Two-dimensional multiplicity fluctuation analysis of target residues in nuclear collisions*

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Multiplicity fluctuation of the target residues emitted in the interactions in a wide range of projectile energies from 500 A MeV to 60 A GeV is investigated in the framework of two-dimensional scaled factorial moment methodology. The evidence of non-statistical multiplicity fluctuation is found in ^{16}O–AgBr collisions at 60 A GeV, but not in ^{56}Fe–AgBr collisions at 500 A MeV, ^{84}Kr–AgBr collisions at 1.7 A GeV, ^{16}O–AgBr collisions at 3.7 A GeV and ^{197}Au–AgBr collisions at 10.7 A GeV.

Keywords: relativistic heavy-ion collisions, target fragmentation, non-statistical fluctuation
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1. Introduction

Since Bialas and Peschanski[1] introduced the concept of intermittency into the study of large non-statistical fluctuation of pions produced in high energy cosmic–ray interactions, a lot of the data analyses have been carried out for the produced pions not only in nucleus–nucleus, but also in lepton–nucleus and hadron–nucleus collisions at high energies. It is believed that pions are the most informative about the collisional dynamics. However, the physics of relativistic interactions is not yet conclusive and therefore we need all probes to analyse thoroughly and meaningfully the experimental data.

Intermittency is defined as the scaled factorial moment growing according to power law with the decrease of phase-space interval size. The unique feature of this moment is that it can be used to detect and characterize the non-statistical density fluctuations in particle spectra, which are intimately related to the dynamics of particle production.

In high energy nucleus–nucleus collisions, target fragmentation also may carry the information about the collisional mechanism. The non-statistical fluctuations of the emission of target fragments in lepton–nucleus, hadron–nucleus and nucleus–nucleus collisions at high energies were investigated not only in one-dimensional phase-space[2−14] but also in two-dimensional phase-space.[15−24] The evidence of non-statistical fluctuation of target fragment production in these investigations was obtained. But in some of these studies[2,4,10] the saturation effect was observed to be that the scaled factorial moment grew according to power law with phase-space interval size decreasing, and in Ref. [16] the scaled factorial moment decreasing according to power law with phase-space interval size decreasing was also observed. These effects indicate that there is not a non-statistical fluctuation in the emission of target fragments. So the study of non-statistical fluctuation of target fragments is not conclusive.

In this paper the non-statistical fluctuations of target evaporated fragments produced in ^{56}Fe–AgBr collisions at 500 A MeV, ^{84}Kr–AgBr collisions at 1.7 A GeV, ^{16}O–AgBr collisions at 3.7 A GeV, ^{197}Au–AgBr collisions at 10.7 A GeV and ^{16}O–AgBr collisions at 60 A GeV are studied in two-dimensional phase space by using not only normal variables $\cos\theta$ and ϕ but also cumulative variables $X_{\cos\theta}$ and X_{ϕ}, where θ is the emissional angle and ϕ is the azimuthal angle of target evaporated fragment.

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2. Experimental details

Five stacks of nuclear emulsions were used in the present investigation. One stack of N4 nuclear emulsions made by the Institute of Modern Physics (Shanxi Normal University, China), was exposed horizontally to HIMAC, NIRS, Japan. The beam was of 500 A MeV ^{56}Fe and the flux was 3000 ions/cm^2. Another four stacks were provided by EMU-01 Collaboration. BA2000 and XSJ-2 microscopes with a 100× oil immersion objective and 10× ocular lenses were used to scan the plates. The tracks were picked up at a distance of 5 mm away from the edge of the plates and carefully followed until they either interacted with emulsion nuclei or escaped from the plates. Interactions which took place within 30 μm from the top or the bottom surface of the emulsion plates were not considered for final analysis. All the primary tracks were followed back to ensure that the events chosen did not include interactions from the secondary tracks of other interactions. When they were observed to do so the corresponding events were removed from the sample.

In each interaction all of the secondaries were recorded which included shower particles, target recoiled protons (grey track particles), target evaporated fragments (black track particles) and projectile fragments. Details of track classification can be found in our previous paper.[25−27]

To ensure that the target nuclei in nuclear emulsion are silver or bromine nuclei, we choose only the events with at least eight heavy ionizing track particles ($N_{\rm h} \geq 8$).

3. Experimental results

The non-statistical multiplicity fluctuation analysis is performed in a two-dimensional phase space. The phase space is divided equally in both directions by assuming it to be isotropic in nature. Denote the two phase space variables as x_1 and x_2, then the qth-order horizontal scaled factorial moment (SFM) is defined as[1]

$$F_{qi}(\delta x_1 \delta x_2) = M^{q-1} \sum_{m=1}^{M} \frac{n_{mi}(n_{mi}-1)\ldots(n_{mi}-q+1)}{n_i(n_i-1)\ldots(n_i-q+1)}, \quad (1)$$

where $\delta x_1 \delta x_2$ is the size of a two-dimensional cell, n_{mi} is the multiplicity in the mth cell of the ith event, n_i is the multiplicity of the ith event, and M is the number of two-dimensional cells into which the considered phase space has been divided. Then the averaged horizontal SFM becomes

$$\langle F_q(\delta x_1 \delta x_2)\rangle = \frac{1}{N_{\rm ev}} \sum_{i=1}^{N_{\rm ev}} F_{qi}(\delta x_1 \delta x_2). \quad (2)$$

Non-statistical multiplicity fluctuation would manifest itself as a power-law scaling of $\langle F_q\rangle$ with the cell size of the form

$$\langle F_q(\delta x_1 \delta x_2)\rangle \propto (\delta x_1 \delta x_2)^{-a_q}, \quad \text{as} \quad \delta x_1 \delta x_2 \to 0, \quad (3)$$

or a linear relation

$$\ln\langle F_q\rangle = -a_q \ln(\delta x_1 \delta x_2) + b_q = a_q \ln M + c_q. \quad (4)$$

The invariant quantity of the scaling $a_q > 0$ is called the intermittency exponent, and it is a measure of the fluctuation strength.

Figure 1 shows the dependences of $\ln\langle F_q\rangle$ on $\ln M$ for target evaporated fragment emitted from ^{56}Fe–AgBr collisions at 500 A MeV (a), ^{84}Kr–AgBr collisions at 1.7 A GeV (b), ^{16}O–AgBr collisions at 3.7 A GeV (c) and ^{197}Au–AgBr collisions at 10.7 A GeV (d) in normal variable space. It can be seen that for lower qth-order $\ln\langle F_q\rangle$ increases linearly with $\ln M$ increasing, but for higher qth-order $\ln\langle F_q\rangle$ increases linearly with the increase of $\ln M$ and then becomes saturated or decreased. So from Fig. 1 we cannot obtain clear evidence of non-statistical multiplicity fluctuation.

The single-particle density distribution in two-dimensional space comprised of emission angle space and azimuthal angle space is non-flat. As the shape of this distribution influences the scaling behaviour of the SFM. Bialas and Gazdzicki[28] introduced 'cumulative' variable which drastically reduces the distortion of intermittency due to non-uniformity of single-particle density distribution. Following them, the cumulative variable $X(x)$ is related to the single-particle density distribution $\rho(x)$ through

$$X(x) = \int_{x_1}^{x} \rho(x')\mathrm{d}x' \Big/ \int_{x_1}^{x_2} \rho(x')\mathrm{d}x', \quad (5)$$

where x_1 and x_2 are two extreme points of the distribution $\rho(x)$. The variable $X(x)$ varies between 0.0 and 1.0, with $\rho(X(x))$ kept almost constant. The values of x_1 and x_2 are -1 and 1 in $\cos\theta$-space, 0 and 2π in azimuthal angle space, respectively.

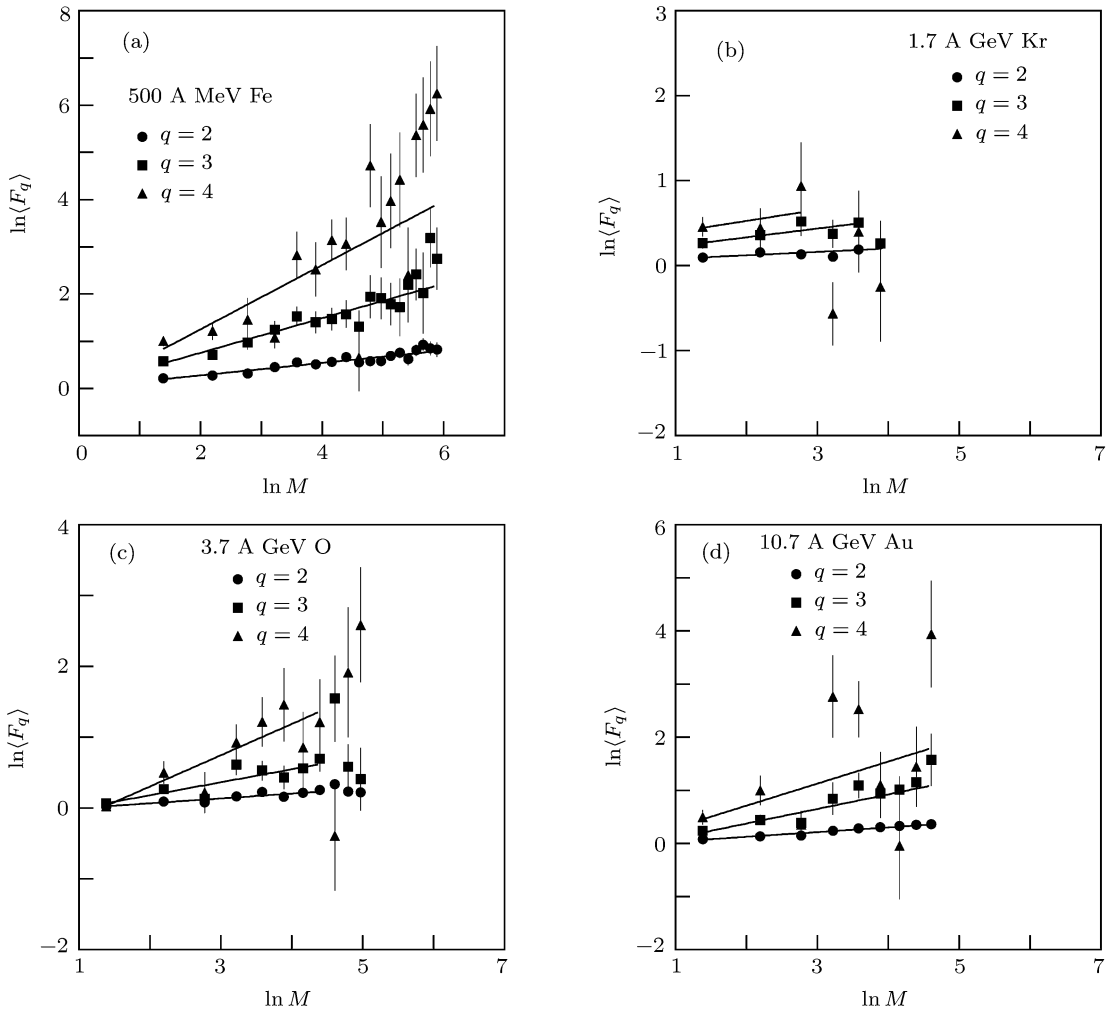


Fig. 1. Plots of $\ln\langle F_q\rangle$ against $\ln M$ for the target evaporated fragments emitted from ^{56}Fe–AgBr collisions at 500 A MeV (a), ^{84}Kr–AgBr collisions at 1.7 A GeV (b), ^{16}O–AgBr collisions at 3.7 A GeV (c) and ^{197}Au–AgBr collisions at 10.7 A GeV (d) in normal variable space.

Figure 2 shows the dependences of $\ln\langle F_q\rangle$ on $\ln M$ for target evaporated fragments emitted from ^{56}Fe–AgBr collisions at 500 A MeV (a), ^{84}Kr–AgBr collisions at 1.7 A GeV (b), ^{16}O–AgBr collisions at 3.7 A GeV (c) and ^{197}Au–AgBr collisions at 10.7 A GeV (d) in cumulative variable space. It can be seen that $\ln\langle F_q\rangle$ decrease linearly with $\ln M$ increasing, which indicates that no evidence of non-statistical multiplicity fluctuation is observed in our data sets. So any fluctuation indicated in Fig. 1 is totally caused by non-uniformity of single-particle density distribution.

The dependences of $\ln\langle F_q\rangle$ on $\ln M$ for target evaporated fragments emitted from ^{16}O–AgBr collisions at 60 A GeV in normal and cumulative variable spaces is shown in Figs. 3(a) and 3(b) respectively. In normal variable space, $\ln\langle F_q\rangle$ increases with $\ln M$ increasing and no saturation is observed. In cumulative variable space, $\ln\langle F_q\rangle$ decreases and then increases with the increase of $\ln M$. So the non-statistical multiplicity fluctuation is observed not only in normal variable space but also in cumulative variable space. These results are totally different from those obtained in Figs. 1 and 2.

In high energy nucleus–nucleus collision, target fragmentation produces 'grey' and 'black' tracks in nuclear emulsion. The grey tracks are formed due to fast target protons of energy reaching up to 400 MeV. The black tracks are images of target evaporated particles of low-energy ($E < 30$ MeV) singly- or multiply-charged fragments. In the cascade evaporation model,[29] the grey tracks are emitted from the nucleus very soon after the instant of impact, leaving the hot residual nucleus in an excited state. Emission of black particles from this state takes place relatively slowly. In a rest system of target nucleus, the directions of the emission of evaporation particles are distributed isotropically. Non-statistical multiplicity fluctuation does not exist in the

state of statistical equilibrium. This is the reason we have obtained in Figs. 1 and 2. If the isotropicity is disturbed by some processes, the non-statistical multiplicity fluctuation can be observed in the emission of target evaporated fragments. This is the result that we have observed in Fig. 3.

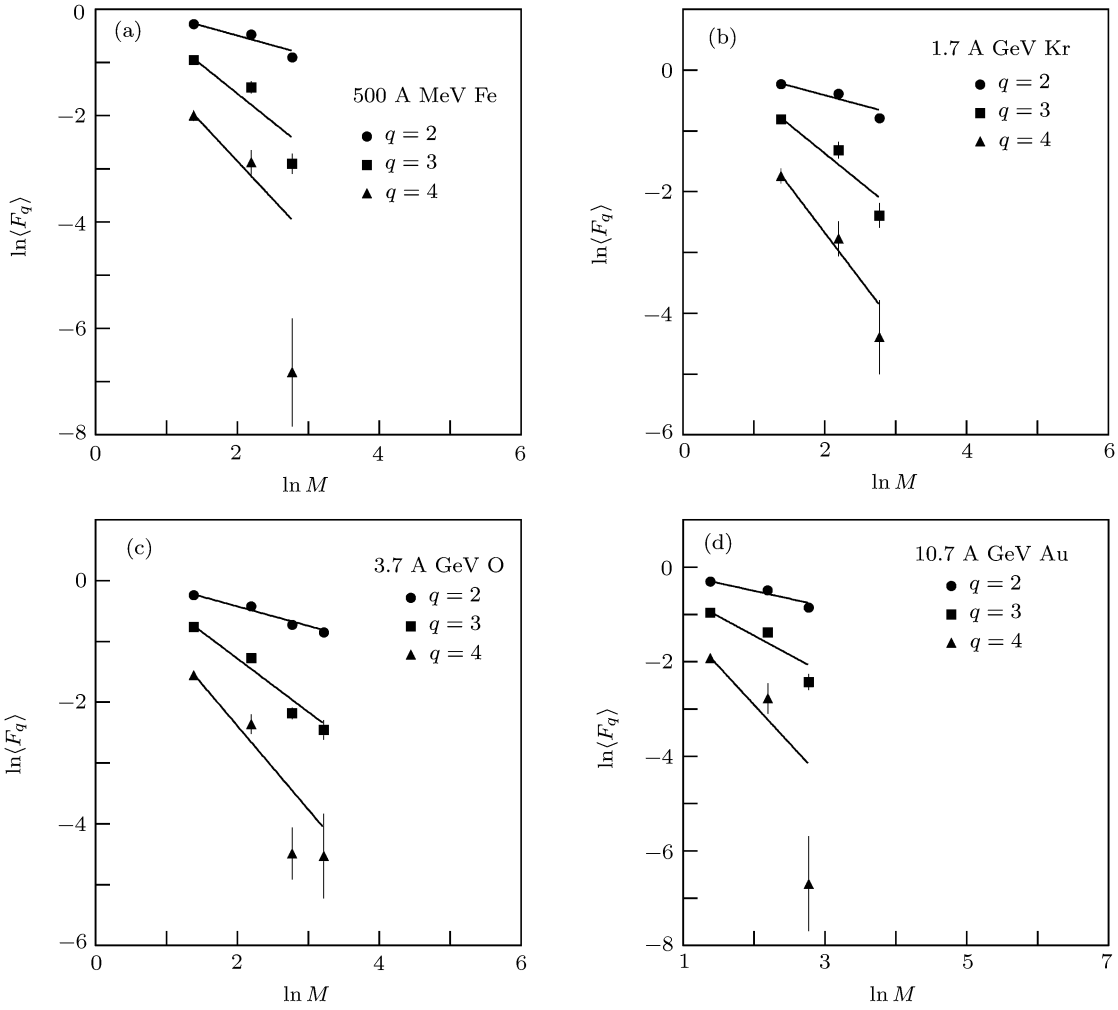


Fig. 2. Plots of $\ln\langle F_q\rangle$ against $\ln M$ for the target evaporated fragments emitted from ^{56}Fe–AgBr collisions at 500 A MeV (a), ^{84}Kr–AgBr collisions at 1.7 A GeV (b), ^{16}O–AgBr collisions at 3.7 A GeV (c) and ^{197}Au–AgBr collisions at 10.7 A GeV (d) in cumulative variable space.

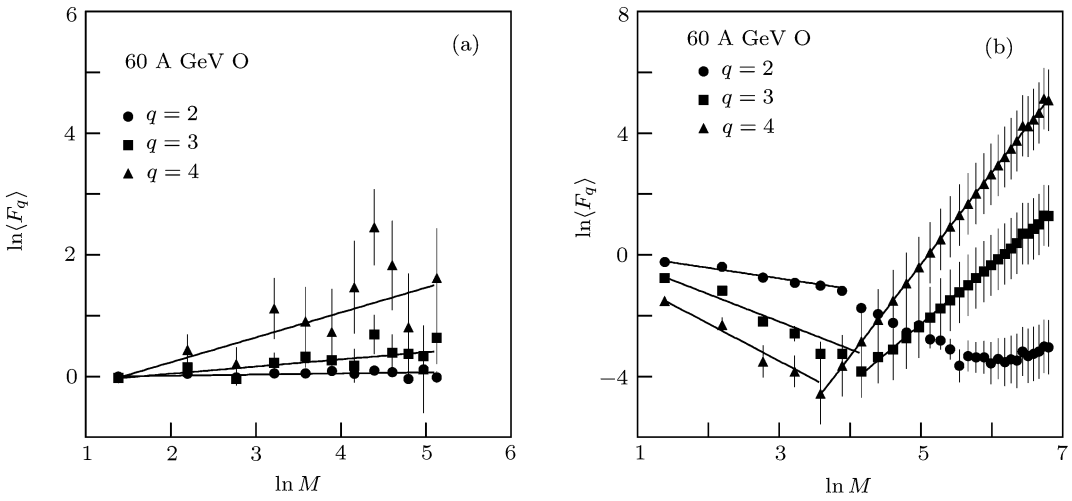


Fig. 3. Plots of $\ln\langle F_q\rangle$ against $\ln M$ for the target evaporated fragments emitted from ^{16}O–AgBr collisions at 60 A GeV in normal variable space (a) and cumulative variable space (b).

The power-law behaviour of the SFM implies the existence of some kind of fractal pattern[30] in the dynamics of the particle production in their final state. Therefore, it is natural to study the fractal nature of target evaporated fragments emitted from ^{16}O–AgBr collisions at 60 A GeV under a self-similar scaling scenario. The anomalous fractal dimension d_q of fractal and multifractal is related intermittency exponent a_q as

$$d_q = \frac{a_q}{q-1}. \tag{6}$$

The variation of d_q with order q is shown in Fig. 4

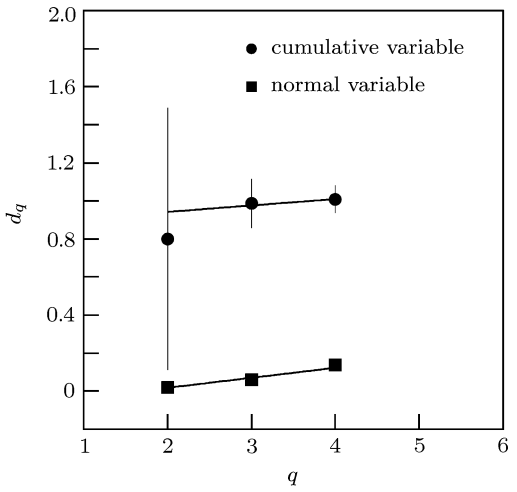


Fig. 4. Plots of d_q against q for target evaporated fragments produced in ^{16}O–AgBr collisions at 60 A GeV.

for target evaporated fragments produced from ^{16}O–AgBr collisions at 60 A GeV. From the plot it is seen that d_q increases with order q increasing, which suggests that a cascade-type mechanism may be responsible for the emission of target evaporated fragments in ^{16}O–AgBr collisions at 60 A GeV.

4. Conclusions

The non-statistical multiplicity fluctuations of target evaporated fragments emitted from ^{56}Fe–AgBr collisions at 500 A MeV, ^{84}Kr–AgBr collisions at 1.7 A GeV, ^{16}O–AgBr collisions at 3.7 A GeV, ^{197}Au–AgBr collisions at 10.7 A GeV and ^{16}O–AgBr collisions at 60 A GeV are investigated not only in normal variable space but also in cumulative variable space. The evidence is found only in ^{16}O–AgBr collisions at 60 A GeV. The anomalous fractal dimension of the intermittency is found to increase with the increase of order of moment q, which suggests the presence of multifractality in the emission of target evaporated fragments from ^{16}O–AgBr collisions at 60 A GeV.

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