H_∞ consensus control of a class of second-order multi-agent systems without relative velocity measurement*

Zhang Wen-Guang(张文广)[a)†], Zeng De-Liang(曾德良)[a)], and Guo Zhen-Kai(郭振凯)[b)]

[a)] *Beijing Key Laboratory of Measurement and Control New Technology and System for Industrial Process, Control & Computer Engineering School, North China Electric Power University, Beijing* 102206, *China*
[b)] *School of Mathematics and Information, Ludong University, Yantai* 264025, *China*

(Received 29 September 2009; revised manuscript received 23 January 2010)

This paper studies consensus control problems for a class of second-order multi-agent systems without relative velocity measurement. Some dynamic neighbour-based rules are adopted for the agents in the presence of external disturbances. A sufficient condition is derived to make all agents achieve consensus while satisfying desired H_∞ performance. Finally, numerical simulations are provided to show the effectiveness of our theoretical results.

Keywords: consensus control, H_∞, second-order multi-agent systems
PACC: 0565, 8710

1. Introduction

Distributed coordination of multiple agents has attracted a great number of researchers in recent years, partly due to recent technological advances in communication and computation, and important applications. During the past decade, numerous results have been obtained for distributed coordination of multiple agents.[1−16] For example, Lafferriere *et al.*[1] studied a method for decentralized stabilization of vehicle formations using techniques from algebraic graph theory. Also, Olfati-Saber *et al.*[2] considered a group of mobile second-order agents moving in the plane and introduced control laws which enable the group to achieve a common velocity while avoiding collisions. Ren[4] proposed several consensus algorithms for second-order multi-agent systems and derived sufficient conditions for state consensus of the system. Moreover, Hong *et al.*[6] investigated multiple second-order agent systems with jointly-connected interconnection topologies. In engineering practice, multi-agent systems are usually subjected to various disturbances such as time-delay and the variation of network topology. Lin *et al.*[16] considered consensus problems for first-order multi-agent systems with external disturbances and model uncertainty on fixed and switching topologies. However, by investigating recent results, little works have been published to address the effects of external disturbances for second-order multi-agent systems.

With this background, we investigate H_∞ conensus control of networks of second-order multi-agent systems with external disturbances. In the analysis, we introduce a new class of positive semi-definite Lyapunov functions, and then based on these functions, we derive a sufficient condition in terms of linear matrix inequalities (LMIs) to make all agents reach consensus while satisfying desired H_∞ performance.

The following notations will be used throughout this paper. The $\mathbb{R}^m$ denotes the set of all m dimensional real column vectors, $\boldsymbol{I}_m$ denotes the m dimensional unit matrix. $\otimes$ denotes the kronecker product. $\mathbf{1}$ represents $[1, 1, \ldots, 1]^{\mathrm{T}}$ with compatible dimensions (sometimes, $\mathbf{1}_n$ is used to denote $\mathbf{1}$ with dimension n), 0 denotes zero value or zero matrix with appropriate dimensions, $\mathcal{L}_2[0, \infty)$ denotes the space of square integrable vector functions over $[0, \infty)$.

2. Graph theory

Let $G(\mathcal{V}, \varepsilon, \mathcal{A})$ be an undirected graph of order n with the set of nodes $\mathcal{V} = \{s_1, \ldots, s_n\}$, set of edges $\varepsilon \subseteq \mathcal{V} \times \mathcal{V}$, and a weighted adjacency matrix $\mathcal{A} = [a_{ij}]$ with nonnegative elements a_{ij} for all $i, j \in \mathcal{I} = \{1, 2, \ldots, n\}$. An edge of G is denoted

*Project supported by the National High Technology Research and Development Program of China (Grant Nos. 2007AA041104, 2007AA041105 and 2007AA04Z163).
†Corresponding author. E-mail: zwg@ncepu.edu.cn
© 2010 Chinese Physical Society and IOP Publishing Ltd

by $e_{ij} = (s_i, s_j)$. The adjacency elements associated with the edges of the graph are positive, i.e., $e_{ij} \in \varepsilon \Leftrightarrow a_{ij} > 0$. Moreover, we assume $a_{ii} = 0$ for all $i \in \mathcal{I}$. The set of neighbours of node s_i is denoted by $N_i = \{s_j \in \mathcal{V} : (s_i, s_j) \in \varepsilon\}$. A diagonal matrix $\boldsymbol{D} = \mathrm{diag}\{d_1, \ldots, d_n\}$ is a degree matrix of G, whose diagonal elements $d_i = \sum_{j=1}^{n} a_{ij}$ for $i = 1, \ldots, n$. Then the Laplacian of the weighted graph G is defined as $\boldsymbol{L} = \boldsymbol{D} - \mathcal{A} \in \mathbb{R}^{n\times n}$.

Correspondingly, the graph Laplacian with the undirected graph is defined as $\boldsymbol{L} = [l_{ij}]$, where $l_{ii} = d_i$ and $l_{ij} = -a_{ij}$, $i \neq j$. An important fact of $\boldsymbol{L}$ is that all the row sums of $\boldsymbol{L}$ are zero and thus $\mathbf{1}_n$ is an eigenvector of $\boldsymbol{L}$ associated with the eigenvalue $\lambda = 0$, and for an undirected graph G, $\boldsymbol{L}$ is symmetric and positive semi-definite. A path is a sequence of ordered edges of the form (s_{i_1}, s_{i_2}), (s_{i_2}, s_{i_3}), $\ldots$, where $s_{i_j} \in \mathcal{V}$. An undirected graph is called connected if there is a path between any distinct pair of nodes.

Lemma 1[17] If the undirected graph G is connected, then its Laplacian $\boldsymbol{L}$ satisfies:

1) rank $(\boldsymbol{L}) = n - 1$, zero is one eigenvalue of $\boldsymbol{L}$, and $\mathbf{1}_n$ is the corresponding eigenvector, i.e., $\boldsymbol{L}\mathbf{1}_n = 0$;

2) the rest $n - 1$ eigenvalues are all positive and real.

3. Problem description

Suppose that the multi-agent system under consideration consists of n agents. Each agent is regarded as a node in an undirected graph $\mathcal{G}$. Each edge $(s_j, s_i) \in \varepsilon$ corresponds to an available information channel from the agent s_i to the agent s_j. Moreover, each agent updates its current state based upon the information received from its neighbours. Suppose the dynamics of the ith agent ($i \in \mathcal{I}$) is

$$\begin{aligned} \dot{x}_i(t) &= v_i(t), \\ \dot{v}_i(t) &= u_i(t), \end{aligned} \tag{1}$$

where $x_i(t) \in \mathbb{R}$ is the position state, $v_i(t) \in \mathbb{R}$ is the velocity state and $u_i(t) \in \mathbb{R}$ is the control input (or protocol).

For the system Eq. (1), we say that all agents achieve consensus, if the states of agents satisfy

$$\lim_{t\to+\infty} [x_i(t) - x_j(t)] = 0 \tag{2}$$

for all $i, j \in \mathcal{I}$.

To solve this problem, we use the following protocol originally proposed by Ref. [4]:

$$\begin{aligned} \dot{p}_i &= -\gamma p_i - k_1 \sum_{s_j\in N_i(t)} a_{ij}(x_j - x_i) + \eta_i, \\ \dot{x}_i &= v_i, \\ \dot{v}_i &= k_2 \sum_{s_j\in N_i(t)} a_{ij}(x_j - x_i) + p_i + w_i, \end{aligned} \tag{3}$$

where $k_1 > 0$, $k_2 > 0$, $\gamma > 0$ are parameters to be designed, $p_i(0) = 0$ and $\eta_i(t), w_i(t) \in \mathbb{R}$ are the external disturbances that belong to $\mathcal{L}_2[0, \infty)$.

Let

$$\begin{aligned} &\boldsymbol{\varphi} = [x_1, v_1, p_1, x_2, v_2, p_2, \ldots, x_n, v_n, p_n]^{\mathrm{T}}, \\ &\boldsymbol{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\gamma \end{bmatrix}, \quad \boldsymbol{B}_1 = \begin{bmatrix} 0 & 0 & 0 \\ k_2 & 0 & 0 \\ -k_1 & 0 & 0 \end{bmatrix}, \\ &\boldsymbol{B}_2 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^{\mathrm{T}}, \quad \boldsymbol{B}_3 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^{\mathrm{T}}. \end{aligned}$$

With the protocol Eq. (3), the network dynamics is

$$\begin{aligned} \dot{\varphi}(t) &= (\boldsymbol{I}_n \otimes \boldsymbol{A} - \boldsymbol{L} \otimes \boldsymbol{B}_1)\varphi(t) \\ &\quad + (\boldsymbol{I}_n \otimes \boldsymbol{B}_2)w(t) + (\boldsymbol{I}_n \otimes \boldsymbol{B}_3)\eta(t), \end{aligned} \tag{4}$$

where $\boldsymbol{w}(t) = [w_1(t), \ldots, w_n(t)]^{\mathrm{T}}$, $\boldsymbol{\eta}(t) = [\eta_1(t), \ldots, \eta_n(t)]^{\mathrm{T}}$ and $\boldsymbol{L}$ is the Laplacian of the graph $\mathcal{G}$.

A natural way to combine the relative information is to define output functions $z_i(t)$ for $i \in \mathcal{I}$ computed from an average of the relative displacements of all agents as follows:

$$\begin{aligned} z_i(t) &= \frac{1}{n}\sum_{j=1}^{n}[x_i(t) - x_j(t)] \\ &= x_i(t) - \frac{1}{n}\sum_{j=1}^{n} x_j(t). \end{aligned}$$

It is clear that the multi-agent system (1) achieves consensus if and only if

$$\lim_{t\to+\infty} z_i(t) = 0, \quad i = 1, \ldots, n.$$

Define the following H_∞ performance index

$$\begin{aligned} J &= \int_0^\infty [z^{\mathrm{T}}(t)z(t) - \gamma_1^2 w^{\mathrm{T}}(t)w(t) - \gamma_2^2 \eta(t)^{\mathrm{T}}\eta(t)]\,\mathrm{d}t \\ &< 0, \end{aligned} \tag{5}$$

where $\boldsymbol{z}(t) = [z_1(t), \ldots, z_n(t)]^{\mathrm{T}}$ and $\gamma_1, \gamma_2 > 0$. In this paper, we will give conditions to make all agents achieve consensus while satisfying desired H_∞ performance.

Let

$$\boldsymbol{C} = \begin{bmatrix} \frac{n-1}{n} & -\frac{1}{n} & \cdots & -\frac{1}{n} \\ -\frac{1}{n} & \frac{n-1}{n} & \ddots & -\frac{1}{n} \\ \vdots & \ddots & \ddots & \vdots \\ -\frac{1}{n} & -\frac{1}{n} & \cdots & \frac{n-1}{n} \end{bmatrix}$$

and

$$\boldsymbol{B}_4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Then the output function $z(t)$ can be written as

$$\boldsymbol{z}(t) = (\boldsymbol{C} \otimes \boldsymbol{B}_4)\varphi(t).$$

Lemma 2[16] Consider the matrix $\boldsymbol{C}$. The following statements hold.

(i) The eigenvalues of $\boldsymbol{C}$ are 1 with multiplicity $n-1$ and 0 with multiplicity 1. The vectors $\mathbf{1}_n^{\mathrm{T}}$ and $\mathbf{1}_n$ are the left and the right eigenvectors of $\boldsymbol{C}$ associated with the zero eigenvalue, respectively.

(ii) There exists an orthogonal matrix $\boldsymbol{U} \in \mathbb{R}^{n\times n}$ such that $\boldsymbol{U}^{\mathrm{T}}\boldsymbol{C}\boldsymbol{U} = \begin{bmatrix} \boldsymbol{I}_{n-1} & 0 \\ 0 & 0 \end{bmatrix}$ and the last column is $\mathbf{1}_n/\sqrt{n}$. Let $\boldsymbol{\Xi}_1 \in \mathbb{R}^{n\times n}$ be the Laplacian of any undirected graph, then $\boldsymbol{U}^{\mathrm{T}}\boldsymbol{\Xi}_1\boldsymbol{U} = \begin{bmatrix} \boldsymbol{\vartheta}_1 & 0 \\ 0 & 0 \end{bmatrix}$, $\vartheta_1 \in \mathbb{R}^{(n-1)\times(n-1)}$.

For convenience of discussion, denote

$$\boldsymbol{U} = [\,\boldsymbol{U}_1 \ \overline{\boldsymbol{U}}_1\,],$$

where $\overline{\boldsymbol{U}}_1 = \mathbf{1}_n/\sqrt{n}$ is the last column of $\boldsymbol{U}$ and $\boldsymbol{U}_1 \in \mathbb{R}^{n\times(n-1)}$ is the rest part.

4. Main results

Theorem 1 Consider an undirected network of agents with fixed topology. The multi-agent system Eq. (4) can reach consensus while satisfying the H_∞ performance index Eq. (5) if there exists a symmetric positive definite matrix $\boldsymbol{P} \in \mathbb{R}^{(3n-3)\times(3n-3)}$ satisfying

$$\boldsymbol{H} = \begin{bmatrix} \boldsymbol{H}_{11} & \boldsymbol{P}(\boldsymbol{U}_1^{\mathrm{T}} \otimes \boldsymbol{B}_2) & \boldsymbol{P}(\boldsymbol{U}_1^{\mathrm{T}} \otimes \boldsymbol{B}_3) \\ (\boldsymbol{U}_1 \otimes \boldsymbol{B}_2{}^{\mathrm{T}})\boldsymbol{P} & -\gamma_1^2\boldsymbol{I} & 0 \\ (\boldsymbol{U}_1 \otimes \boldsymbol{B}_3{}^{\mathrm{T}})\boldsymbol{P} & 0 & -\gamma_2^2\boldsymbol{I} \end{bmatrix} < 0, \tag{6}$$

where $\boldsymbol{H}_{11} = \boldsymbol{P}(\boldsymbol{I}_{n-1} \otimes \boldsymbol{A} - \bar{\boldsymbol{L}} \otimes \boldsymbol{B}_1) + (\boldsymbol{I}_{n-1} \otimes \boldsymbol{A} - \bar{\boldsymbol{L}} \otimes \boldsymbol{B}_1)^{\mathrm{T}}\boldsymbol{P} + \boldsymbol{I}_{n-1} \otimes \boldsymbol{B}_4$ and $\bar{\boldsymbol{L}} = \boldsymbol{U}_1^{\mathrm{T}}\boldsymbol{L}\boldsymbol{U}_1$.

Proof To make the analysis become easier, we need first define some new variables. Let

$$\begin{aligned} W_1(t) &= \frac{1}{n}\sum_{i=1}^{n}\int_0^t \mathrm{e}^{\boldsymbol{A}(t-s)}\boldsymbol{B}_2 w_i(s)\mathrm{d}s, \\ W_2(t) &= \frac{1}{n}\sum_{i=1}^{n}\int_0^t \mathrm{e}^{\boldsymbol{A}(t-s)}\boldsymbol{B}_3 \eta_i(s)\mathrm{d}s, \\ \bar{\varphi}(t) &= \varphi(t) - \mathbf{1}_n \otimes W_1(t) - \mathbf{1}_n \otimes W_2(t), \\ \delta(t) &= (\boldsymbol{U}_1 \otimes \boldsymbol{I}_2)^{\mathrm{T}}\bar{\varphi}(t), \\ \bar{\delta}(t) &= (\bar{\boldsymbol{U}}_1 \otimes \boldsymbol{I}_2)^{\mathrm{T}}\bar{\varphi}(t), \end{aligned} \tag{7}$$

where $W_1(t)$ and $W_2(t)$ describe the average effect of external disturbances $w(t)$ and $\eta(t)$ on each agent, respectively, $\bar{\varphi}(t)$ describes the states of all agents, which takes out the average of external disturbances, $\bar{\delta}(t)$ describes the average states of all agents, and $\delta(t)$ depicts the disagreement states of all agents.

Define a Lyapunov function for Eq. (4) as follows:

$$V = \varphi^{\mathrm{T}}(t)\boldsymbol{P}\varphi(t),$$

where $0 \le \boldsymbol{P} = \boldsymbol{P}^{\mathrm{T}}$, $\mathrm{rank}(\boldsymbol{P}) = 3n-3$ and $\boldsymbol{P}(\mathbf{1}_n \otimes \boldsymbol{I}_3) = 0$.

Let $\bar{\boldsymbol{P}} = (\boldsymbol{U}_1 \otimes \boldsymbol{I}_3)^{\mathrm{T}}\boldsymbol{P}(\boldsymbol{U}_1 \otimes \boldsymbol{I}_3)$, then it is easy to see that

$$\begin{aligned} V &= \varphi^{\mathrm{T}}(t)\boldsymbol{P}\varphi(t) \\ &= \begin{bmatrix} \delta(t) \\ \bar{\delta}(t) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \bar{\boldsymbol{P}} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta(t) \\ \bar{\delta}(t) \end{bmatrix} \\ &= \delta^{\mathrm{T}}(t)\bar{\boldsymbol{P}}\delta(t) > 0. \end{aligned} \tag{8}$$

By Lemma 1, we have $\boldsymbol{L}\mathbf{1}_n = 0$. Calculating $\dot{V}$, we have

$$\begin{aligned}\dot{V} &= 2\varphi^{\mathrm{T}}(t)\boldsymbol{P}[(\boldsymbol{I}_n\otimes\boldsymbol{A}-\boldsymbol{L}\otimes\boldsymbol{B}_1)\varphi(t)+(\boldsymbol{I}_n\otimes\boldsymbol{B}_2)w(t)+(\boldsymbol{I}_n\otimes\boldsymbol{B}_3)\eta(t)]\\ &= 2\bar{\varphi}^{\mathrm{T}}(t)\boldsymbol{P}[(\boldsymbol{I}_n\otimes\boldsymbol{A}-\boldsymbol{L}\otimes\boldsymbol{B}_1)\bar{\varphi}(t)+(\boldsymbol{I}_n\otimes\boldsymbol{A})[\boldsymbol{1}_n\otimes W_1(t)\\ &\quad+\boldsymbol{1}_n\otimes W_2(t)]+(\boldsymbol{I}_n\otimes\boldsymbol{B}_2)w(t)+(\boldsymbol{I}_n\otimes\boldsymbol{B}_3)\eta(t)]\\ &= 2\begin{bmatrix}\delta(t)\\ \bar{\delta}(t)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\bar{\boldsymbol{P}} & 0\\ 0 & 0\end{bmatrix}\left(\begin{bmatrix}\boldsymbol{I}_{n-1}\otimes\boldsymbol{A} & 0\\ 0 & \boldsymbol{A}\end{bmatrix}-\begin{bmatrix}\bar{\boldsymbol{L}} & 0\\ 0 & 0\end{bmatrix}\right)\begin{bmatrix}\delta(t)\\ \bar{\delta}(t)\end{bmatrix}^{\mathrm{T}}\\ &\quad+2\begin{bmatrix}\delta(t)\\ \bar{\delta}(t)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\bar{\boldsymbol{P}} & 0\\ 0 & 0\end{bmatrix}(\boldsymbol{U}^{\mathrm{T}}\otimes\boldsymbol{I}_3)(\boldsymbol{I}_n\otimes\boldsymbol{A})[\boldsymbol{1}_n\otimes W_1(t)+\boldsymbol{1}_n\otimes W_2(t)]\\ &\quad+2\begin{bmatrix}\delta(t)\\ \bar{\delta}(t)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\bar{\boldsymbol{P}} & 0\\ 0 & 0\end{bmatrix}(\boldsymbol{U}^{\mathrm{T}}\otimes\boldsymbol{I}_3)[(\boldsymbol{I}_n\otimes\boldsymbol{B}_2)w(t)+(\boldsymbol{I}_n\otimes\boldsymbol{B}_3)\eta(t)]\\ &= 2\delta^{\mathrm{T}}(t)\bar{\boldsymbol{P}}(\boldsymbol{I}_{n-1}\otimes\boldsymbol{A}-\bar{\boldsymbol{L}}\otimes\boldsymbol{B}_1)\delta(t)+2\delta^{\mathrm{T}}(t)\bar{\boldsymbol{P}}(\boldsymbol{U}_1^{\mathrm{T}}\otimes\boldsymbol{B}_2)w(t)+2\delta^{\mathrm{T}}(t)\bar{\boldsymbol{P}}(\boldsymbol{U}_1^{\mathrm{T}}\otimes\boldsymbol{B}_3)\eta(t). \qquad (9)\end{aligned}$$

On the other hand, by Lemma 2, it is easy to see that

$$\begin{aligned}z(t) &= (\boldsymbol{C}\otimes\boldsymbol{I}_2)\varphi(t)\\ &= (\boldsymbol{C}\otimes\boldsymbol{B}_4)(\boldsymbol{U}\otimes\boldsymbol{I}_3)(\boldsymbol{U}\otimes\boldsymbol{I}_3)^{\mathrm{T}}\varphi(t)\\ &= \mathrm{diag}\{\boldsymbol{U}_1\otimes\boldsymbol{B}_4,0\}\begin{bmatrix}\delta(t)\\ \bar{\delta}(t)\end{bmatrix}.\end{aligned}$$

It follows that $\lim\limits_{t\to\infty} z(t)=0$ when $\lim\limits_{t\to\infty}\delta(t)=0$. So, whether the multi-agent system Eq. (4) can reach consensus is only related to the component $\delta(t)$. If $w(t)\equiv\eta(t)\equiv 0$ and $\bar{\boldsymbol{P}}(\boldsymbol{I}_{n-1}\otimes\boldsymbol{A}-\bar{\boldsymbol{L}}\otimes\boldsymbol{B}_1)+(\boldsymbol{I}_{n-1}\otimes\boldsymbol{A}-\bar{\boldsymbol{L}}\otimes\boldsymbol{B}_1)^{\mathrm{T}}\bar{\boldsymbol{P}}<0$, then it follows from Eqs. (8) and (9) that consensus can be asymptotically achieved.

Define the following cost performance index

$$J_T=\int_0^T[z^{\mathrm{T}}(t)z(t)-\gamma_1^2w^{\mathrm{T}}(t)w(t)-\gamma_2^2\eta(t)^{\mathrm{T}}\eta(t)]\mathrm{d}t,$$

where $T>0$ and $w(t)\in\mathcal{L}_2[0,\infty), t\in[0,\infty)$. From the linear superposition theorem, we can study the H_∞ performance of the protocol Eq. (3) by assuming $\varphi(0)=0$. This means that $\delta(0)=0$ and hence $V(0)=0$. Then it follows that

$$\begin{aligned}J_T &= \int_0^T[z^{\mathrm{T}}(t)z(t)-\gamma_1^2w^{\mathrm{T}}(t)w(t)-\gamma_2^2\eta(t)^{\mathrm{T}}\eta(t)]\mathrm{d}t\\ &= \int_0^T[z^{\mathrm{T}}(t)z(t)-\gamma_1^2w^{\mathrm{T}}(t)w(t)-\gamma_2^2\eta(t)^{\mathrm{T}}\eta(t)\\ &\quad+\dot{V}(t)]\mathrm{d}t-(V(T)-V(0))\\ &\le \int_0^T[g^{\mathrm{T}}(t)\boldsymbol{H}g(t)]\mathrm{d}t-V(T),\end{aligned}$$

where $g(t)=[\delta(t)^{\mathrm{T}}\ \ w^{\mathrm{T}}(t)\ \ \eta^{\mathrm{T}}(t)]^{\mathrm{T}}$. Note that $V(T)\ge 0$. Then, $J_T<0$ when Eq. (6) holds. Letting $T\to+\infty$, we have $\int_0^\infty z^T(t)z(t)\mathrm{d}t<\int_0^\infty\gamma_1^2w^{\mathrm{T}}(t)w(t)\mathrm{d}t+\int_0^\infty\gamma_2^2\eta(t)^{\mathrm{T}}\eta(t)\mathrm{d}t$.

Remark 1 In Theorem 1, we only study the fixed topology case but the obtained result can be extended to the case of switching topology and time-delay since the approach is based on the Lyapunov function.

Remark 2 Compared with Ref. [16], our approach is based on a new class of positive semi-definite Lyapunov functions and does not need to perform any model transformation.

Remark 3 It should be noted that a necessary condition for the condition (6) is that the graph $\mathcal{G}$ is connected. Under the condition Eq. (6), it is easy to see that $\boldsymbol{P}(\boldsymbol{I}_{n-1}\otimes\boldsymbol{A}-\bar{\boldsymbol{L}}\otimes\boldsymbol{B}_1)+(\boldsymbol{I}_{n-1}\otimes\boldsymbol{A}-\bar{\boldsymbol{L}}\otimes\boldsymbol{B}_1)^{\mathrm{T}}\boldsymbol{P}<0$, which requires that all eigenvalues of $\bar{\boldsymbol{L}}$ should be positive. Hence, the communication topology $\mathcal{G}$ is connected by Lemma 1.

5. Simulation results

Here numerical simulations will be given to illustrate the theoretical results obtained in the previous sections. These simulations are performed with four agents. Figure 1 shows the communication topology graph which is connected.

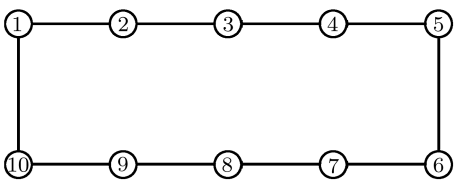


Fig. 1. The communication topology graph.

Suppose that the weight of each edge is 1. Let $\gamma=1$, $k_1=1$, $k_2=5$ and $\gamma_1=\gamma_2=1$. By using the LMI Toolbox in Matlab, it can be solved that there exists

an positive definite matrix $\boldsymbol{P} \in \mathbb{R}^{27\times 27}$ such that the matrix inequality (6) holds. Take the external disturbances as $w(t) = [1\ 2\ -1\ 3\ 0.5\ 3\ 6\ -3\ -2\ -1.5]^{\mathrm{T}}\bar{\omega}(t)$ and $\eta(t) = [1.5\ -1\ 2\ -1\ 3\ 3\ 3.2\ -1\ 7\ -4]^{\mathrm{T}}\bar{\omega}(t)$, where $\bar{\omega}(t) = \begin{cases} 1, & 0 \le t \le 1, \\ 0, & \text{otherwise.} \end{cases}$ Figures 2 and 3 show the position and velocity trajectories of all agents, whereas Fig. 4 shows the energy trajectories of the output function $z(t)$ and the disturbances. It is found that all agents reach consensus. Also, from Fig.4, it is observed that $\int_0^\infty z(t)^{\mathrm{T}}z(t) - w(t)^{\mathrm{T}}w(t) - \eta(t)^{\mathrm{T}}\eta(t)\mathrm{d}t < 0$. All these suggest that under the condition given by Theorem 1, all agents asymptotically achieve consensus while satisfying desired H_∞ performance.

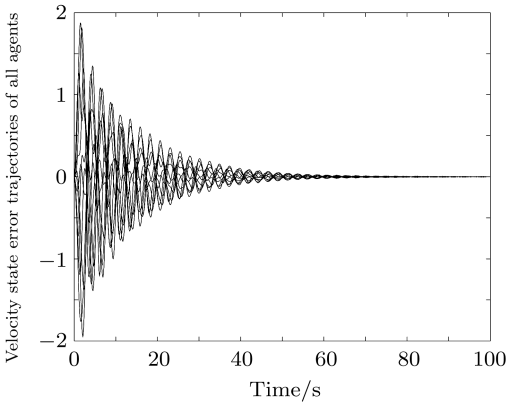


Fig. 2. Position state error trajectories of all agents.

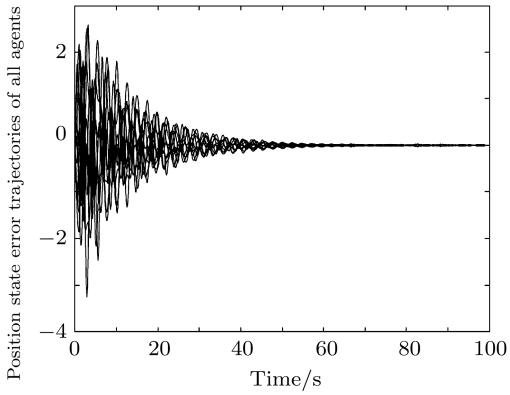


Fig. 3. Velocity state error trajectories of all agents.

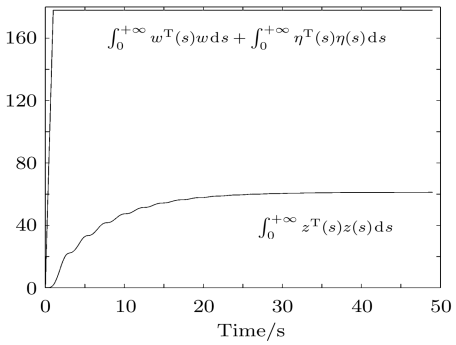


Fig. 4. The energy trajectories of $z(t)$ and the disturbances.

6. Conclusions

In this paper, consensus control problems are studied for a class of second-order multi-agent systems without relative velocity measurement in the presence of external disturbances. A sufficient condition is derived to make all agents achieve consensus while satisfying desired H_∞ performance. Finally, numerical simulations are provided to show the effectiveness of our theoretical results.

References

[1] Lafferriere G, Williams A, Caughman J and Veerman J 2005 *Syst. Control Lett.* **54** 899
[2] Olfati-Saber R 2006 *IEEE Trans. Automat. Cibtr.* **51** 410
[3] Olfati-Saber R and Murray R M 2004 *IEEE Trans. Automat. Cibtr.* **49** 1520
[4] Ren W 2007 *Proceedings of the 46th IEEE Conference on Decision and Control* p2295
[5] Ren W and Beard R 2005 *IEEE Trans. Automat. Cibtr.* **50** 655
[6] Hong Y, Gao L, Cheng D and Hu J 2007 *IEEE Trans. Automat. Cibtr.* **52** 647
[7] Hu J and Hong Y 2007 *Physica* A **374** 853
[8] Wang L and Xie G 2007 *Int. J. Robust Nonlin.* **17** 941
[9] Shi H, Wang L and Chu T 2006 *Physica* D **213** 51
[10] Lin P and Jia Y 2008 *Physica* A **387** 303
[11] Lin P and Jia Y 2009 *Automatica* **45** 2154
[12] Hu J P and Yuan H W 2009 *Chin. Phys.* B **18** 3777
[13] Li Y M and Guan X P 2009 *Chin. Phys.* B **18** 3355
[14] Pei W D, Chen Z Q and Yuan Z Z 2008 *Chin. Phys.* B **17** 373
[15] Li H , Lin P and Zhang C 2009 *Acta Phys. Sin.* **58** 158 (in Chinese)
[16] Lin P, Jia Y and Li L 2008 *Syst. Control Lett.* **57** 643
[17] Godsil C and Royle G 2001 *Algebraic Graph Theory* (New York: Springer-Verlag)