The error curve for the ChT of the Gaussian function is plotted in Fig. 3(a), with maximal normalized error $e_{\mathrm{ChT}} \approx 10^{-15}$.

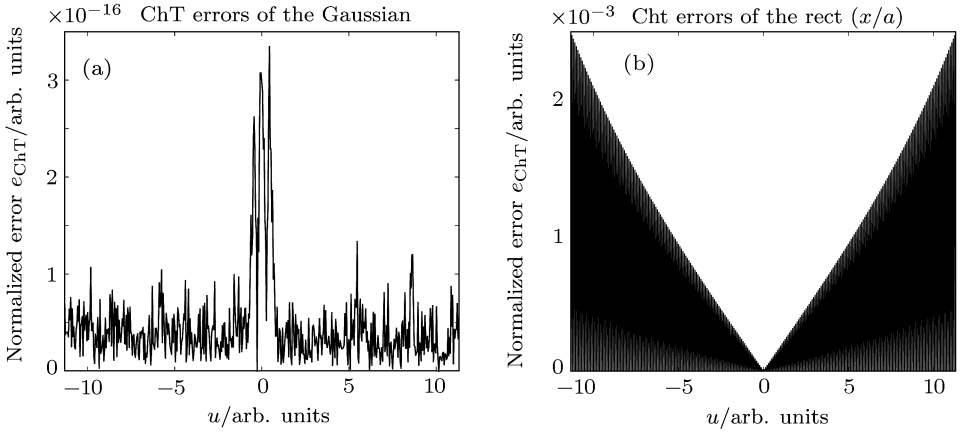


Fig. 3. (a) Errors of ChT of $f\ (x) = \exp(-x^2/a^2)$ normalized by the analytical transform with $a = \sqrt{\pi}/2$ versus u, and (b) errors of the ChT of rect(x/a) normalized by the analytical transform with $a = \Delta x/2$ versus u.

A rectangle function, rect (x/a), is also used to evaluate the accuracy of this algorithm. Its Fourier transform is denoted by $F\left\{\text{rect}\,(x/a)\right\} = \sin(2\pi au)/(\pi u)$. With the same scaling factors as the above except for $a = \Delta x/2$, the errors are shown in Fig.3(b) with maximal normalized error $e_{\text{ChT}} \approx 10^{-3}$, which is much higher than that of Gaussian function.

However, the validity and the performance of the algorithm do not reduce. By comparison with the results in Ref. [8], most of the errors are observed to accumulate in the FFT, so that one can optimize the FFT parameters to gain a higher accuracy.

5.2. Fresnel diffraction of a rectangular window

The Fresnel diffraction optical system is one of the simplest physical response systems. A rectangle function's Fresnel diffraction is used to test the algorithm here. The analytical form of the FnT of rectangle function rect (x/a) is

$$\begin{aligned}\text{FnT}\left\{\text{rect}\,(x/a)\right\} &= \int_{-\infty}^{+\infty} \text{rect}\,(x/a)\exp\left[\frac{\mathrm{j}\pi}{\lambda f}(x-u)^2\right]\mathrm{d}x \\ &= \frac{\sqrt{\mathrm{j}\lambda f}}{2}\left\{\text{erf}\left[\sqrt{\frac{\pi}{\mathrm{j}\lambda f}}\left(\frac{a}{2}-f\right)\right] + \text{erf}\left[\sqrt{\frac{\pi}{\mathrm{j}\lambda f}}\left(\frac{a}{2}+f\right)\right]\right\},\end{aligned}$$

where $\text{erf}\,(x) = 2/\sqrt{\pi}\cdot\int_0^x \exp\left(-t^2\right)\mathrm{d}t$ is the error function. The values of scaling factors are $a = \Delta x/2$, $\Delta x = \Delta u = \sqrt{N}$, $N = M = 512$ and $L = 512$.

Deviations of those computed value from exact analytical value can be observed in Fig. 4(a) for the amplitude and in Fig. 4(b) for the phase, showing better agreement between the computed results and the exact values than in Ref. [27].

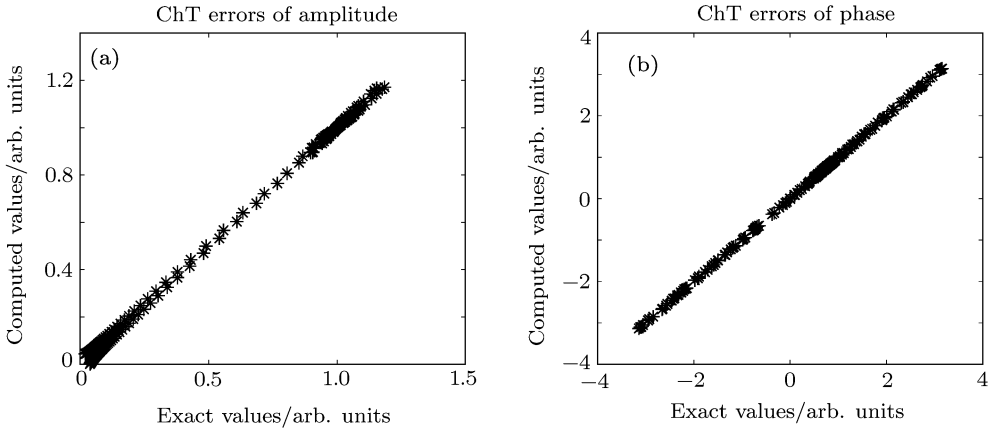


Fig. 4. (a) Deviations of computed amplitude value from the exact analytical amplitude value, and (b) deviations of computed phase value from the exact analytical phase value.

5.3. Fractional Fourier transform of a rectangular window

The FrFT of a square function is also be used to test the algorithm. Using the same scaling factors as those used above, yields an analytical form of the FrFT of square function

$$\begin{aligned}\mathrm{FrFT}\left\{\mathrm{rect}\left(x/a\right)\right\} &= \int_{-\infty}^{+\infty} \mathrm{rect}\left(x/a\right) \exp\left[\mathrm{j}\pi\left(\frac{x^2}{\lambda f_1 \tan\theta} - \frac{2xu}{\lambda f_1 \sin\theta} + \frac{u^2}{\lambda f_1 \tan\theta}\right)\right] \\ &= \sqrt{\frac{\mathrm{j}\lambda f_1 \tan\theta}{2}} \left\{\mathrm{erf}\left[\sqrt{\frac{\pi}{\mathrm{j}\lambda f_1 \tan\theta}}\left(\frac{a}{2} - \frac{u}{\cos\theta}\right)\right] + \mathrm{erf}\left[\sqrt{\frac{\pi}{\mathrm{j}\lambda f_1 \tan\theta}}\left(\frac{a}{2} + \frac{u}{\cos\theta}\right)\right]\right\},\end{aligned}$$

where $\theta = p\pi/2$ and p is the factional-order of Fourier transform, and $p = 0.5$ has been used.

Deviations of the computed value from analytical value are shown in Fig. 5(a) for the amplitude and in Fig. 5(b) for the phase. The amplitudes match better than the phases. Furthermore, the FrFT parameters can be optimized to achieve a higher accuracy.

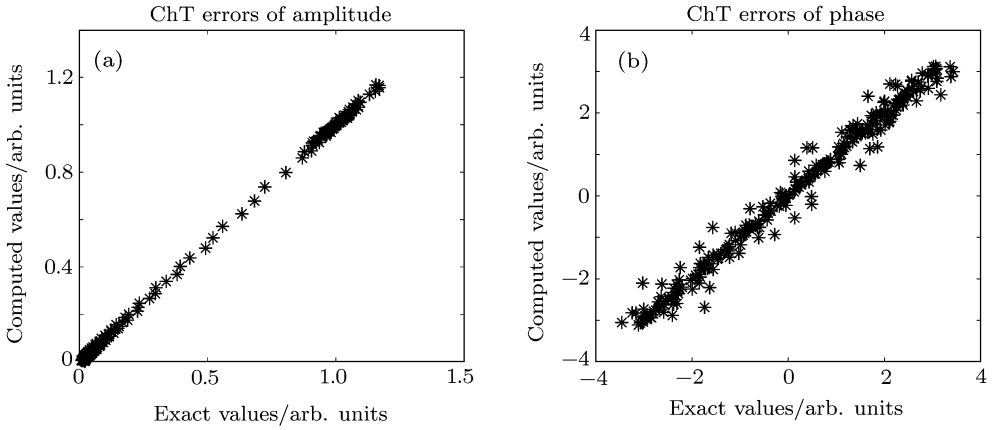


Fig. 5. (a) Deviations of computed amplitude value from the exact analytical amplitude value, and (b) deviations of computed phase value from exact analytical phase value.

6. Conclusion

Two physical interpretations of ChT related to Fresnel diffraction and WDF are presented. The ChT can be regarded as a Fresnel diffraction observed on a spherical tangent to the diffraction plane with radius $R = 1/\left(\lambda\left(\beta^2/(4\alpha) - \gamma\right)\right)$, or it can also be considered to be a rotation and stretching transformation of the WDF space. Employing two FFTs a general fast algorithm for numerical calculation of ChT is attained. The algorithm has a good evaluation when $\Delta x^2\left|\alpha + \beta/2\right| \leq L \leq \Delta x^2\left|\beta/2\right|$ is fulfilled. The computation complexity of this algorithm lies in $2L\left[\log L + O\left(1\right)\right]$, which is better than a fast convolution. The Fourier transform of a Gaussian function and the Fourier transform, FnT, the FrFT of a square function are calculated by using the algorithm to evaluate the performance. The analytical results and the calculated results both in the amplitude and in phase are in good agreement and the accuracy of this algorithm is better than that in Ref. [27]. Other ChT systems can also be accurately calculated with this algorithm, which makes it a handy tool for solving many general scalar diffraction problems.

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